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Fundamentals of Springs Mechanics

Third Edition

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Foreword

Technical springs are well-known machine components that can be reversibly deformed under load and cyclic or oscillating forces. Springs convert kinetic energy into potential energy, store energy, and return it to the system almost without loss when relieved. In order to take advantage of these properties for optimized applications, two essential aspects must be considered: the characteristics of the material used and a spring design that is adapted and optimized for the application. Spring steel alloys have the appropriate material characteristics for springs, which are usually highly stressed. In addition, the well-designed shape of the spring allows the technical requirements and characteristics such as stiffness, fatigue life, etc. to be met. Various spring designs are classified according to their shape and type of loading, which in most cases also provides the basic understanding for their technical calculation. Due to their various technical characteristics and functions, springs are still almost irreplaceable components in every new and modern machine concept, in airplanes, ships, buildings, trains, or automobiles. In order to meet all these high requirements, standards, and specifications, accurate calculation methods are needed, which take into account all important physical effects of springs.

The purpose of this script is to explain the mechanical and physical properties of specific steel alloy springs and to present complementary analytical calculation methods based on existing and summarized calculation models. Approaches for characteristic spring data such as weight and package, life and crack growth, creep and relaxation rate, and lateral vibrations and natural frequencies are presented for specific spring shapes. The script includes calculations for coil springs, disk springs, wave springs, and thin-walled rods with semi-open cross-section. Due to the analytical approach of all calculation models, ambitious development and design engineers get a helpful review and overview of existing and complementary calculation methods for springs.

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Preface

This book focuses on the mechanics of elastic elements made of steel alloys, with an emphasis on metal springs for the automotive industry. Industry and scientific organizations are intensively studying the fundamentals of spring element design and constantly improving the mechanical properties of spring materials. The development tasks of spring manufacturing companies include the optimal application of existing material types. The task is therefore to evaluate the mechanical properties in a targeted manner and then to design springs that make full use of the achievable material properties. The first edition of this book (Kobelev, 2018) was revised to reflect the current trends and the state of the art in the modeling of springs. The second edition includes several corrections and improvements of the theory from the first edition. Most of the methods for fatigue assessment of spring materials have been revised. Experimental data for creep and relaxation under temperature have been added. The theory of simultaneous creep and plastic flow has been added to describe the time-delayed presetting of coiled springs. This theory is essential for the fine tuning of presetting processes in machines.

The third edition was considerably extended compared to the second edition (Kobelev, 2021). Chapters 5 and 6 deal with the new theory of disk springs with fixed inner and outer edges. In Chap. 5, disk springs with constant material thickness are studied using the models of thin and moderately thick isotropic shells. The deformation behavior of the disk spring is treated as a one-dimensional inversion of a circular ring of rectangular section about certain inversion centers. The calculation of the disk springs examines the free-gliding edges and the edges with constrained radial motion. The formulations are used to derive load-displacement formulae for the disk springs with different constraints on radial movement on the inner and outer surfaces.

Based on this model, the disk spring with non-constant thickness is calculated in Chap. 6. The disk spring with radially or tangentially variable material is studied. The equations provided are sufficiently accurate to evaluate relatively flat disk springs with smooth thickness variations along the latitude and longitude of the conical central surface. The derived formulae are applicable for design purposes. With these two new chapters, the formulae cover most conceivable applications of disk springs in heavy industry, mechanical engineering and automotive engineering.

Another reorganized topic deals with the calculation of damage during fatigue loading. There are several independently invented criteria for calculating damage during loading with different amplitudes and an average force in the cycle. The engineer must occasionally recalculate the damage from one damage model to another. At first sight, the recalculation appears to be difficult. The new Appendix D provides methods for recalculating the most common damage criteria. This is an important task for the correct application of finite-element methods to damage assessment. A correct description of fatigue processes is important for machinery where potential damage can become a hazard to people. This applies to the aerospace, energy, and automotive industries, among others. In particular, the fatigue life of springs has not yet been fully evaluated. The merging of the multiaxial stress tensor to the equivalent scalar value is one open question. The mean stress sensitivity is the other. Various strength hypotheses have been proposed to answer these questions. The strength hypotheses lead to relationships for a reasonable curve expressed in stress or strain. From this curve, the final allowable amplitudes of stress or strain are obtained. The final allowable amplitudes correspond to a certain number of cycles to failure. The dominant stresses in leaf and disk springs are normal stresses. For shear compression or helical tension springs, the shear stresses should be used instead of the normal stresses. Fatigue characteristics are determined by analyzing the failure parameters and fatigue properties of the materials. The appendix discusses in detail the mean stress sensitivity of the common spring material and compares the different approaches to evaluating the fatigue characteristics of the spring.

The motivation of the previous editions was mainly the fatigue phenomena of the spring elements. The scope of the manuscript was considerably extended in the 2nd and 3rd editions of the book, entitled *Durability of Springs*. Two-thirds of the text of the current volume deal with design and manufacturing aspects of spring technology. This status quo argues for an extension of the title. Since the 3rd edition, the text has been given the new title *Fundamentals of Spring Mechanics*. The logical structure of the text remains largely unchanged. Readers familiar with the old editions will find the updated, corrected, and expanded chapters in the new edition as well.

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Introduction

Aims and Methods of the Book

The integral parts of many mechanical systems are elastic elements or springs (Juvinall & Marshek, 2017, Chap. 12). The springs make it possible to maintain a tension or force in a mechanical system, to absorb the shocks and to reduce the vibrations. Highly stressed spring elements in modern industrial and transport equipment must survive a very high number of cycles with high average stress and high amplitude stress. These springs are manufactured from high-quality wires and by means of special mechanical and heat treatment processes. The standard designations of various spring steels are summarized in (ASTM DS67A, 2002).

The spring is the widespread resilient element, which is used in the industrial machinery and automotive systems:

- coil tension springs and/or torsion springs in disk or drum brakes, locks or locking or blocking systems,
- torsion or bending springs for belt tensioners, safety belts, and for load compensation in clutch pedals,
- disk springs with or without slots for use in clutches, bearings, and for load pre-tensioning,
- leaf springs for chassis suspension,
- helical, or coil, spring for reducing impact events in passenger cars, some heavy trucks, and railroad cars
- coil springs for nozzle holders, in transmissions, as valve springs, injection regulators, or as vibration dampers in clutches and brake cylinders, as diesel fuel pumps, valvetrains, brakes, seats, doors, and control elements.

Coil, of helical, springs are formed by winding a wire or rod of uniform cross-section around a cylinder. A fixed distance is maintained between successive coils of a spring so that the axis of the wire forms a helix. The standard design procedures for helical springs are (SAE, 1996) and (DIN EN, 2012, 2013, 2015). Efficient design procedures for spring elements are based on modern simulation and optimization methods. Achieved by these methods, the reduction of the weight of suspension springs causes the reduction of the unsprung mass of the axle. This reduction has a positive effect on the comfort, traction, and steering characteristics of the car. The development of modern passenger cars has also highlighted the trend toward reduced packaging of suspension components to maximize space for occupants and cargo. Such requirements lead to a reduction in spring dimensions and wire cross-section. Springs can also be found in high-precision test equipment, where springs play the role of energy harvesters.

One of the most important applications for heavy-duty springs is in the valve train of internal combustion engines. Valve springs in internal combustion engines provide forced contact between all moving valve train components during the valve lift up to maximum engine speed. Assuming annual production of 100 million cars having roughly 20 valve springs per engine, one gets a rough number of 2000 million valve springs produced. In Europe and North America, the valve spring is produced mainly from high-tensile wire alloyed with the elements chrome and vanadium. The extremely high oscillating stresses on the surface of the wire achieve the peak values up to 2000 MPa. The requirements on failure rate must be below 1.5% for engine operation test. Hereby, they are subjected to extreme vibration stresses and must endure up to 3×10^8 cycles without failure (Muhr, 1992).

There are some common characteristics of high-quality and properly designed springs:

- the high homogeneity of stresses on the surface of the spring; thus, the absence of stress-concentrators;
- the considerable residual stresses, which, if properly induced, significantly prolong the operation life;
- high sensitivity to imperfections, flaws, inclusions, and corrosion;
- high amount of specific stored elastic energy.

The spring industry has developed specialized materials and sophisticated manufacturing processes to meet the above requirements.

The springs are mostly made from oil-quenched and tempered steel wire, which is wire formed by drawing hot-rolled steel rod through a drawing die and oil-quenching the resulting wire. Oil quenching is a term of art that identifies a process that generally involves heating the wire to austenitizing temperatures, quenching it in oil, quenching it by heating, and recoiling it. This sequence of manufacturing steps increases the tensile strength of the material. However, the ductility of the austenitized material is reduced. The material behaves almost elastically until the moment of fracture. The influence of these two effects, the requirement to increase the ultimate stress and the reduction in ductility, on the fatigue life of the material is contradictory.

The oil-quenched and tempered low-alloy chrome-silicon spring steel wire achieves strengths in excess of 2600 MPa. Even higher strengths can be achieved with chrome-silicon-vanadium alloy steels for valve applications. The wire is peeled or ground before drawing and subjected to non-destructive crack testing after annealing. At the same time, high strength requires high purity and surface finish of the semi-finished product, good formability in cold-forming steels, and corrosion resistance.

High purity also improves ductility while maintaining static strength, which is particularly important for cold-formed springs. The latest phase of materials development for spring steels has therefore been strongly characterized by efforts to achieve very good purity and surface qualities on the one hand, and high-strength steels with the best possible ductility on the other. A metallurgy specially adapted to high-strength spring steel wires allows a very good degree of purity (Hagiwara et al., 1991; Kawahara et al., 1992; Wiemer, 1998). Several publications are devoted to the improvement of wire properties and surface conditions (O'Malley & Hayes, 1990; Postma, 1993). Also included are developments in the direction of thermo-mechanical forming, improvement of toughness properties for a given strength, considerations of the most suitable heat treatment equipment, and the special manufacturing and treatment processes for stainless spring semi-finished products (Lehnert, 1997; Illgner, 1995; Schmitz-Cohnen, 1994).

This setting reduces relaxation and improves creep behavior of springs at operating temperature. It is well known that curing affects the static residual stresses in the spring and changes the cyclic fatigue properties of the spring. In modern spring manufacturing, both cold and hot settings are used. Heat setting refers to the process of applying a time-dependent load to the spring at an elevated temperature. The main physical process during spring setting is material creep.

Shot peening is another mechanical surface treatment used to improve spring performance. Local plastic deformation occurs on the wire surface, resulting in an improvement and strengthening of the properly machined surface. Shot peening significantly increases the fatigue life of springs.

As a result, springs are a highly sophisticated element of modern machines with the highest fatigue life for relatively intensive cyclical stresses. To achieve the required fatigue life, the production of springs involves several specific procedures. The production of modern springs includes several special techniques, which make springs stand out from ordinary machine components. It is worth mentioning that occasionally occurring fatigue failure of springs can cause damage to the entire machine component. In this case, the failure provokes costs that are incompatible even with the price of the highest quality spring.

Structure of the Book

The book reviews the advanced theory of elastic elements from the point of view of structural and material mechanics. The book examines the principal problems that are essential for clarifying the manufacturing and performance of the metallic spring elements. The elements of creep, plasticity, and fatigue serve as the building blocks of the physical background. What all the problems considered have in common is that they are solved in closed form.

The content of the book is divided into three parts. Part I studies springs from the design point of view. The spring elements of machines and vehicles can be roughly divided into helical, leaf, disk, and twist beams. The models of each family of elastic elements and their optimization are discussed in Part I of the book, which covers Chaps. 1–7. Part II explains the basic processes of spring manufacturing. Plastic cold working is the main manufacturing process. Coiling and plastic presetting are described in Chaps. 8–10. Part II examines the creep and relaxation of springs during presetting. Part III of the manuscript estimates the service life of the springs. This part deals with the fatigue phenomena that usually limit the spring life. This part consists of Chaps. 11–14. The appendix includes the mathematical formulary for fatigue estimation of springs and different presentations of fatigue diagrams.

Part I begins with the determination and evaluation of the spring design. The optimization of springs is studied in Chap. 1. The helical springs are the typical energy-storing elements of the valve train in automotive engines, in the suspensions of passenger cars and railway wagons, and in mechanical engineering. The design formulas for linear helical springs with inconstant wire diameter and variable mean spring diameter are presented. These formulas are used to optimize the spring for a given spring rate and wire strength. The basic principles of leaf spring design are also briefly discoursed.

Chapter 2 presents analytical solutions for the torsion problem of an incomplete torus with circular and non-circular cross-sections. The hollow cross-sections of the shape demonstrate a closed form of the analytical solution. The solution is useful for the analysis and design of helical springs with non-circular wires. The torsion problems for straight cylinders with circular and elliptical cross-sections allow for the well-known closed-form solutions that will be needed in the next chapters. Two main load applications, axial force and axial moment, are analyzed.

Chapter 3 explains a powerful method for simplifying the helical spring equations. Instead of a full treatment of the helical wire, the deformation of the virtual center line is studied. The virtual centerline has effective strain, torsion, and bending stiffness. It behaves like an initially straight elastic rod or column. This simplification allows a straightforward solution of several practically important problems. These include the load dependence of the transversal vibrations of helical springs and the transformation of transversal vibrations into buckling mode. Lateral buckling of the spring is considered in the context of dynamic stability as a limit case for vibration analysis.

For proper consideration of dynamic effects, models of flexible springs with solid wire are required. In some cases, such as when the spring is uniform, analytical models for dynamics and buckling can be developed. However, in typical springs, only the central coils are uniform; the ends are often not (e.g., they have a varying helix angle or cross-section). Thus, obtaining analytical models in this case can be very difficult, if possible. A variety of theories can be found in the literature to describe the dynamic behavior of helical springs, which includes the interaction of bending (flexural), torsion, and longitudinal waves. In addition, various approximate methods are used to determine the fundamental frequencies of spring vibrations. The methods used to determine the fundamental frequencies can be roughly divided into three groups:

- analysis methods, based on the concept of an equivalent column;

- exact analysis methods, based on the theory of spatially curved bars;
- numerical methods, based on finite-element formulation for spatially curved bars.

The governing equations for the transversal vibrations of the axially loaded linear helical springs are developed. The method is based on the traditional concept of an equivalent column. The effect of the axial load on the fundamental frequency of the transversal vibrations is shown. The natural frequency of the transverse vibrations of the spring depends on the variable length of the spring. If the number of active coils remains constant, the frequency gradually decreases as the length of the spring is shortened. Finally, when the frequency becomes zero, the lateral buckling of the spring occurs by the divergence mode. Note that progressive springs have the opposite behavior. The number of active coils in compressed progressive springs decreases and the transverse frequency increases. Therefore, the progressive springs rarely became transversely unstable.

The combination of tension, compression, and torque loads often causes deformation of spring elements. These loads sometimes lead to spatial buckling of the elements. The stability of helical springs under the combined tension, compression, and torsion is elucidated.

Disk springs (also known as Belleville washers) are studied in Chap. 4. Disk springs are examples of highly stressed machine elements. The workpiece for disk springs is in the form of a sheet or roll. The disk springs are usually produced by stamping. Other processes to produce disk springs are blanking, stamping, perforating, cutting, drawing, notching, lancing, and bending. Multiple punches can be used together to produce a part in one step. The “Conical Disk Spring,” “Belleville Spring,” or “Belleville Washer” is typically used as a spring or to apply a preload or flexible quality to a bolted joint or bearing. These springs are the typical energy-storing elements of the valve train in clutches and automatic transmissions of automobiles. Belleville springs are generally made of spring steel and can be subjected to static loads, rarely alternating loads, and dynamic loads. Belleville springs must meet stringent fatigue life and creep requirements. Because the basic characteristics of disk springs include high fatigue life, better space utilization, low creep tendency, high load capacity with small spring deflection. From a mechanical point of view, Belleville washers are flat conical rings subjected to axial load. Normally, the thickness of the ring is constant, and the applied load is evenly distributed between the upper inner edge and the lower outer edge. In this chapter, the equilibrium equations of thin and moderately variable thickness disk springs are obtained. Variational principles for conical shells are used for the derivation. Simplification is based on common deformation hypotheses. Closed-form analytical solutions of thin and thick truncated cone shells are obtained.

Disk wave springs are also analyzed in Chap. 4. Wave springs are coiled, resilient parts made of flat material. The special feature of these springs is the reduction of their spring height under the conditions of spring force and travel. This feature makes wave springs suitable for compact installation. The production of wave springs from wire, as opposed to the punching of disk springs from flat or rolled sheet metal.

This allows a considerable amount of scrap to be saved, which is typical of stamping operations. Both linear and nonlinear disk wave springs are deliberated.

Chapter 5 examines fixed-edge disk springs using the thin and moderately thick isotropic shell models. The thickness of the material is kept constant in this chapter. The calculation of the disk springs examines the free-gliding edges and the edges with constrained radial motion. The variation formulas are used to derive load-displacement formulas for the disk springs with different radial constraints on the inner and outer surfaces. The kinematic hypothesis is used for the shell models of conical shells. The motivating feature of the presented theory is its ability to calculate the cup springs with free-gliding edges and the edges with constrained radial motion. The equations developed here are based on general assumptions and are suitable for disk springs made of isotropic materials, such as spring steel and light metal alloys. The advantage of the methodology is the derivation of closed-form solutions for several common restrictions on the radial motion of the inner and outer edges. The developed formulas are recommended for industrial calculations of free and restricted disk springs and Belleville washers.

Chapter 6 observes the disk spring with variable thickness. The thickness of the material is variable along the meridional and parallel coordinates of the conical coordinate system. The calculation of disk springs includes the cases of free-gliding edges and edges on cylindrical curbs, which restrict the radial movement. The equations developed here are based on common assumptions and are simple enough to be applied to industrial calculations.

The analysis of thin-walled, semi-open-section beams is performed in Chap. 7. An essential characteristic of this class of thin-walled beam-like structures is their closed but flattened profile. In this book, an intermediate class of thin-walled beam sections is studied. The cross-section of the beam is closed, but the shape of the cross-section is elongated and curved. The walls forming the section are nearly equidistant. The unusual shape of semi-open thin-walled beams allows for efficient optimization due to the large variability of shapes. The automotive application of semi-open thin-walled beams is conversed later in Chap. 7. The main application of the theory of semi-open thin-walled beams is the twist beam of the semi-solid trailing arm axle. The analytical expressions for the effective torsional stiffness and effective bending stiffness of the twist beam are derived in terms of the section properties of the semi-open section twist beam. Based on the stiffness coefficients of the twist beam, the roll rate, chamber, and lateral stiffness of the suspension are derived.

Mechanical problems encountered in the manufacture of helical springs are examined in Part II, which includes Chaps. 8 and 9. In Chap. 8, we analyze the coiling of helical springs. For this purpose, we study the plastic flow and the appearance of residual stresses. It is well known that the excessive stresses during the coiling of helical springs can cause the rod to break. In addition, the high level of residual stress in the formed coil spring significantly reduces its fatigue life. For practical estimation of the residual and coiling stresses in the helical springs, the analytical formulas are required. In this chapter, the analytical solution of the problem of elastic-plastic deformation of cylindrical bar under combined bending and torsional moments is found for a special nonlinear stress-strain law. The obtained solution allows the analysis of the active stresses during the combined bending and torsion. In addition, the residual stresses in the beam after spring-back are also derived in a closed analytical form. The obtained results agree with the reported measured values. The developed method does not require numerical simulation and is perfectly suitable for the programming of coiling machines, the estimation of loads during the production of cold-wound helical springs and for the dimensioning and wear calculation of coiling tools.

In Chap. 9, the prestressing calculation for helical springs is developed. The method is based on the deformation formulation of the plasticity theory and common kinematic hypotheses. From a mathematical point of view, the governing equations of prestressing are somewhat analogous to the equations of the coiling process. Two main types of helical springs are studied—compression springs and torsion springs. For the first type (axial compression or tension springs), the spring wire is twisted. The basic approach neglects the pitch and curvature of the coil and replaces the helical wire with a straight cylindrical rod. The elastic-plastic torsion of the straight bar with circular cross-section is studied. The analysis is based on the hypothesis of St. Venant. In the second type (torsion helical springs), the helical wire is in the state of bending. The model analyzes the delayed prestressing, which is accompanied by a significant creep.

The last, Part III, includes the life cycle of the elastic elements, the high static stresses lead to the residual deformation. The loss of sag leads to the gradual reduction of spring forces with the resulting failure if the spring's working length continues. Spring breakage due to static creep is the extremely rare event. Moderate cyclic loading is accompanied by some creep and cyclic sag loss. Severe cyclic loading sooner or later leads to fatigue failure of the spring. These two sources of possible damage to elastic elements are summarized in Chaps. 10–14.

Understanding the long-term behavior of springs under high static loading is essential to their proper design. Creep and relaxation of springs is the subject of Chap. 10. Stress analysis for creep has a long history in engineering mechanics, driven by the need to design for elevated temperatures. In solid mechanics, creep is the tendency of materials to deform gradually or continuously under the action of external mechanical stresses. At stresses below the yield strength of the material, slow inelastic deformation occurs. In the spring industry, this is called creep when a spring under constant load loses length, and it is called relaxation when a spring under constant compression loses load. Creep and relaxation rates depend on temperature, stress in the metal, yield strength, and time. Increased temperature, stress, and time significantly increase the creep and relaxation rates. In particular, temperature and stress have the greatest influence. An accurate description of creep is essential for proper spring design. Finally, Chap. 10 demonstrates the evaluation of creep constants in a wire twisting experiment. In addition, the exact analytical expressions for the torsional and bending creep of bars are derived using the common constitutive models. One of the common creep constitutive models is the Norton-Bailey law, which gives a power law relationship between minimum creep rate and (constant) stress. The power law can be found in high-temperature design and creep numerical codes.

Other common creep laws are the exponential and Garofalo laws, which more adequately describe the stress dependence over a wide range of working stresses. For all these laws, we derive the analytical formulas for creep caused by steady or oscillating loading. In Chap. 10, the generalized expression for the creep law is studied. The new expression is based on experimental data and unifies the primary, secondary, and tertiary regions of the creep curve. The relaxation functions for bending and torsion depend only on the maximum stress in the cross-section, which occurs at the outer surface of the coil. Finally, we explain the temperature dependence for the creep of spring materials.

Chapters 11 and 12 briefly consider the fatigue effects of springs. We begin with the deterministic approach. The results presented show the average fatigue characteristics and evaluate the stress levels at which the majority of springs fail. In this case, we speak about mean S-N lines. The durability of springs under oscillating loads is the subject of Chap. 11. Traditional fatigue design methods are based on collecting a large amount of experimental data in cyclic tests, structuring the data, and extracting empirical formulas. The method of analyzing crack growth under repeated loading is reviewed in Chap. 11. The expressions for the spring length over the number of cycles are derived in terms of a higher transcendental function. The proposed method starts from the micromechanically inspired effects of crack propagation, explains the history of crack propagation, and finally provides the stress-life curves.

Several effects on fatigue life, mainly the effects of stress ratio and multiaxiality, are deliberated in Chap. 12. The solutions presented are used to estimate the fatigue life of springs for asymmetric harmonic loading with substantial mean stress. An attempt is made to unify various traditional methods into a unified Bergmann-Walker formula. Different settings of two fitting parameters in the unified criterion result in the common fatigue criteria.

It is noteworthy that the high-quality springs differ from the low-quality products mainly by the scattering ranges. The evaluation of scatter requires the statistical methods conversed in the next chapters. The statistical effects on fatigue life are further discussed in Chaps. 13 and 14. We study the probability descriptions for the fatigue limit of heterogeneously stressed members. The proposed approach for stress gradient sensitivity of fatigue life is based on the "weakest link" concept. This method is applicable to the exceptionally brittle materials that fail immediately after the failure of the first constituent element. The weakest link approach is used to calculate the number of cycles to complete failure under different probability levels. The effect of fluctuating stresses on the fatigue life of springs is combined with the influence of heterogeneous stress distribution (stress gradient) over the wire cross-section and time-varying stresses. The stress field is inhomogeneous over the cross-section of the spring wire. The stress distribution is uniquely defined by the ratio of the diameter of the wire to the diameter of the spring body. The calculated lifetimes are compared with the lifetimes of helical springs subjected to cyclic loading.

Chapter 14 examines the stochastic effects on the fatigue life of springs. Stochastic crack propagation is typical of the low-stress and high-cycle fatigue regions. The deviation and branching of the crack are caused by the high inhomogeneity of the polycrystalline structure at the micro level. For the low amplitude of stress, the crack extension per cycle is less than the typical size of inhomogeneities. The stochastic differential equation for the traveling crack is derived. The stochastic equation is similar to the forced Brownian motion equation. The methods are based on the unified fatigue laws. These laws lead to analytical solutions for the crack length at the mean value and the range of cyclic variation of the stress intensity factor. In this chapter, we demonstrate the closed-form expressions for the number of cycles to failure as a function of the initial crack size.

Target Audience of the Book

This book was written as a script for the courses "Applied Mechanics of the Automobile," "Automotive Engineering, Chassis, II, III," "Structural Optimization in Automotive Engineering," and "Powertrain Modeling and Optimization," which the author has been teaching at the University of Siegen, North Rhine-Westphalia, Germany, since 2001.

This book is primarily intended for engineers involved in the design and development of springs who have graduated from automotive or mechanical engineering programs at technical colleges or other engineering schools. Researchers working on elastic elements and energy harvesting devices will also find a broad overview of the fundamentals of spring methodology.

The current book presents powerful methods for the analysis of elastic elements made of steel alloys. The focus is on metal springs for the automotive industry.

It is well known that the industry researches the design of spring elements and constantly develops the quality of spring materials. New materials are developed in the factories of material suppliers. The task of the spring manufacturing companies is the optimal application of the existing and newly developed types of materials. The technologically advanced method consists in the target-oriented evaluation of the mechanical properties and the subsequent design of the springs, which makes full use of the measured material properties. Thus, the full development of the improved materials is only possible if their essential properties are rigorously acquired. Design and manufacturing must fully exploit all available capabilities of the semi-finished product.

An enormous number of papers have been written on this and related subjects. This does not mean that the science of spring mechanics and strength has become completely useless. Rather, it is necessary to have a thorough understanding of mechanics and metallic disciplines in order to elucidate the wide range of possibilities. Therefore, much of this book is devoted to these topics.

The precise methods for the design of different types of springs are summarized in the relevant standards. Industrial research has developed reliable methods for estimating fatigue life and creep effects. The purpose of this book is not to replace the established methods of design and pragmatic methods of life assessment of springs. The aim of this book is to qualitatively explain the mechanical behavior of the spring as a unique elastic element, which has some very specific properties. We try to survey the vast and fragmented landscape of springs from a single point of view of classical mechanics. We try to compare the different methods of service life evaluation and to point out the most effective and general methods. There are many experimental values that are currently the subject of speculation. It is possible that these values will be obtained in future studies. Most of the methods presented are recognized and applicable to other heavily loaded structural elements.

A few words about solution methods. There are several recognized commercial finite-element codes, e.g., (ANSYS, 2020; ABAQUS, 2020). However, for the modeling of technical systems, analytical solutions often offer important advantages. First, the transparency of closed-form solutions. Because analytical solutions are represented as mathematical expressions, they provide an understandable view of how variables and relationships between variables affect the result. Second, performance: algorithms and models expressed in terms of analytical solutions are often more effective than the corresponding numerical applications. For example, to compute the solution of an ordinary differential equation for different values of its parametric inputs, it is often faster, more accurate, and more appropriate to evaluate an analytical solution than to integrate numerically. Third, numerical solutions are sometimes extremely abundant. The main reason is that sometimes we either don't have an analytical approach, or the analytical solution is too slow, and instead of computing for hours and getting an exact solution, we rather compute for seconds and get a good approximation. Finally, numerical solutions can seldom contribute to the invention of new ideas. For this reason, the treatment of the material in this book resolves the problems studied to closed-form solutions in the form of mathematical expressions.

The content of this book is logically related to the work *Design and Analysis of Composite Structures for Automotive Applications* by the same author. The latter book is an extension of the present book and covers the subject of composite materials. The manuscript (Kobelev, 2019a) examines the special properties of composite materials, such as their anisotropy, inhomogeneity, load direction dependence, stress coupling, and stacking capabilities.

Contents

Part II Manufacturing of Springs

Symbols[1]

Symbol	Description	Unit[a]
Q_a	Activation energy of diffusion	kJ/mol
χ	Angle between meridian and principal material axis	rad
ϕ_Q	Angle of inclination of the bent axis	rad
α	Angle of pitch of helical spring	rad
ϑ	Angle of twist per unit length	rad
$\bar{\vartheta}$	Angle of twist per unit length after spring-back	rad
$\psi = H/(r_e - r_i)$	Angle, of slope, deformed middle surface of disk spring	rad
$\alpha = h/(r_e - r_i)$	Angle, of slope, free middle surface of disk spring	rad
ϕ	Angle, rotation of the middle surface disk spring	rad
A	Area of the material part of the cross-section	m^2
A_m	Area, enclosed by the curve L_m	m^2
$\phi_1(r), \phi_2(z)$	Auxiliary functions	1
$k_1,\ k_2$	Auxiliary functions for survival probabilities and ratio of cycles to failure of spring to straight rod	1
$\sigma_0,\ N_0$	Auxiliary scaling constants, $\sigma'_f = \sigma_0(2N_0)^{-b_0}$	1
$T,\ B$	Axes of wire cross-section (thickness and width)	m
$\bar{R} = 1/\bar{\kappa}$	Bending radius after unloading (after spring-back)	m
$R = 1/\kappa$	Bending radius in during active coiling	m
$\theta_a = 2\pi n_a$	Circumferential angle along wire length	rad

(continued)

[1] For the sake of continuous consistency in the manuscript, the designations may be different from those used in the standards.

(continued)

$x_e \le x \le x_i$	Coordinate on the meridian of free conical shell	m
$x_c,\ y_c$	Coordinates of the center of mass of the cross-section	m
$\alpha_x,\ \alpha_y$	Coordinates of the twist center of the cross-section	m
$x = \rho \cos\phi,$ $y = \rho \sin\phi$	Coordinates, Cartesian of the circular cross-section	m
$0 \le \rho \le r,$ $0 \le \phi \le 2\pi$	Coordinates, polar of the circular cross-section	m rad
$k_i = k_i(w)$	Correction factors for stress, $i = 1, .., 4$	1
c_τ	Creep constant, for shear strain	$\frac{s^{-\varsigma}}{Pa^{\xi+1}}$
c_σ	Creep constant, for uniaxial strain	$\frac{s^{-\varsigma}}{Pa^{\xi+1}}$
t_c	Creep, average time	s
$\bar{t}$	Creep, Norton-Bailey constant	s
$\bar{\varepsilon},\ \bar{\gamma}$	Creep, strain rate constants	1/s
ξ	Creep, stress exponent	1
$\bar{\sigma},\ \bar{\tau}$	Creep, stress scaling constants	Pa
ς	Creep, time exponent	1
κ	Curvature in moment of plastic deformation	1/m
κ_i	Curvature principal, $i = 1,\ 2$	1/m
N_L	Cycles number to the failure for a given stress amplitude, highest (failure event of the last homogeneously stressed specimen)	1
N_F	Cycles number to the failure for a given stress amplitude, lowest (failure event of the first homogeneously stressed specimen)	1
$\mu_+^*,\ \mu_-^*$	Deflection at loading and unloading, critical	1
ρ_m	Density of material	kg/m^3
$D^{\widehat{\alpha}}$	Derivative, fractional of order $\widehat{\alpha}$	$1/s^{\widehat{\alpha}}$
$d = 2r$	Diameter of circular wire or bar	m
D_e	Diameter of middle surface of free spring, external (outside)	m
D_i	Diameter of middle surface of free spring, internal (inside)	m
d_{opt}	Diameter of wire, optimal	m
μ^*	Dimensionless length, character	1
$\tilde{s}$	Displacement, axial, measured from upper inside edge to lower outside edge	m
s_b	Displacement, caused by bending moment	m
s_s	Displacement, caused by shearing force	m
s_Q	Displacement, lateral	m
τ_e	Endurance limit for completely reversed stress	Pa
K_{th}	Endurance threshold limit	$N\sqrt{m}$

(continued)

(continued)

$U_e,\ U_1 \ldots U_5$	Energy, elastic strain	J
U_f	Energy, potential of applied forces	J
Π	Energy, total potential	J
$\langle EI_B \rangle$	Equivalent bending stiffness	Pa m^4
$\langle GS \rangle$	Equivalent shear stiffness	Pa m^2
F^*	Euler's critical load for compression	N
$m_2 > 1$	Exponent, at short-term limit	1
$m_1 > 1$	Exponent, endurance limit	1
$p > 1$	Exponent, fatigue	1
$c(R_\sigma)$	Exponent, fatigue ductility	1
p_S	Exponent, fatigue in high-cycle range	1
p_L	Exponent, fatigue in moderate cycle range	1
$b_0(R_\sigma) = -1/p_\sigma$	Exponent, of fatigue strength	1
k	Exponent, secant	1
k_f	Extent constant of failure region	–
S_f	Factor for safety	1
$\gamma'_f(R_\sigma)$	Fatigue ductility coefficient, shear	1
$\varepsilon'_f(R_\sigma)$	Fatigue ductility coefficient, uniaxial	1
$\lambda \equiv \frac{1}{2}\left(\sigma'_f\right)^{-\frac{1}{b_\sigma}}$	Fatigue equation, constant	1
$\sigma'_f\ (R_\sigma)$	Fatigue strength coefficient for normal stress	Pa
$\tau'_f(R_\sigma)$	Fatigue strength coefficient for shear stress	Pa
F_R	Force acting on the upper middle surface, radial	N
$F_\tau(z,\ s)$	Force tangential, pro unit length	N/m
F	Force, axial on the spring	N
F_θ	Force, Circumferential in the wire direction	N
$\tilde{F}_{1Z}$	Force, corrected total axial	N
$F_\sigma(z,\ s)$	Force, normal, pro unit length	N/m
$F_z(t)$	Force, of spring as the function of time	N
Q	Force, shear	N
F_z^0	Force, Spring at the moment $t = 0$	N
F_{min}	Force, spring loads at lengths L_{comp}	N
F_{max}	Force, spring loads at lengths L_{ref}	N
F_z	Force, total axial acting on the upper middle surface	N
F_{AL}	Force, total axial due to Almen and Laszlo	N
F_{zDIN}	Force, total axial, DIN standard	N
$N_1,\ N_2,\ N_{12}$	Forces, meridional, circumferential and shear direct	N

(continued)