Mathematical Engineering

Yuri N. Skiba
David Parra Guevara

Mathematical Methods for the Assessment and Control of Industrial Emissions

Springer

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Preface

Industrial pollution is a serious problem worldwide, both in developed and developing countries. The rapid growth of the population and significant development of production and transport have led to the release of toxic pollutants that have a negative impact on the environment and human health. The problem of environmental pollution is exacerbated by large industrial complexes whose emissions exceed sanitary standards, as well as accidental or hidden discharges of pollutants. This applies equally to the processes taking place on the continent and in the ocean. Since many gases and aerosols affect the environment in one way or another, the immediate task is to assess the strength and localization of emissions in order to reduce their negative impact.

In recent years, the problem of protecting and restoring the environment has become one of the most important tasks of many sciences, the development of which is stimulated by the increasing pace of technological progress in all countries. There are two approaches to reduce emissions and protect the environment in the major industrial regions. The former, so-called "technological way" deals with "green" technologies that clean air or remove pollutants. The second approach including analytical methods of prevention is of great mathematical interest and is more cost-effective than the application of end-of-pipe treatment technologies. The methods use mathematical models to describe the dispersion of pollutants, and the concentrations of harmful pollutants are regulated by a control of the intensity of emissions from the pollution sources. The advantage of this approach is that it makes it possible to simulate on a computer the concentrations of various primary and secondary pollutants, to determine the sources of excessive pollution of particularly vulnerable areas (residential areas, forests, etc.), and to develop emission control strategies for operating industrial enterprises. Also, this approach makes it possible to plan the placement of a new industrial zone and the installation of safety devices in high-risk areas to prevent an accident or unauthorized discharge.

This book is intended to introduce the reader to modern mathematical methods for preventing dangerous environmental pollution. It discusses various dispersion models and emission control strategies. The overall goal of all strategies is to limit emissions from pollution sources in order to comply with sanitary standards and prevent risks to human health. Since sanitary standards are usually associated with temporal and spatial averages, the proposed control strategies aim to reduce average concentrations of pollutants to acceptable levels in a particular zone and time interval. An effective control strategy should be applied whenever a short-term forecast model predicts a violation of sanitary standards. The focus is on various mathematical aspects related to industrial pollution, including the dispersion and assessment of pollutants, the control of their emissions, and the identification of the basic parameters of unknown sources. Because these problems are interdisciplinary, the reader needs a basic knowledge of differential equations, mathematical modeling, optimization theory, and numerical methods. The attached bibliography contains extensive literature on these environmental issues.

To illustrate the basic ideas of mathematical control, we will often use a simple two-dimensional (vertically integrated) model that describes the dispersion of quasi-passive pollutants. Despite this, any control strategy can also be applied to a three-dimensional dispersion model. Various examples of air and marine pollution control are presented in Chaps. 5–7. In particular, the application of dual assessments to oil spill problems is given in Sects. 4.1 and 4.2, while the assessment of vehicular emissions is described in Sect. 4.3. A method of detection of industrial plants that violate established emission rates is proposed in Sect. 4.4, while a method of optimal location of a new industrial plant is described in Sect. 4.5. Chapter 5 is devoted to methods of control of industrial air pollution, and Chap. 6 considers various mathematical problems of bioremediation of marine oiled shorelines. Chapter 7 is devoted to solving inverse problems related to the identification of the location and intensity of unknown point sources, and Chap. 8 describes numerical methods used to simulate the dispersion of pollutants in 2D and 3D problems, which are based on a coordinate-wise splitting of the model operator.

Given its breadth of coverage, the book will benefit a wide variety of readers, including researchers, professionals, aspirants, and students.

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Contents

Chapter 1
Introduction

Air pollution is a general term that covers a broad range of contaminants (gases and particles) in the atmosphere. Pollution can occur from natural causes or from human activities. The most characteristic sources of air pollution have always been combustion proceses that produce such primary pollutants as sulfur dioxide or smoke [1]. Although the environment has its own mechanisms to assimilate wastes and reintegrate nature, these mechanisms become ineffective in the conditions of growing anthropogenic activity [2]. Today, many of the key pollutants in the urban atmospheres are secondary pollutants produced by processes initiated through photochemical reactions. These pollutants form mixtures that cause various damages in humans and ecosystems [2–4].

Each air pollutant, depending on its concentration and toxicity, entails various public health problems, ranging from discomfort of airways to an increase in deaths among the most vulnerable population (cardiac patients, children and elderly, etc.). In either event, contamination is a factor that worsens the quality of life of humans. Unfortunately, the impact of a mixture of pollutants on ecosystems is not only local, as in the case of urban photochemical smog [5]. It can also be regional (acid deposition) [6, 7], or global (destruction of the ozone layer and global climate change) [8]. Today's society cannot return to pre-industrial times to solve the problems caused by air pollution, and therefore it is important to control emissions in order to reduce the concentration of harmful substances in the atmosphere to recommended maximum permissible levels.

Water from seas and rivers has traditionally been used as a means of removing human waste, and the biological cycle of water ensures the reabsorption of this recyclable organic wastes. But today, in addition to these organic wastes, a large number of harmful chemicals are released into rivers and seas, which destroy aquatic flora and fauna, cancel the action of bacteria and algae in the process of biodegradation of organic and chemical water pollutants. In particular, petroleum products are often from ships or coastal sites and are absorbed by marine life. Being carcinogenic and

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mutagenic, polycyclic aromatic hydrocarbons contained in crude oil, pose a danger to aquatic organisms and, ultimately, to humans (when consuming fish and shellfish).

One way of regulation of concentrations of harmful pollutants in the atmosphere is based on using mathematical models of dispersion of polluting substances [9, 10]. The advantage of this approach is that it permits a computer simulation of concentrations of various primary and secondary pollutants and identification of the sources that cause the excessive pollution of particularly sensitive areas (residential areas, forests, etc.). Besides, the use of adjoint models and estimates enables the development of strategies to control emissions of operating industrial plants [11–14]. Also, this approach makes it possible to plan the location of a new industrial zone [9, 15] and installation of safety devices in high-risk areas to prevent an accident or illicit dumping.

In this book, various dispersion models and emission control strategies are considered. The overall objective of all strategies is to limit emissions from industrial sources in order to meet air quality standards and reduce the risks to human health. As the health standards are usually related with temporal and spatial averages, the proposed control strategies aim at reducing mean concentrations of pollutants to acceptable levels in a particular region and time interval. A control strategy should be applied whenever prognostic model predicts a violation of sanitary standards.

Generally, there are two approaches to reduce emissions and protect the environment in the major industrial regions. The former, so-called “technological path”, deals with “green” technologies (based on various filtering methods [16–18]) which allow maintaining the lowest levels of emissions of harmful pollutants. The second approach provides control of emission rates from the main pollution sources and is of considerable mathematical interest. To illustrate the basic ideas of mathematical control, we will often use a simple two-dimensional (vertically integrated) model that describes the dispersion of quasi-passive pollutants (in this model, the chemical reactions are ignored or described by a linear law). Of course, every control strategy can also be applied to any three-dimensional dispersion model. Besides, experience with these strategies has pointed to new areas of application. Various examples of pollution control in the atmosphere and the marine environment are presented in Chaps. 5 and 6.

This chapter provides general information on the most important air and water pollutants (Sects. 1.1–1.3) and the main processes of their dispersion (Sect. 1.4). The main features of the long-term and short-term programs of environmental pollution control are explained in Sect. 1.5.

1.1 Pollution and Adverse Health Outcomes

In a historical sense, air pollution has become a serious problem since the smoke generated by the incomplete combustion of wood began to enter the atmosphere. For civilizations in Western Europe, the introduction of bituminous coal as an energy source in the fourteenth century, added a new aspect to the problem of air pollution.

Coal remains an important energy source, due to its low cost and abundance when compared to other fuels, particularly for electricity generation. Air pollution from coal-fired power plants is large and varied and contributes to a significant number of negative environmental and health impacts. A typical coal plant generates 3.5 million tons of CO_2 per year. Coal combustion releases nitrogen oxides, sulfur dioxide, particulate matter, mercury and dozens of other substances known to be hazardous to human health. Burning coal is also a leading cause of smog and acid rain.

In general, the atmospheric pollutant is a substance that natural or anthropogenic sources emit into the atmosphere in quantities that in the short, medium or long term have adverse effects on living organisms, ecosystems and materials. Natural events, such as volcanic eruptions, forest fires and anaerobic fermentation processes, can produce large quantities of pollutants such as sulfur dioxide (SO_2), carbon dioxide (CO_2), nitrogen oxides (NO_x), methane (CH_4), hydrochloric acid (HCl), ammonia (NH_3), hydrosulfuric acid (H_2S), solid particles, etc. [1]; however, these events are sporadic. Due to the diversity and intensity of emitted substances, human activities such as agriculture, power generation, transportation, mining, metallurgy, chemical industry, electronics, and waste treatment are the main generators of the following pollutants: sulfur dioxide (SO_2), carbon monoxide (CO), carbon dioxide (CO_2), nitrogen oxides (NO_x), methane (CH_4), hydrochloric acid (HCl), ammonia (NH_3), ozone (O_3), sulfuric acid (H_2SO_4), nitric acid (HNO_3), heavy metals (Pb, Cd, Zn), hydrocarbons, etc. [4].

Analysis of the phenomenon of air pollution at the regional level and in the medium term shows that the five of the total number of pollutants emitted into the atmosphere are the most important, since they account for 95% of pollution [2, 3]. These pollutants are carbon monoxide (CO), sulfur dioxide (SO_2), nitrogen oxides (NO_x), hydrocarbons and particulate matter. Of these, the first three are chemicals that have a defined composition; hydrocarbons are a family of substances; and finally, particulate matter involves liquid or solid aerosols which are suspended in the air with dimensions ranging from 0.001 to 10,000 microns. Carbon monoxide, nitrogen oxides and hydrocarbons originate mainly from the combustion of gasoline in motor vehicles, while sulfur dioxide and particulate matter are emitted by industrial sources such as power plants, incinerators and cement works. The concentrations of these five contaminants, together with the concentration of ozone, whose formation and accumulation depends on the presence of nitrogen oxides, hydrocarbons and solar radiation with a wavelength of 430 nm [5] are used as an index of air quality [3].

There is no doubt that excessive levels of pollution are causing a lot of damage to human and animal health, tropical rainforests, as well as the wider environment. Pollution effects depend on the toxicity, reactivity and amount of pollutants, and on the environmental conditions (atmospheric stability, moisture, etc.). The reactivity is important, as common air pollutants can react with one another to form highly reactive and toxic substances [19]. Typical examples of secondary pollutants are chlorine gases, ozone and sulfuric and nitric acids. In particular, the sulfuric and nitric acids are formed when sulfur dioxide and nitrogen dioxide react with the radicals OH and water [6]. These acidic substances may impact far from the source, since the reactions which form them are slow [7]. Thus, emissions occurring in

Table 1.1 Health standards for the principal pollutants

Compound	Health standards ($\mu g/\mathrm{m}^3$)	Time interval
CO	10000 40000	Average value for 8 h Average value for 1 h
NO_x	100 200	Average value for 1 year Average value for 1 h
Hydrocarbons	160	Average value for 3 h
SO_2	80 365	Average value for 1 year Average daily
Particulate matter	75 260	Average value for 1 year Average daily
Ozone	240	Average value for 1 h

Germany can cause acid rain in Sweden. In contrast, the reactions involved in the formation of ozone are fast, and therefore their impact is local. For example, when the sun shines on air polluted with automobile exhaust, photochemical reactions of nitrogen dioxide, non-methane hydrocarbons and carbon monoxide produce “Los Angeles-type smog” containing high concentrations of ozone and other oxidants [5].

Mathematical modelling of the dispersion and control of pollutants is a very difficult problem, because the equations describing the reactions of substances are usually nonlinear [11]. For the sake of simplicity, in the dispersion models considered in this text, the reactivity is parametrized by a linear term. In order to control air pollution, the maximum allowable average concentrations (sanitary standards) are set for priority pollutants [3, 4]. Table 1.1 shows the health standards for the six principle pollutants.

Moreover, it is noteworthy that the impact of various contaminants on the health of humans is a subject that is still only partially known, because the effects of substances in large quantities within a few hours are understood, but in general the impact that the medium or low concentrations produce over years or decades is unknown.

For example, experiments carried out on animals have shown that concentrations of 2 ppm (parts per million) of ozone for 3 h are lethal [3]. Moreover, the concentrations of 0.1 ppm during episodes of photochemical smog in cities for periods of 4–8 h in a day, cause eye irritation and asthma attacks. However, the health effects when humans are exposed to this pollutant for years are unjustly ignored. The polynuclear hydrocarbons such as benzopyrene are another example. These substances are formed during combustion of coal, fuel oil and gasoline, and at high concentrations are identified as potent cancerogens. However, their impact on humans is not known when they are continuously present at concentrations of a few nanograms per cubic meter, which is typical for large cities.

One factor which complicates the impact of pollutants on living beings is the fact that hundreds of harmful substances coexist in the environment at the same time, and their combined effect (known as the synergistic action) can be greater than the sum of their individual effects. The most striking example is the air pollution episode in December 1952 in London, where nearly 4000 people died as a result of the synergistic action of the two substances formed during the combustion of coal in homes and factories: sulfur dioxide and solid particulate matter [4]. Finally, it should be noted that contaminants from natural sources may represent serious short-term air quality problems when they are generated in significant amounts near human settlements.

1.2 Main Atmospheric Pollutants

We now consider various types of air pollutants in more detail.

Smog. The term *smog* was first coined during the 1950s when it was used to describe a mixture of smoke and fog experienced in London. Smog occurs when emissions from industry, motor vehicles, incinerators, open burning and other sources accumulate under certain climatic conditions. The term smog, as used now, is widely applied in the pollution conditions, characterized by a significant reduction in visibility.

Haze. The term *haze* is traditionally an atmospheric phenomenon where dust, smoke and other dry particles obscure the clarity of the sky. Actually, haze and smog are closely related. Both conditions represent a reduction in visibility. They differ in terms of the intensity and geography. Haze generally refers to the reduction of visibility that is not as intense as smog. The term smog is often used to describe the marked reduction of visibility in cities or large metropolitan areas. On the other hand, haze refers to a wide scale of unfavorable pollution levels, which cause reduced visibility during the summer months.

Gases and particles. Due to important impact on the atmosphere, vegetation, human health and materials, the traditional air pollutants, gases and particles have received considerable research and attention. Although the terms gases and particles are used, they actually represent the three phases of matter. The point is that the particles represent two phases: solid and liquid. When fine solid particles and liquid droplets are dispersed in the atmosphere, the resulting suspension is characterized as an aerosol. Aerosols may reduce visibility, stain materials and affect health. Aerosols can be natural or not. Examples of natural aerosols are fog, forest exudates and geyser steam. Examples of artificial aerosols are haze, dust, particulate air pollutants and smoke. Aerosols can be generated in different ways. For example, gaseous aerosols are produced by the combustion and subsequent condensation of metal vapors. And photochemical aerosol is produced by the condensation of gases which results from atmospheric photochemical reactions. Most acid aerosols which contribute to the formation of the slight haze and acidification of precipitation are apparently also produced by photochemical reactions.

Although the particles are the most visible manifestation of air pollution, they constitute only a relatively small part of the emissions, since 90% of the pollution substances released into the atmosphere are gases. These gaseous pollutants can be produced by burning fuels and other materials, smelting an ore, or evaporation of volatile liquids or solids.

Sulfur dioxide. Sulfur dioxide (SO_2) is historically the most prominent of pollutant gases, having achieved this reputation as a respiratory irritant. Globally, most of SO_2 is produced by volcanoes and by the oxidation of sulfur gases, caused by the decomposition of plants. Since the "natural" sulfur dioxide is emitted to the atmosphere mainly at high altitudes or away from the densely populated centers, the background concentration of gas in the clean air is quite small (about 1 ppm). However, at present a considerable amount of sulfur dioxide is emitted at ground level, particularly, in inland areas of the Northern Hemisphere. The main anthropogenic source of SO_2 is the combustion of coal, a solid that, depending on the geographical area where it is extracted, contains from 1 to 9% of sulfur dioxide. In many countries, the use of coal is important for the generation of electricity. Typically, half or more of the sulfur is retained in the coal; if coal is pulverized prior to combustion, this type of sulfur can be removed mechanically. Many point sources of SO_2 are associated with the smelting industry of nonferrous metals.

Nitrogen dioxide. Nitrogen dioxide is the chemical compound with the formula NO_2. It is considered harmful to plants and is known to be toxic to humans, but not at concentrations that normally occur in the atmosphere. The only physical property of nitrogen oxides which is of great interest in the air pollution is its color, NO_2 is the yellow/brown colored gas [20]. This typical color becomes visible when the NO_2 emissions in the chimney exceed the 75 ppm. Exposure to 20 ppm is immediately dangerous to life and health. People with asthma, as well as children and the elderly are generally at greater risk for the health effects of NO_2. In addition, its negative effect is the enormous corrosion it can cause due to acidification of the environment.

Carbon monoxide. Carbon monoxide (CO) is the specific chemical compound, the simplest among the six principal pollutants. Carbon monoxide is a colorless, odorless and tasteless gas that is slightly less dense than air. The most important property of CO is its ability to reduce the oxygen carrying capacity of our blood. Carbon monoxide is the result of inefficient combustion of carbonaceous fuels. It is not produced in large quantities by most stationary sources of fuel, such as boilers and heaters because they are designed and tuned to operate as efficiently as possible. The largest CO emissions result from unplanned combustions such as wildfires and solid waste combustion, which are inefficient because the fuel is not uniform and has high moisture content. A part of CO also comes from various industrial processes, particularly smelters, refineries and paper mills. The main source of carbon monoxide is the combustion of gasoline in motor vehicles. Combustion in a motor is very inefficient, in part by the intrinsic properties of the motor, and partly because the cars are designed to produce velocity and power instead of economy. The cars produce more than half of the nearly 100 million tons of carbon monoxide emitted in a year [20].

Ozone. Ozone (O_3) is colorless or slightly bluish gas. It is a normal constituent of the atmosphere with peak concentrations in the middle stratosphere. As a consequence of anthropogenic influences, O_3 represents two different but very important roles. In the stratosphere, the quasi-stationary concentration of ozone acts as a filter for ultraviolet radiation. But in the troposphere, at slightly elevated levels, ozone is a pollutant. In both atmospheres, urban and non-urban, the concentrations of O_3 are often higher than those produced by photolysis of NO_2. The key to high levels of tropospheric ozone is the chemical reactions that convert NO into NO_2 without consuming O_3. In relatively clean atmosphere, during the hottest months of the year, the concentrations of background level of O_3 are estimated in the range of 20–50 ppb (parts per billion). The main natural sources of O_3 are the photochemical processes and movement of stratospheric ozone in regions named *tropopause folds* in the lower atmosphere, particularly during the months of spring.

Pollution sources. Pollutants are classified as primary or secondary. Primary pollutants are emitted directly from a source (ash from a volcanic eruption, carbon monoxide gas from motor vehicle exhaust, sulfur dioxide from a factory, etc.). Secondary pollutants are not emitted directly. Rather, they are formed in the air when primary pollutants react or interact.

The sources of primary pollutants can be classified as mobile or stationary sources, combustion or non-combustion sources, point or area sources, direct or indirect sources. Major point sources include power or industrial plants, refineries, wastewater treatment plants, chemical or nuclear explosions, oil spill from a damaged oil tanker, etc. Emissions from motor vehicles, aircrafts, trains and ships along their routes are examples of mobile line sources. While point sources of pollution were studied since the late nineteenth century, linear sources did not receive much attention from scientists until the late 1960s, when environmental regulations for highways and airports began to emerge. An area source is a collection of individually small emission sources within a single geographical area that produce similar air pollutants. Locomotives operating on certain linear tracks are examples of a line source, whereas locomotives operating within a railyard are an example of an area source of pollution. Other area sources of air pollution are open burning, forest fires, evaporation losses from large spills of volatile liquids. The sources that do not emit themselves, but indirectly contribute to high pollution levels, are indirect sources. As examples we can call malls or stadiums, which contribute to a high level of vehicular traffic in the zones of their location.

1.3 Water Pollutants

Water is essential for life as we know it. Without water is hard to imagine any life. The water cycle in our planet, also called hydrological cycle, in essence consists in the fact that the water enters the atmosphere by evaporation or breathability and returns to the soil by condensation and precipitation. Fortunately, the Earth is almost flooded: a total volume of about 1400 million cubic kilometers covers 71% of the land surface. Yet in many parts is not easy to obtain the necessary quantities of sufficiently pure water. The fact that terrestrial ecosystems, humans and agriculture depend more on accessible fresh water, which constitutes only 0.4% of the total volume, implies that the problems of scarcity and water quality are multiplied. An estimated 580 people in India die of water pollution related illness every day.

The highest concentration of the world's human population occurs on the banks of rivers and coastal areas, with the consequent impact of human activity on water. Practically all the wastewater (85%) are discharged into the coastal strip, not only in developing countries but also in developed countries such as US [21], particularly in estuaries and bays where heavy contamination is common near large cities because of nutrients, toxic substances and microorganisms. As occurs in the atmosphere, diffuse sources of marine pollution are difficult to register. One must distinguish between what is denoted by point source and diffuse source. Point source is any single identifiable source that discharges substances harmful to aquatic life, recreation and human consumption. Examples of a point source in a bay are a ship, a municipal treatment plant, the river mouth or a submarine diffuser. A diffuse source of water pollution is commonly more difficult to identify and to control than a point source because it arises in a catchment from many different sources that do not have an obvious discharge point. Diffuse source water pollution is caused when pollutants from a range of dispersed urban and rural land use activities contaminate our waterways. Atmospheric deposition of pollutants is an important diffuse source of acid, nutrients, metals and other substances. In the coastal strip, the berths areas, drains of untreated water, as well as rain runoffs that drag oil, grease, dust, dirt, debris, animals, microorganisms, and chemicals such as metals, pesticides, and fertilizers, can be considered as diffuse sources. The contributions of highly toxic contaminants constitute a serious threat, contribute to the deterioration of coastal ecosystems and represent a huge challenge for health authorities, as it is very difficult to quantify and monitor their effects. When the ground cover has not been disturbed by human activity, rainwater percolates through the soil, which absorbs and detoxifies many pollutants. But when the soil is not sufficiently covered with natural vegetation, water drains freely, washing on its way to coastal waters solids and liquids.

According to the definition of a pollutant, it is considered that pollution is generated in water by the addition of any substance in sufficient quantity to cause measurable harmful effects on flora and fauna (including human). The presence in the environment of one or more contaminants, or any combination thereof, impair or disrupt the life, health and human welfare, flora and fauna, or degrade the quality of air, water, land, assets or resources of the nation. One can speak of four basic types of contamination: physical contamination (noise, infrasound, thermal pollution and radioisotopes), chemical contamination (hydrocarbons, detergents, plastics, pesticides, heavy metals, sulfur derivatives and nitrogen), biological contamination (bacteria, fungi, viruses, parasites, introduction of animals and plants from other areas) and elements that damage the aesthetics (landscape degradation and the introduction of industries).

Often the taste, smell and appearance indicate that water is contaminated, but the presence of hazardous pollutants can only be detected by special and precise chemical and biological tests. Among the factors that generate pollution and characterize industrial civilization are: the growth of production and excessive energy consumption, the growth in the metallurgical industry; the growth of road, air and water circulation, and the growth of the amount of waste and scrap being thrown and/or incinerated.

The water of the seas and rivers has traditionally been used as a means of disposal of human waste, and the biological cycles of water ensure the reabsorption of this recyclable organic waste. But today, apart from these organic waste, large quantities of harmful chemicals are thrown into rivers and seas, which destroy aquatic plant and animal life, and cancel the action of bacteria and algae in the process of biodegradation of organic and chemical pollutants of water.

The most serious is that petroleum derivatives are often ejected from ships or coastal industries and absorbed by marine fauna and flora. While being carcinogenic and mutagenic (Table 1.2), polycyclic aromatic hydrocarbons, the components of crude oil, represent a risk to aquatic organisms and ultimately to humans (through fish and shellfish consumption).

Physical contaminants. These contaminants affect the appearance of the water, and when they float or sink they interfere with aquatic flora and fauna. They are insoluble liquids or solids of natural origin and synthetic products that are thrown into the water as a result of human activities, as well as foams, oil residues and heat (thermal pollution).

Chemical contaminants. Chemical pollutants include organic and inorganic compounds dissolved or dispersed in the water. Inorganic contaminants are various products that come from domestic, agricultural and industrial discharges or soil erosion. The main ones are chlorides, sulfates, nitrates and carbonates, and also acidic waste, alkaline waste and toxic gases dissolved in the water, such as sulfur oxides, nitrogen, ammonia, chlorine and hydrogen sulfide. Many of these pollutants are released directly into the atmosphere and fall washed away by rain. This acid rain has harmful effects which can be observed both in vegetation and buildings and monuments in the industrialized cities.

Organic contaminants. Organic pollutants also dissolve or disperse in water coming from domestic, agricultural, industrial waste and soil erosion. They are human and animal waste, waste from slaughterhouse, waste processing of food for humans and animals, various industrial chemicals of natural origin, such as oils, greases, tars and dyes, and various synthetic chemicals such as paints, herbicides, insecticides, and so on. Organic contaminants consume oxygen dissolved in the water and affect aquatic life (eutrophication). Abnormal levels of nitrogen compounds in the water, such as ammonia or chloride, are used as an index of the presence of these contaminating impurities in the water.

Biological contaminants. Biological pollutants include fungi, bacteria and viruses that cause diseases, algae and other aquatic plants. Some bacteria are harmless and others participate in the degradation of organic matter in the water. There are certain bacteria that decompose inorganic substances. The elimination of the viruses that are transported in water is a very difficult and expensive job.

Other contaminants. It should also be noted such important contaminants as heavy metals (lead, cadmium, mercury), certain pesticides, cyanides, hydrocarbons, arsenic and phenol, which provoke the destruction of aquatic ecosystems and serious damage to people who drink water or its products contaminated with this class of chemicals.

Table 1.2 Toxic ingredients in everyday products that contaminate water

Product	Ingredients	Effect
Household cleaners	Powder abrasive cleaners and sodium phosphate, ammonia, ethanol	Corrosive, toxic and irritating
Cleaners with ammonia	Ammonia, ethanol	Corrosive, toxic and irritating
Bleachers	Sodium hydroxide, potassium hydroxide, hydrogen peroxide, sodium hypochlorite or calcium	Toxic and corrosive
Disinfectants	Ethylene and methylene glycol, sodium hypochlorite	Toxic and corrosive
Openers	Sodium hydroxide, potassium hydroxide, sodium hypochlorite, hydrochloric acid, petroleum distillates	Extremely corrosive and toxic
Polishers of floors and furniture	Ammonia, diethylene glycol, petroleum distillates, nitrobenzene, naphtha and phenols	Flammable and toxic
Cleaners and polishers of metals	Sulfuric acid	Corrosive and toxic
Oven cleaners	Potassium hydroxide, sodium hydroxide, ammonia	Corrosive and toxic
Toilet bowl cleaners	Oxalic acid, muriatic acid, paradichlorobenzene and sodium hypochlorite	Corrosive, toxic and irritating
Carpet cleaners	Naphthalene, perchlorethylene, oxalic acid and diethylene	Corrosive, toxic and irritating
Aerosol products	Hydrocarbons, flammable	Toxic and irritating
Pesticides and insect repellents	Organophosphates, carbonates and pyrethrins	Toxic and poisonous
Adhesives	Hydrocarbons	Flammable and irritant
Antifreeze	Ethylene glycol	Toxic
Gasoline	Lead tetraethyl	Toxic and flammable
Engine Oil	Hydrocarbons, heavy metals	Toxic and flammable
Transmission fluid	Hydrocarbons, heavy metals	Toxic and flammable
Wiper fluid	Detergents, methanol	Toxic
Batteries	Sulfuric acid, lead	Toxic
Brake fluid	Glycols, glycol ethers	Flammable
Car body wax	Naphtha	Flammable and irritant

According to the origin, it is considered that pollution is of two types:

(a) the pollution caused by natural or geochemical reasons, and
(b) the pollution caused by human activities and called anthropogenic pollution.

Among the harmful effects on microorganisms, populations and ecosystems, are the following:

(a) damage to human health (poisoning, infectious and chronic diseases, death),
(b) damage to flora and fauna (eutrophication, disease and death),
(c) alterations of ecosystems (erosion, eutrophication, persistent accumulation of harmful compounds, destruction), and
(d) aesthetic discomfort (bad smells, tastes and unpleasant appearance)

Ocean pollution. The ocean is currently the “dustbin of the world”, which will bring adverse effects in the future. Most of the world’s coastal areas are polluted mainly due to discharges of sewage, chemicals, waste, radioactive waste, oil and sediment. The countries whose seas are the most polluted are Bangladesh, India, Pakistan, Indonesia, Malaysia, Thailand and the Philippines. In particular, dolphins, sea lions and sea turtles die when they eat waste or are trapped in bags, ropes and other plastic garbage thrown into the sea.

Biofilms. You might not be familiar with the word “biofilm”, however every day you are in contact with these substances. The plaque that forms on your teeth and cause dental caries and periodontal disease is a type of biofilm. Also, the biofilm causes blockage in the pipes and covers stones in rivers or streams. The biofilm is formed when bacteria adhere to the surfaces of some kind of water environment and begin to secrete a slimy, sticky substance that can stick all kinds of materials—metal, plastic, soil particles, medical implant materials, biological tissues. Biofilms may be formed of a single species of bacteria but generally consist of many species of bacteria, fungi, algae, protozoa, debris and corrosion products. Essentially, biofilms may form on any surface exposed to bacteria and water.

Associations of microorganisms participate in many biotechnological processes, including wastewater treatment, where they form a structured community, enclosed in an extracellular polymer matrix (biofilm). The introduction of microsatellites into biofilms (bioaugmentation) stimulates the activity of microorganisms-destroyers and protects them from the extremal environmental factors. Under natural conditions, the majority of microorganisms exist as structured associations called biofilms. Similar associations are created in many biotechnological processes: soil bioremediation, microbial leaching of metals from ores, in the process of wastewater treatment, and in many other cases.

For example, in the biofilters that represent primary systems of wastewater treatment, water is filtered through a solid support covered with a dense microbial biofilm, whose microorganisms remove various contaminants, oxidize ammonia, absorb phosphates and heavy metals, and restore nitrates.

Oil spills. Oil spills represent a serious problem for aquatic systems and can eliminate or adversely affect fish, other aquatic organisms, birds and mammals. Spills can kill or reduce populations of organisms living in coastal sands and rocks, and can kill the worms and insects that serve as food for birds and other animals. When oil is introduced into the coastal marshes, it damages or kills fish, shrimp, birds and other animals. An oil spill may also contaminate beaches used for swimming and recreation, and even when sophisticated techniques of collection of hydrocarbons are applied, they generate waste contaminated areas (beaches, seabed). In the study of oil degradation with microorganisms that oxidize hydrocarbons, it has been shown that in the reconstructed biofilms (containing besides microorganisms their natural satellites which are able to oxidize oil), the rate of oxidation of hydrocarbon is many times higher than the rate of oxidation in the absence of satellites [22]. This is due to the fact that the satellites emit in the environment substances that activate the vital functions of the oil oxidants. The role of satellites in a biofilm can also be in the protection of microorganisms-destructors from environmental stressors. Therefore, the controlled application of biofilms can improve the performance of sewage systems and industrial water systems without significant additional costs. Using this approach, it is equally important both the formation of biofilm, and the regulation of the composition of its community (bioaugmentation).

Therefore, one way to remove oil residues is through biostimulation, consisting of the addition of nutrients (nitrogen, phosphorus) to facilitate the increase of native oil-degrading microorganisms in the environment (bacteria, fungi). Such microorganisms consume hydrocarbons and produce water, carbon dioxide and biomass, and thereby the contaminants are reincorporated to the environment. An important factor to achieve maximum growth rates of microorganisms is reaching critical concentrations of nutrients and keeping them for a suitable time. This is a difficult goal, since the concentration in contaminated areas depends on the place where nutrients are supplied, the rate of discharge and the physical influences of the aquatic system (dispersion conditions). For example, the distribution and movement of dissolved nutrients, and hence their concentration and residence time in the field affected by oil also depend on the water waves [23]. In particular, the authors note that in the presence of water waves, the residence time of nutrients in the tidal zone is reduced by about 25%.

Bioaugmentation is the addition of bacterial cultures required to speed up the rate of degradation of a contaminant. In case of petroleum, microorganisms capable of degrading hydrocarbons are used if some contaminants cannot be degraded by the existing microorganisms. There are many different types of hydrocarbon degrading microorganisms, with over 200 known species of bacteria, fungi, and yeast that are capable of degrading different compounds, from a simple such as methane up to compounds with more than 40 carbon atoms [24, 25].

Since most natural environments contain microorganisms, bioaugmentation is not as effective for cleaning oil polluted regions, since the addition of non-native organisms into the environment often leads to their competition with the existing beneficial microorganisms. That is why bioaugmentation has never given a positive effect in cleaning oil contaminated waters [26]. Also, bioaugmentation has been criticized, since the introduction of non-native species into the environment is not safe. Unlike bioaugmentation, the primary advantage of biostimulation is that the environment is modified to stimulate already present native microorganisms to achieve maximum degradation rate of toxic oil compounds.

1.4 Main Processes of Pollution Dispersion

Since humans discovered the usefulness of fire, the atmosphere has been used as a repository of waste products of their activities. For thousands of years, the atmosphere, with its huge volume and constant movement, has readily accepted and dispersed relatively small amounts of polluting substances. The dispersion process depends on the local weather conditions (speed and direction of winds, turbulence, atmospheric stability, etc.) and the topography of the region [27]. But in the modern era of big cities and intense industrialization, the ability of the atmosphere to disperse pollutants has been exceeded. Adverse weather conditions lead to a deterioration of the dispersion of pollutants and relatively high pollution levels. This causes significant human health and environmental damages.

Wind-induced advection. Advection is the horizontal movement of the atmosphere, and prevailing winds are known as advective winds. The horizontal winds play a significant role in the transport of pollutants. With the wind speed increases, the volume of air passing per unit time through a unit area also increases. If the emission rate from a source is relatively constant, but the wind speed becomes twice as much, then the concentration of contaminants is reduced by half, i.e., the pollution concentration is inversely proportional to the wind speed.

In the surface (boundary) layer, horizontal wind velocity depends on the friction. Therefore, boundary layer is also known as the friction layer, where friction is an important force. The friction force is proportional to the surface roughness, which in turn is determined by terrain topography (presence of mountains, valleys, rivers, lakes, forests, crop areas, buildings, etc.). On a clear, calm night, the boundary layer over a lake may extend only a few hundred meters upward, but may extend a few thousand meters upward over a city on a hot, windy afternoon. On average, wind speed over smooth surfaces such as lakes and crop areas is generally higher than over the mountains and buildings. Dispersion of pollutants is also affected by wind direction variability. If the wind direction varies significantly, the pollutants will be distributed over a larger area than in the case when the wind direction is relatively constant. In this case, the concentration of pollutants in a given area in the first case will be less. Moreover, the concentration of pollutants over a given area will be smaller in the first case. It should be noted that substantial changes in the wind direction can occur in a short period of time. For example, a change in the wind direction of 30° in one hour is typical, and during a period of 24 h the wind direction can be changed by 180°. Similarly, seasonal changes may result in wind direction changes by as much as 360° [27].

Atmospheric turbulence. A plume emitted from a source is carried by mean winds. But this process is also affected by atmospheric turbulence. Atmospheric turbulence is small-scale, irregular air movements characterized by winds that vary in speed and direction. Turbulence is important because it mixes and churns the atmosphere and causes water vapor, smoke, pollutants and other substances, as well as energy, to become distributed both vertically and horizontally. Thus, under the influence of atmospheric turbulence, a plume spreads over a large area with time. There are two main mechanisms for generating atmospheric turbulence. These are mechanical and convective turbulence.

The mechanical turbulence is generated as the air flows over obstacles on the earth surface, such as trees, buildings, and hills. Its intensity increases with increasing wind speed and with increasing surface roughness and decreases with height above the ground. When vertical heat flux is small, so that most of the atmospheric turbulence is mechanically generated, the atmosphere is said to be neutral or in a state of neutral stability.

The convective turbulence is related with atmospheric instability. As solar radiation heats the earth surface, the lower layers of the atmosphere increase in temperature and convection begins, driven by buoyancy forces. The convective turbulence is usually associated with large eddies, the effects of which are often visible on plumes from stacks. At night, when the surface of the earth cools, temperature increases with height, and turbulence tends to be suppressed.

Atmospheric precipitation. With the exception of CO_2, clorofluorocarbons *CFCs*, and some other chemically stable compounds, no appreciable accumulation of pollutants in the atmosphere occurs, even if the pollutants are continuously emitted into the atmosphere. This is due to the processes of dry and wet deposition. Dry deposition is the removal of gaseous or particulate material from the pollution plume by gravitational sedimentation of particles of relatively large size and absorption of particles of gas or liquid in liquid or solid material. Wet deposition is the removal of pollution plume components by the action of rain. For example, the wet deposition of radionuclides in a pollution plume by a burst of rain often forms so called hot spots of radioactivity on the underlying surface. Meteorological phenomena that increase the frequency and intensity of rainfall in a region significantly reduce pollutant levels and improve air quality. Precipitations associated with migratory cyclones are particularly effective in cleaning up the atmosphere. On the other hand, pollution problems are aggravated in the regions that get little precipitation.

Sedimentation. Sedimentation is the tendency for particles in suspension to settle out of the fluid in which they are entrained, and come to rest against a barrier. For example, the atmospheric particles fall to the ground from the air. Washout of particles by snowflakes, rain, hail, sleet, mist or fog is a common form of agglomeration and sedimentation. Particles with diameters less than 20 microns are treated as a gas, and their effects, despite their velocity, are generally ignored, while the larger particles already have perceptible sedimentation rates.

Inversion layer. Atmospheric inversion layer is a layer, within which an atmospheric property is inverted, i.e., its change with height is deviated from the normal pattern. Usually, within the lower atmosphere the air near the Earth is warmer than the air above it, largely because the atmosphere is heated from below as solar radiation warms the earth surface, which in turn then warms the layer of the atmosphere directly above it. Thus, the atmospheric temperature normally decreases with increasing altitude. The inversion almost always refers to a temperature inversion, i.e., an increase in temperature with height. Therefore, the air cannot rise in an inversion layer, since it is cooler and thus denser at the bottom. In other words, temperature inversion stops atmospheric convection (which is normally present) from happening in the affected area. This can become a problem in big cities (for example, Greate Smog of 1952 in London).

1.5 Short-Term and Long-Term Pollution Control Problems

Control of air pollution control in cities is a complex issue that deserves attention due to the negative impact of high concentrations of pollutants on human health. One approach to solving this problem is to implement regional air pollution control programs. Pollution control is regulated by various environmental agencies, which set limits on the release of pollutants into the air, water, and land. It should be noted that each country has different environmental agencies and a completely different set of laws to limit emissions of industrial pollution. In addition, any efforts to measure and study health and environment impacts may prove important in the future, and every individual should take responsibility for preserving the environment.

In general, the goal of air pollution control programs is usually to meet a set of air quality standards, or at least reduce the number of days per year when these standards are violated. Depending upon programs length, they can be divided into two categories [28]: short-term control and long-term control. These control programs supplement each other, however, the air quality objectives of long- and short-term control strategies may be quite different.

The duration of a short-term control problem can vary from several hours to several days, usually under atmospheric inversions that promote the accumulation of contaminants. The actions that characterize this type of control include immediate reduction of pollutant emissions and even suspension of production. A goal of short-term control is ordinarily to keep the maximum concentration of a certain pollutant below a given value (existing sanitary standard) on that particular day or during the whole period of atmospheric stability. Weather forecasts and pollutant concentrations measured continuously at a number of stations are used to regulate the intensity of pollution sources and prevent the occurrence of dangerously high concentrations. An ideal control system should be able to predict the weather and pollution levels from few hours to a couple of days in advance, and use the information from this prediction in deciding what action to take.

Examples of short-term control strategies are:

(a) introducing restrictions on the use of vehicles,
(b) prohibiting the use of certain fuels (replacement of coal or fuel oil to natural gas),

(c) prohibiting certain activities, such as incineration of refuse, and
(d) restricting emission rates from industrial sources (see Chap. 5)

Item d) is an interesting mathematical problem. It is important to note that a mathematical control strategy is only useful if it is able to determine the necessary reduction in emissions from pollution sources to prevent excessive concentrations of harmful pollutants. At present, such preventive methods are not adequately incorporated into the existing short-term control programs to combat air pollution, as most control strategies are applied only after the concentrations of pollutants have already reached dangerous levels [29].

On the other hand, the duration of a long-term control program can vary from several months to several years, and such a program usually focuses on the strategies that reduce the total mass of pollutants emitted during the whole period. Some examples of long-term air pollution control policies are:

(a) enforcing standards that restrict the pollutant content of combustion exhaust,
(b) introducing new technologies of production and cleaning (filters or catalytic converters),
(c) requiring motor vehicles to meet certain emissions standards,
(d) prohibiting or encouraging the use of certain fuels in power plants, and
(e) closing or relocating some industrial plants in a region

For long-term control, a typical objective might be to reduce to a specified value the expected number of days per year that the maximum hourly average concentration of a certain pollutant exceeds a given value (air quality standard). Thus, the use of long-term strategies does not necessarily prevent dangerous levels of contamination, which may occur on certain days. This is because the set of actions are generally designed with the aim not only to maintain air quality standards, but also to meet cost–benefit criteria. In other words, the set of long-term policies are designed to minimize

$$C = C_e + C_s$$

where C_e is the cost of implementation of control activities, and C_s is the cost of damage to human health and ecosystems. Furthermore, this minimization process is realized under the constraint that the number of days per year when air quality standards J_α^0 are violated must not exceed a maximum allowable number $N_{\max}$, i.e.

$$N(J_\alpha > J_\alpha^0) \leq N_{\max}$$

When J_α^0 decreases, cost C_e increases, while cost C_s decreases, and therefore an optimal set of actions corresponds to a minimal value of C. Obviously, in this long-term control problem, the requirement to reduce emission rates was included only indirectly.

1.6 Mean and Point Pollution Control

As has already been noted, the goal of air pollution control programs is to meet a set of air quality standards, or at least reduce the number of days per year when these standards are violated. We also noted that all control programs can be divided into two categories: long-term and short-term control strategies [28]. These programs supplement each other, but differ in the methods and duration of their implementation. For example, the use of long-term strategies does not necessarily prevent dangerous levels of contamination, which may occur on certain days. For such short events (hours to days), short-term local (city) control should be applied to keep pollutant concentrations below air quality standards. Such control actions must include immediate reduction of pollutant emissions and even suspension of certain activities. In the event of an unfavorable forecast from a numerical air pollution model, the first step in a short-term control strategy is to determine the amount of reduction in the intensity of each pollution source so that a reforecast with reduced sources intensities would no longer violate air quality standards. The second step is the technical implementation of such reduction, which may consist of different procedures depending on the type of emission source. Typically, emission reduction parameters (damping parameters) are calculated using mathematical models that combine an air quality model with an optimization model.

The idea of using mathematical programming models for environmental quality control arose in the late 60 s [30–33]. In particular, the linear programming models proposed by Teller [34] and Kohn [35] for air pollution control are the first optimal strategies that take into account economic and environmental variables. Since then, many air pollution control models have been developed for different purposes and polluted regions. The following elements characterize the optimization model: (i) the air pollutants considered, i.e. primary or secondary substances, which determine the complexity of chemical reactions that should be considered in the dispersion model; (ii) an objective or cost function that estimates the cost of changes in emission rates; (iii) environmental restrictions that must be observed in a specific limited area (control zone); and (iv) technological constraints that define the upper and lower limits of scarce resources or fuel materials. Of course, different formulations of these elements create linear or nonlinear programming models with many variables and constraints. However, the common goals of all air pollution control models are to meet environmental goals, technological constraints, and minimize the costs associated with reducing the emission intensity of sources [36–41].

It is important to note that while many studies have analyzed long-term air quality plans, fewer studies have addressed the issue of air pollution episodes [42]. Short-term emission control is a complex problem because its optimal solution requires a series of simulations using a regional air quality model to determine pollutant concentrations for different emission control scenarios [42–44]. In practice, such a procedure can increase response time to short-term pollution events to the extent that mitigation actions can be determined after a pollution crisis. In such cases, rapid response is critical.

To formulate a control strategy, it is assumed that the short-term forecast obtained using the dispersion model is unfavorable, that is, the concentration of a pollutant emitted from several sources exceeds the corresponding air quality standard in the control zone. An optimization problem is then formulated to calculate the damping coefficients and determine the optimum emission reduction at the lowest cost.

Control strategies can be implemented to achieve average pollutant concentrations or point concentrations in an area Ω. In the first case, consider the following functional to estimate the average concentration

$$J(\phi) = \frac{1}{\tau|\Omega|} \int\limits_{T-\tau}^{T} \int\limits_{\Omega} \phi(r,t)drdt \tag{1.6.1}$$

Here $\phi(r,t)$ is the pollutant concentration at point r and moment t, and $(T-\tau, T)$ is the time interval in which the forecast of the average concentration is unfavorable, i.e., exceeding the air quality standard: $J(\phi) > J_0$. Chapter 5 presents some optimal and non-optimal short-term control strategies. Each of these strategies describes a procedure for reducing the intensity of emissions from pollution sources in order to comply with air quality standard $J(\phi) \le J_0$ in the area Ω and time interval $(T-\tau, T)$.

On the other hand, control of point concentrations is a more rigorous strategy that requires more computational effort and time to obtain an answer. This strategy is now briefly described. The corresponding analysis is shown in detail in [40].

We assume that in a region D there are N sources with emission rates $f_i(r,t)$ $(i = 1, \ldots, N)$. It is also assumed that meteorological conditions and air quality are predicted in D over a time interval $(0, T)$ using an appropriate dispersion model in combination with a weather forecast model. Let us assume that the atmospheric air quality forecast obtained with the emission rates $f_i(r,t)$ $(i = 1, \ldots, N)$ is unfavourable, that is, $\phi(r,t) > J_0$ at some points of a zone $\Omega \subset D$ and at some points in time interval $(0, T)$, where ϕ is the concentration of a pollutant in the region D, and J_0 is the corresponding air quality standard. Then, in order to prevent excessive concentration of the pollutant in the zone Ω, it is necessary to apply a short-term control to establish the appropriate intensity of all pollution sources within $(0, T)$. In other words, we should determine in the time interval $(0, T)$ reduced emission rates, optimal in the sense that the air quality standard will be satisfied:

$$\phi(r,t) \le J_0\,, \text{ for any } r \in \Omega \text{ and } t \in (0,T) \tag{1.6.2}$$

To simplify the discussion, we write the linear dispersion model in the form

$$\frac{\partial\phi}{\partial t} + A\phi = \sum_{i=1}^{N} f_i(r,t),\ \phi(r,0) = \phi^0(r) \text{ in } D \tag{1.6.3}$$

The boundary conditions of the problem are omitted because they do not depend on the forcing and initial distribution of the pollutant $\phi^0(r)$ in$D$, which are the main parameters in the control strategies. It is assumed that model (1.6.3) is well-posed, i.e., its solution is unique and continuously depends on the forcing and the initial condition. We will assume that the solutions of the dispersion model for various forcing and initial conditions are continuous functions. Moreover, the turbulent diffusion, included in the definition of the linear operator$A$, guarantees not only the continuity of the solution of model (1.6.3), but also the continuity of its derivatives.

Short-term control of emission rates of pollutant sources consists of finding nonnegative damping factors $\lambda_i \leq 1$ that define new emission rates as

$$\lambda_i f_i(r,t), \quad i = 1,\ldots,N \tag{1.6.4}$$

where $(1-\lambda_i) \times 100$ represents the percentage reduction in emissions of the ith pollution source. The purpose of introducing the coefficients λ_i is to fulfil the environmental condition (1.6.2) for the solution φ of the following dispersion problem:

$$\frac{\partial\varphi}{\partial t} + A\varphi = \sum\nolimits_{i=1}^{N} \lambda_i f_i(r,t), \quad \varphi(r,0) = \phi^0(r) \ \text{ in } D \tag{1.6.5}$$

Since this goal can be achieved with different sets of such parameters, the optimal values will be obtained with a suitable optimization process.

Let us now introduce concentration functions $C_i = C_i(r,t) \ \ i = 0,1,\ldots,N$, as solutions to the following dispersion problems:

$$\frac{\partial C_0}{\partial t} + AC_0 = 0, \ \ C_0(r,0) = \phi^0(r) \ \text{in}\, D \tag{1.6.6}$$

and

$$\frac{\partial C_i}{\partial t} + AC_i = f_i(r,t), \ \ C_i(r,0) = 0 \text{ in } D, \quad i = 1,\ldots,N \tag{1.6.7}$$

Note that if $C_j(r,t) = 0$ for any $r \in \Omega$ and any instant $t \in (0,T)$ then the jth source can be excluded from the control problem. In the future, we believe that such sources have already been identified and eliminated, i.e. all N sources in the dispersion model are responsible for the contamination of the zone Ω. Since the dispersion model (1.6.3) is linear and has a unique solution, the solution to problem (1.6.4) can be written as a linear combination of the concentration functions C_i:

$$\varphi(r,t) = C_0(r,t) + \sum_{i=1}^{N} \lambda_i C_i(r,t), \ \ r \in D \ \text{ and } \ t \in (0,T) \tag{1.6.8}$$

for any set of nonnegative damping coefficients, $\lambda_i \leq 1, \ i = 1,\ldots,N$. By virtue of Eq. (1.6.8), the environmental condition (1.6.2) is equivalent to the following relation: