

Challenges in Physics Education

Marco Giliberti  
Luisa Lovisetti

# Old Quantum Theory and Early Quantum Mechanics

A Historical Perspective Commented  
for the Inquiring Reader

*With a foreword by* Helge Kragh



 Springer

# **Challenges in Physics Education**

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# Foreword I

In 1999 I published a book on Princeton University Press on the development of physics, broadly conceived, in the twentieth century. I chose the title *Quantum Generations* in order to highlight the massive influence of quantum theory on most (if far from all) aspects of physics research during the period, an influence that has not diminished since then. Indeed, today—when we are approaching the centenary of quantum mechanics—physics is to a large extent based on the quantum concepts and techniques introduced in the first three decades of the previous century and which are discussed in penetrating detail in the present book.

Not only has physics to some extent become quantum physics, but the theory that Heisenberg introduced in 1925 has also had a considerable impact on a broad spectrum of other sciences ranging from chemistry to neurology and computer science. (In this respect, quantum mechanics differs drastically from the other foundational theory of the twentieth century, Einstein's general theory of relativity.) Even classical observational sciences such as geology and meteorology nowadays take into account effects based on quantum mechanics, witness the new field known as 'neutrino geology' that emerged a few decades ago. And of course, the impact of the now century-old theory is not limited to the natural sciences as it has for long been discussed by philosophers, linguists, theologians, and other scholars in the humanities.

When physicists (and presumably philosophers too) prepare for the 2125 bicentenary a century from now, in all likelihood the theory will not have changed materially. My guess is that will be celebrated for basically what it is today, a cornerstone of modern physics, a lasting chapter in the long history of natural philosophy, and the basis for a class of remarkable technologies ranging from the transistor of the 1940s to the quantum computer of the immediate (?) future. Perhaps, by that time the old dream of a truly unified theory of quantum mechanics and general relativity, a problem that physicists have fought with since about 1930, will no longer be just a dream. As an alternative scenario might also speculate that by 2125 quantum mechanics has been replaced by a better and entirely different theory, but the possibility that present quantum mechanics should in some fundamental sense be wrong seems utterly remote.

It may be worth pointing out that physicists and physics students typically employ the term ‘quantum mechanics’ in the meaning of what historians of physics prefer to call the ‘old quantum theory’ or just quantum theory, that is, the development from 1900 to the advent in 1925 of proper quantum mechanics. According to the former usage, Einstein’s light-quantum hypothesis of 1905 and Bohr’s 1913 atomic theory both belong to the history of quantum mechanics, which is slightly anachronistic given that the term ‘quantum mechanics’ had not yet been coined. The first reference to ‘quantum mechanics’ that I know of dates from 1921 and was due to the British physicist Charles G. Darwin who used it as just a synonym for ‘quantum theory.’ Only in the fall of 1925 did modern quantum mechanics come into existence and the term (in its German version *Quantenmechanik*) came to denote a new and radically different theory of the atomic and subatomic realm. For a while Schrödinger’s favoured ‘wave mechanics’ (*Wellenmechanik*) was used as frequently as ‘quantum mechanics,’ but from about 1932 the latter term was almost universally adopted for the new theory. It is also worth mentioning, as an indication of the public and cultural visibility of quantum mechanics, that some of the terms associated with the theory have been adopted as metaphors in our common language. When a new commercial product is advertised as a ‘quantum jump’ it is a metaphor with roots in the spontaneous transitions between energy states that Bohr, to the consternation of many contemporary physicists, introduced in his new atomic theory.

Many working physicists are presumably ignorant of or indifferent to the historical development of quantum theory, believing that what happened in the past cannot possibly be relevant to current research problems. However, even if knowledge about history is clearly not a prerequisite for a career in physics, in some cases it opens up for new perspectives relevant also to modern research problems. Besides, many didactically oriented studies have shown that the historical approach is helpful when it comes to students’ understanding of what quantum physics is all about. Even more importantly, it is only through the lens of history that one can fully appreciate that quantum mechanics is and always has been an integral part of the more general history of culture.

As is well known, contrary to most other fields of physics philosophers have since the 1930s been intensely occupied with the conceptual problems of quantum mechanics, sometimes in an abstract sense but at other times taking the historical dimension seriously. The history of quantum physics provides an instructive case study of theory change and the role that experimental anomalies and conceptual reconfigurations play in such change. No wonder that the transition from the old to the new quantum has been scrutinized within the frameworks of rival philosophical views of theory change such as due to Karl Popper, Thomas Kuhn, Imre Lakatos as well as several later philosophers.

Quantum history is today a well-established and sophisticated branch of history of science cultivated by historians, philosophers and practising physicists with the necessary dual competences, that is, both in history and in physics. The scholarly study of the quantum revolution—or better quantum revolutions in plural—started in earnest in the 1960s. It was indebted to the ambitious pioneering project called *Sources for the History of Quantum Physics* (SHQP) sponsored by the American

Philosophical Society and completed in 1967 with a report authored by Thomas Kuhn, John Heilbron, Paul Forman and Lini Allen. The same year followed the mathematician Bartel L. Van der Waerden's *Sources of Quantum Mechanics* with full translations of key papers in the old quantum theory and the new quantum mechanics through the period from 1917 to 1926. Building in part on material in SHQP, in 1966 Max Jammer published his magisterial *The Conceptual Development of Quantum Mechanics*, the first comprehensive history of the birth and early development of quantum theory in the modern tradition.

Without going into further detail, Jammer's important book was eventually followed by a series of equally detailed works either covering the entire early quantum history or focusing on particular theories and themes. Suffice to mention a few of these works such as Edward MacKinnon's *Scientific Explanation and Atomic Physics* from 1982 and Olivier Darrigol's innovative *From c-Numbers to q-Numbers* from 1992 in which the author emphasizes the formal analogies between classical physics and quantum theory such as Bohr's correspondence principle. As far as completeness and details are concerned, no work surpasses the six large volumes of *The Historical Development of Quantum Theory* published by Jagdish Mehra and Helmut Rechenberg in the years from 1982 to 2001. But of course, not even the Mehra-Rechenberg work comprising a totality of some 4,500 pages is a final history of quantum mechanics. Indeed, for very general reasons there is no such thing as a final or definitive history of anything. About two decades later, Anthony Duncan and Michel Janssen published another work in two massive volumes titled *Constructing Quantum Mechanics* (2019, 2023) which is a more manageable and in some respects more appealing (if also quite demanding) alternative to the one presented by Mehra and Rechenberg. While the latter work goes up to about 1940, Duncan and Janssen decided to end their quantum history in 1927, shortly before Dirac's extension of quantum mechanics to the relativistic regime.

It is in this tradition of quantum historiography that *Old Quantum Theory and Early Quantum Mechanics* by Luisa Lovisetti and Marco Giliberti should be located. Their meticulously documented work differs in several ways from the existing literature on the history of quantum physics, for example by including solid chapters on topics which are often disregarded or considered to be peripheral to the 'real' history of how quantum mechanics came into being. Thus, as a historian of chemistry and not only of physics I am pleased to see in the first part of the book detailed accounts of nineteenth-century chemical atomic theories including the fascinating but today largely forgotten vortex theory of the atom (which played a most important role to J. J. Thomson in his construction of the first classical atomic model based on electrons). Like Mehra and Rechenberg, the authors deal comprehensively with Dirac's 1928 theory of the electron and its surprising consequences such as the antielectron (which in 1932 became the positron) and antimatter in general. They also cover the early phase of nuclear quantum mechanics and the philosophical discussions up to and including the famous EPR paper and Bohr's response to it. Generally, they pay more attention than most other authors to the role played by experiments, which they carefully analyse and relate to the theoretical development of quantum and atomic physics.



Last but not least, *Old Quantum Theory and Early Quantum Mechanics* is throughout composed with an eye on teaching physics students how quantum mechanics developed historically and, at the same time, clarifying the fundamental concepts and ideas behind the new theory. For this purpose, the authors incorporate a series of so-called ‘pedagogical detours’ in which they deviate from the strict chronology and attentively discuss some of the important concepts related to the history of quantum physics. To mention but two examples, among the first of the pedagogical detours is a fairly detailed account of Thomson’s influential but pre-quantum atomic model sometimes described metaphorically as the ‘plum pudding model.’ Later in the book there is another detour, pedagogical as well as illuminating, concerning the non-classical concept of spin and its intimate connection to Dirac’s relativistic theory of the electron. Altogether, this is a welcome contribution to the historical quantum literature which exemplarily combines the scientific content of quantum theory not only with its rich history but also with sections of didactical interest.

Copenhagen, Denmark

Helge Kragh

## Foreword II

It is a pleasure for me, representing the Italian Physical Society (SIF), to write a short forward to this very interesting book since it gives me the chance to enlighten a few points related to the motivations that brought the authors to write it.

As written also in the preface, physicists largely use quantum mechanics but many of them do not know well enough the historical context in which this theory was born and was developed also thanks to ingenious experiments carried out during the years.

The SIF has always been engaged via its journals and books in addressing various aspects of the history of physics, primarily from the physicists' perspective. With its publications in this area the SIF aims to foster and disseminate an awareness and understanding of the historical development of ideas in physics and on 'how Nature works'. This specific book, that has the patronage of SIF, addresses the fundamental questions motivating the quantum mechanics, to describe atomic and subatomic properties and phenomena. To read about the discussions and steps made to arrive to the formulation of the quantum mechanics principles allows us to get a deeper grasp and understanding of the fascinating quantum world.

This book is very timely also in view of the celebration of 2025 as the International Year of Quantum Science and Technology (IQST). The proposal for IQST has been endorsed by leading international scientific societies, academies, and unions throughout the world, including the SIF. The year 2025 recognizes 100 years since the initial development of quantum mechanics. Over the past century, quantum science and technology has become more central to a wide variety of scientific and engineering fields ranging from physics, chemistry, material science, biology, and information science. Looking forward, quantum science and technology will be the key cross-cutting scientific field of the twenty-first century. This book fits well into the rich portfolio of foreseen initiatives with goals that encompass science, education, outreach, and particularly the promotion of physics education.

To conclude I like to underline that SIF is grateful to the authors for the excellent work they had done and it recognizes the high quality and the pedagogical value of their endeavor. The SIF also expects that this book will stimulate fruitful interactions between working physicists and historians of sciences.

Enjoy the reading!

Milan, Italy

Angela Bracco  
SIF President

# Preface

2024 marks one hundred years since the name “quantum mechanics” was used by Bohr for indicating a theory that, at the time, was still unborn, and 2025 will be a century since the first formulation of such a theory was made by Heisenberg. The 2022 Nobel Prize for Physics was awarded to Aspect, Clauser, and Zeilinger “for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”. The European Union and the United States of America are financing major projects to bring together research institutions, university, industry, and policymakers, in a collaborative initiative concerning the so-called “second quantum revolution”, and the creation of a learning environment to inform and educate society about quantum technologies. Indeed, many popular books on quantum mechanics and its “strange” features, and about the wonders of quantum computers and cryptography can be found.

However, too often, in our research activity on the teaching of modern physics we still find graduates in STEM subjects (including physics) with only a very approximate knowledge of the historical and epistemological reasons that led to the birth of quantum mechanics and, moreover, with even a poor understanding of its methods and contents.

On the one hand, this situation highlights the general wide interest in quantum mechanics but, on the other hand, it also shows the great difficulty in understanding current presentations of quantum mechanics. Besides more effective didactical proposals, we believe we need—this is the working hypotheses at the root of this book—, a coherent cultural presentation that begins with historical aspects and progressively addresses the physical structure of quantum mechanics, replete with insightful commentaries for the inquiring reader; namely, a reader who is passionate about exploring the construction of our world, the laws governing the universe, and about how these factors influence our self-perception and world view.

This book has precisely the purpose to reach out to such a reader, providing a historical presentation of old quantum theory and early quantum mechanics integrated with comments and examples that help contextualize and understand the physics discussed. In fact, it contains a detailed analysis of the usual topics that have most contributed to the birth and the development of quantum mechanics (black-body

spectrum, Bohr's hydrogen atom, the uncertainty principle, EPR paradox, etc.), but also deals with ideas, concepts and results that are not usually treated (vortex atoms, the birth of the term "electron", non-quantum models of the Compton effect, etc.). The time span taken into consideration goes mainly from the 1880s to the 1940s, but a special chapter is entirely devoted to cutting edge problems of entanglement, and there are also brief notes on more recent developments.

We based our work on nearly eight hundreds primary sources—books, papers, letters, newspapers, etc.—whose content is not only partially reported, but also explained, and inserted in the historical, social and disciplinary context of the time, and also on about three hundreds secondary sources.

Our intentions are essentially those of starting a discussion "orthogonal" to pure erudition. Indeed the book, alongside a rigorous and commented historical framework, engages the reader in an educational dialogue about the presented physical aspects, featuring specific sections and subsections with pedagogical observations.

This is the reason why this book is intended for students pursuing STEM degrees, particularly those seeking an understanding of the genesis and rationale behind quantum mechanics. But it is surely also addressed to professional physicists who are eager to reconsider the cultural foundations underlying the quantum view of the world without getting stuck in the pseudo-historical reconstructions, so present in many textbooks, which prevent the correct understanding of the problems and of the given solutions. We are thus thinking of inquiring minds, people who teach quantum physics, and individuals involved in quantum technologies.

Presented as a scientific tool with a humanistic touch, this book invites engaging in a personal exploration of the physical knowledge developed between the late nineteenth and early twentieth centuries. It seeks to facilitate discussions on cultural and historical foundations, motivations for supporting and understanding physics research, and the evolution of the powerful theory known as "quantum mechanics."

The narrative encompasses the thoughts and words of key figures, emphasising the significance of original sources in forming a comprehensive understanding of the addressed problems. Each quote is accompanied by pedagogical considerations, promoting a deeper understanding or providing a contemporary interpretation.

We adopted a somewhat "smiling" approach, balancing formalism with a light presentation style, ensuring accessibility without compromising rigour. We, therefore, avoided speculative interpretations of quantum mechanics, focusing instead on well-sounded scientific considerations, complemented by insights into social, economic, and cultural contexts. In the book, there can be found also some interludes that are not purely decorative but serve to understand in a deeper way the personality of some scientists, and the physics research context of the time, in order to make it possible the understanding of why things evolved the way they did.

Moreover, we also pose questions and reflections, stemming from our extensive research in quantum physics education, and addressing difficulties and gaps in traditional presentations across all school levels.

Several features distinguish this book's approach. Great attention has been paid to historical accuracy with precise and specific references for anyone interested in efficiently and quickly retrieving the quoted or referenced text; for this reason, the footnotes have been placed at the bottom of the page—so as not to impose unnecessary efforts on the reader seeking bibliographic references.

When deemed helpful for understanding, the mathematical context has been developed in sufficient detail in sections marked with an asterisk (\*). In all chapters, in sections called “A pedagogical detour” rather personal observations and heuristic considerations have been proposed—the kind that find little or no place in physics textbooks and which, often, do not even find a place in reconstructions that are more attentive to history.

Even within the quotes we have sometimes felt the need to make observations or quick and precise comments; in such cases, they have been inserted in square brackets.

Concerning the structure, the book is divided into two parts.

The first part concerns the old quantum theory. Despite the name, the old quantum theory is that set of interpretations and models developed in response to problems posed by the unexpected results of some experiments or of conceptual nature, without being “offspring” of a major theory. Rather, they developed in a “classical” context with the addition of more or less reasonable *ad hoc* hypotheses. Examples include Planck's solution to the problem of the black-body spectrum and Bohr's model of the hydrogen atom.

The second part of the book concerns the theory of quantum mechanics; starting from Heisenberg's seminal work in 1925 and up till the EPR paradox of Einstein, Podolski, and Rosen in 1935. As said, treatment follows the chronological development as much as possible, but keeping in mind that the entire discussion focuses on quantum aspects. Therefore, except for some brief recalls, most of the important non-quantum physics discoveries of the same period are not presented (*e.g.*, the theory of relativity is not discussed).

A final chapter addresses the important quantum problem of entanglement. Although based on original, fundamental sources, this chapter deviates somewhat from the rest of the book, mainly due to the impossibility of keeping up with the vastness of research on this topic from 1935 to the present. Therefore, the reader will find only a small, personal selection of the vast research on the subject.

We were not left alone in this endeavor. We would like to particularly thank Michela Cavinato and Leonardo Gariboldi for the careful review of the book and their timely and valuable suggestions and comments, as well as for all the physical, historical, and epistemological discussions had together. We also thank Stefano Olivares and Bassano Vacchini with whom we have often reasoned about quantum physics, including issues discussed in this book. We express profound gratitude to all four of them, friends and colleagues of ours at the Physics Department “Aldo Pontremoli” of the University of Milan. Of course, any errors or inaccuracies are entirely our responsibility.

Moreover, this book would not have been possible without the entire staff at Springer-Nature. Our heartfelt thanks especially to Marina Forlizzi, for her professionalism, courtesy, and friendliness. Our thanks extend to Sridevi Purushothaman and Jegadeeswari Diravidamani for their great support and kindness. Last but certainly not least, we acknowledge to Marisa Michelini (series editor of “Challenges in Physics Education”), whose suggestions have significantly enhanced the approach to writing this book.

Milan, Italy  
January 2024

Marco Giliberti  
Luisa Lovisetti

# Prelude

*An indispensable hypothesis, even though still far from being a guarantee of success, is however the pursuit of a specific aim, whose lighted beacon, even by initial failures, is not betrayed.*<sup>1</sup>

Max Planck

In the last decades of the nineteenth century, classical physics seemed to rest on absolutely solid foundations consisting of Newton's mechanics for the motion of macroscopic bodies, and on Maxwell's electromagnetic theory for radiation and manifestations of its interactions with matter.

On 25 October 1871, in the inaugural lecture as first Cavendish Professor in Cambridge, James Clerk Maxwell (Edinburgh, 1831–Cambridge, 1879) reported the current opinion and wrote that:

This characteristic of modern experiments—that they consist principally of measurements—is so prominent, that the opinion seems to have got abroad, that in a few years all the great physical constants will have been approximately estimated, and that the only occupation which will then be left to men of science will be to carry on these measurements to another place of decimals.<sup>2</sup>

However, as a profound thinker that he was, he immediately warned of the danger of this mental attitude by adding just below:

If this is really the state of things to which we are approaching, our Laboratory may perhaps become celebrated as a place of conscientious labour and consummate skill, but it will be out of place in the University, and ought rather to be classed with the other great workshops of our country, where equal ability is directed to more useful ends. [...] But the history of science shews that even during that phase of her progress in which she devotes herself to improving the accuracy of the numerical measurement of quantities with which she has long been familiar, she is preparing the materials for the subjugation of new regions, which

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<sup>1</sup> Planck M (1967) *The Genesis and Present State of Development of the Quantum Theory*. Nobel Lectures, Physics 1901-1921. Elsevier Publishing Company, Amsterdam, p. 407.

<sup>2</sup> Clerk Maxwell J (1871) *Introductory Lecture on Experimental Physics*, October 25, 1871. Macmillan, London & Cambridge, p. 9.



would have remained unknown if she had been contented with the rough methods of her early pioneers.<sup>3</sup>

Twenty-three years later, in 1894<sup>4</sup>, Albert Abraham Michelson (Strelno, 1852–Pasadena, 1931)—risen to prominence in 1887 thanks to what is commonly called *the most famous failed experiment in history* (namely, the Michelson-Morley experiment)—said that again, with similar words:

It is never safe to affirm that the future of physical science has no marvels in store which may be even more astonishing than those of the past; but it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. [...] An eminent physicist has remarked that the future truths of physical science are to be looked for in the sixth place of decimals.<sup>5</sup>

Also Max Planck (Kiel, 1858–Göttingen, 1947), in a 1924 lecture, recalled that:

When I began my physical studies [in Munich in 1874] and sought advice from my venerable teacher Philipp von Jolly [...]. He described physics to me as a highly developed, almost fully mature science, which now, after it had been crowned with the discovery of the principle of conservation of energy, would probably soon have assumed its final stable form. There might still be a speck or a small bubble in one corner or another to be examined and classified, but the system as a whole is fairly secure, and theoretical physics is noticeably approaching that degree of perfection that geometry, for example, has had already for centuries.<sup>6</sup>

At the time, the strong faith in the universal validity of classical physics was further deeply reinforced by the great successes of its technological applications, which highlighted a turning point in the industrial revolution of the second half of the nineteenth century. However, we must not confuse the faith in the laws of classical physics with the idea that the physicists of the second half of the nineteenth century held a Newtonian mechanistic vision. In fact, the development of electromagnetism had prompted the emphasis on the idea of a *luminiferous* ether to which properties were ascribed which, it was hoped, could also explain other fundamental phenomena, such as gravitation, for example<sup>7</sup>. Was electromagnetism, in some sense, more fundamental than mechanics?

In addition, the recent thermodynamic discovery of the conservation of energy had led some renowned scientists—in *primis*, the Nobel Prize for Chemistry Wilhelm

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<sup>3</sup> Clerk Maxwell J (1871) Introductory Lecture on Experimental Physics, October 25, 1871. Macmillan, London & Cambridge, p. 9.

<sup>4</sup> Michelson A (1894) Some of the objects and methods of physical science. Quarterly Calendar 3(2). University of Chicago, Chicago, pp. 12-15.

<sup>5</sup> Michelson A (1894) Some of the objects and methods of physical science. Quarterly Calendar 3(2). University of Chicago, Chicago, p. 15 [the same passage is reported also in: Michelson A (1896) The department of Physics. Introductory. Annual register, July 1895 – July 1895. The University of Chicago Press, Chicago, p. 159].

<sup>6</sup> Planck M (1924) Vom Relativen zum Absoluten: Gastvorlesung, gehalten in der Universität München am 1. Dezember 1924. S. Hirzel, Lipzeig, pp. 3-4.

<sup>7</sup> Kragh H (1999) Quantum Generations: A History of Physics in the Twentieth Century. Princeton University Press, Princeton, NJ, p. 4.

Ostwald (Riga, 1853–Lipsia, 1932)—to consider an “energetic” type program (later failed) in which it was hoped to make the properties of the natural world descend from those of a hypothetical substance-like primordial energy.

Impossible not to think also of the work of Ernst Mach (Brno, 1838–Haar, 1916), masterfully exposed in his 1883 famous book *The Science of Mechanics: a critical and historical account of its development*<sup>8</sup>, aimed at a reformulation of mechanics.

It is in this conceptual framework that the more or less revolutionary proposals that we are going to discuss have to be inserted in order to understand their difficulty in being accepted by the physics community.

In summary, the situation was not as peaceful as it seemed, and new difficulties were also emerging in classical physics, requiring new horizons to be opened. Some subtle elements of contradiction, concerning various experimental facts that classical physics was unable in any way to frame in its theoretical structure, were crucial in cracking these sedimented and apparently steadfast certainties...

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<sup>8</sup> Mach E (1883) *Die Mechanik in ihrer Entwicklung: historisch-kritisch dargestellt*. F.A. Brockhaus, Leipzig.

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**Part I**  
**The Twilight and the Dawn: The  
Inadequacy of Classical Physics  
and the Gradual Arose of Quantum Theory**

# Chapter 1

## Black-Body Radiation



*As soon as we step beyond the established boundaries of pure thermodynamic theory, we enter a trackless region confronting us with obstacles which even the most astute of us are almost at a loss to tackle [Wien (1967), p. 275].*

Wilhelm Wien

**Abstract** The problem of theoretically explaining the thermal emission spectrum of a so-called black body had a long and difficult gestation but, in the end, led to astonishing results that revolutionized our understanding of the world. In this chapter, we will start from the analysis of the characteristics of a black body, with the description provided by G. Kirchhoff in the second half of the nineteenth century; then, we will deal with the Stefan-Boltzmann law and, finally, we will discuss the formula obtained by W. Wien, with its limitations.

### 1.1 Physics at the End of the Nineteenth Century

The story we are going to tell is rooted back in ancient times. In fact, the study of the relationship between light and matter and the study of the optical properties of bodies are very ancient. Even before the famous experiments of Isaac Newton (Woolsthorpe-by-Colsterworth, 1642–Kensington, 1727) in the second half of the seventeenth century, it was known that a prism produces a spectrum of colours when crossed by sunlight.

Although many important theories had been previously proposed, it was only at the beginning of the nineteenth century that a solid and consistent wave theory of the light spectrum began to be seriously considered. One of the main contributors was the British polymath Thomas Young (Milverton, 1773–London, 1829),<sup>1,2</sup> who, for instance, was able to attribute the correct order of magnitude to the frequency of each of the visible “colours”. Curiously, despite great experimental evidence,

---

<sup>1</sup> Young (1802).

<sup>2</sup> Young (1804).

Young's work did not arouse particular interest in the physics community of the early 1800s, as it remained substantially confined within the English borders. At the time, Newton was considered an undiscussed authority in the field of optics, and Young was perfectly aware that being in contrast with Newton's ideas meant to be (almost certainly) severely criticized, especially for what concerned the very nature of light (a matter which was considered as fundamental).

I have indeed been accused of insinuating "that Sir Isaac Newton was but a sorry philosopher." But it is impossible that an impartial person should read my essays on the subject of light without being sensible that I have as high a respect for his unparalleled talents and acquirements as the blindest of his followers, and the most parasitical of his defenders [...]. But, much as I venerate the name of Newton, I am not therefore obliged to believe that he was infallible. I see, not with exultation, but with regret, that he was liable to err, and that his authority has, perhaps, sometimes even retarded the progress of science.<sup>3</sup>

However, a few years later, between 1815 and 1816, the French engineer Augustin Jean Fresnel (Broglie, 1788–Ville-d'Avray, 1827) began to deal with experimental optics: by presenting convincing evidence in support of wave theory, he managed to force its acceptance even among the staunchest supporters of the Newtonian (corpuscular) model, spreading the idea of a wave theory of light also across the Channel,<sup>4,5,6,7,8</sup> Although much of his work had been independently anticipated by Young, Fresnel decided to publish his studies at the Paris Academy of Sciences. It should be noted that the relationship between Young and Fresnel was always polite and smooth, even if, in their epistolary correspondence, Young sometimes used a little paternalistic tone, and Fresnel appeared too modest about the priority of his intuitions.<sup>9</sup>

Sir, I beg you to accept the homage that I pay you with a copy of my memoir on diffraction. When I submitted it to the Institute, I did not know your experiences and the conclusions you had drawn from them, and I presented as new the explanations that you had already given a long time ago. I removed them from the printed memoir which I have the honour to send you, and I left there only that of the coloured fringes of the shadows, because I added something to what you had already said on this phenomenon.<sup>10</sup>

Moreover, in the same years, radiations not visible to the human eye were discovered: infrared radiation, by the German-born British astronomer Friedrich Wilhelm Herschel<sup>11</sup> (Hannover, 1738–Slough, 1822) in 1800, and ultraviolet radiation, by the

<sup>3</sup> Young (1855), p. 201.

<sup>4</sup> Fresnel (1816).

<sup>5</sup> Fresnel (1826).

<sup>6</sup> Mémoire sur les couleurs développées dans les fluides homogènes par la lumière polarisée. Présenté à l'Académie, le 30 Mars 1818 [Fresnel (1866), pp. 655–683].

<sup>7</sup> Fresnel (1821), pp. 127–141.

<sup>8</sup> Fresnel (1821), pp. 441–454.

<sup>9</sup> Correspondance d'Augustin Fresnel avec le docteur Thomas Young, et lettres y relatives (1816–1827) [Fresnel (1868), pp. 737–778].

<sup>10</sup> Young (1855), p. 376. Translated by the authors.

<sup>11</sup> Herschel (1800).