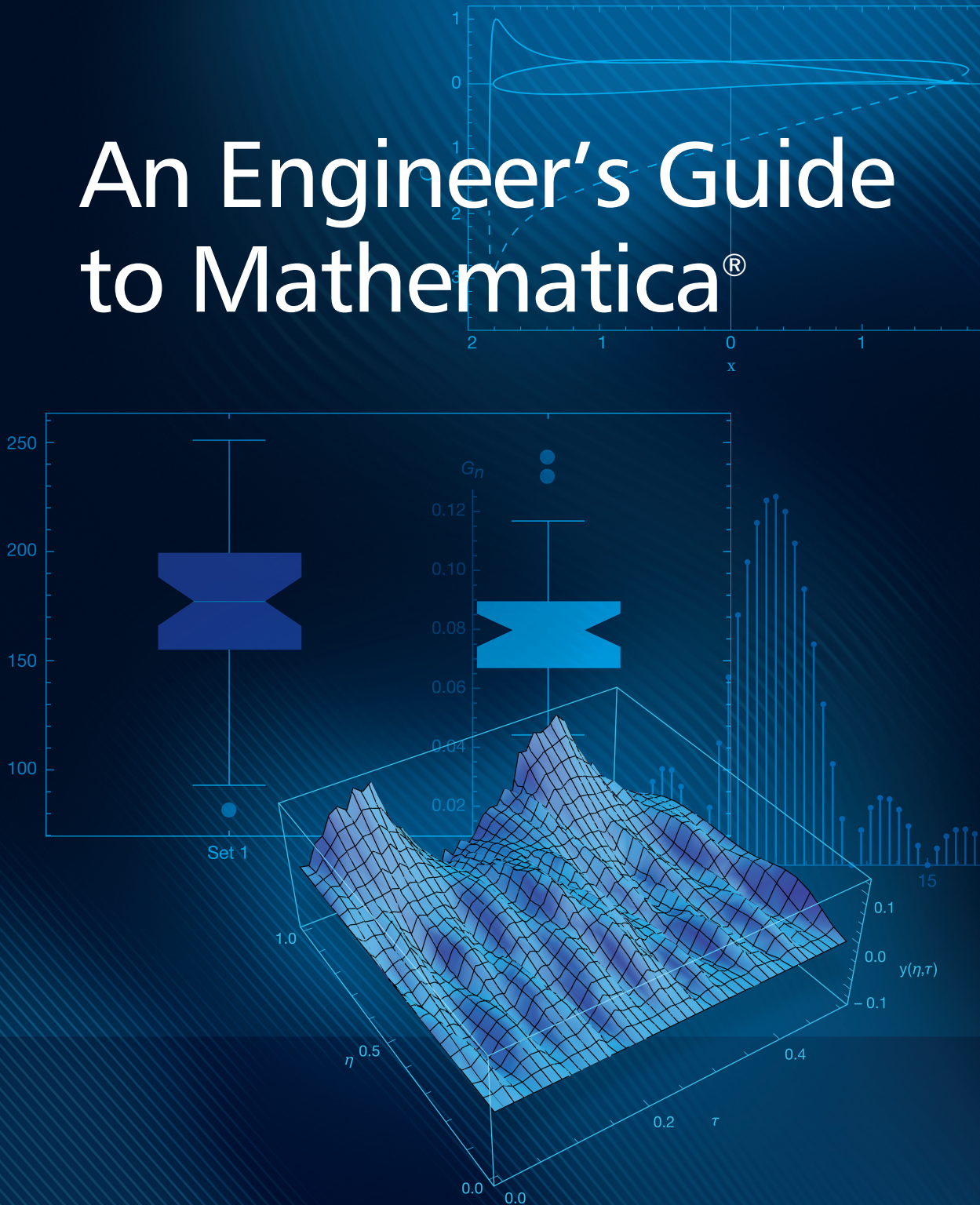


Edward B. Magrab

# An Engineer's Guide to Mathematica®



WILEY



# **AN ENGINEER'S GUIDE TO MATHEMATICA®**



# AN ENGINEER'S GUIDE TO MATHEMATICA®

**Edward B. Magrab**

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**WILEY**

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*For  
June Coleman Magrab*



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# Preface

The primary goal of this book is to help the reader attain the skills to create Mathematica programs that obtain symbolic and numerical solutions to a wide range of engineering topics, and to display the numerical results with annotated graphics.

Some of the features that make the most recent versions of Mathematica a powerful tool for solving a wide range of engineering applications are their recent introduction of new or expanded capabilities in differential equations, controls, signal processing, optimization, and statistics. These capabilities, coupled with its seamless integration of symbolic manipulations, engineering units, numerical calculations, and its diverse interactive graphics, provide engineers with another effective means of obtaining solutions to engineering problems.

The level of the book assumes that the reader has some fluency in engineering mathematics, can employ the engineering approach to problem solving, and has some experience in using mathematical models to predict the response of elements, devices, and systems. It should be suitable for undergraduate and graduate engineering students and for practicing engineers.

The book can be used in several ways: (1) to learn Mathematica; (2) as a companion to engineering texts; and (3) as a reference for obtaining numerical and symbolic solutions to a wide range of engineering topics involving ordinary and partial differential equations, optimization, eigenvalue determination, statistics, and so on.

The following aids have been used to make it easier to navigate the book's material. Different fonts are used to make the Mathematica commands and the computer code distinguishable from text. In addition, since Greek letters and subscripts can be used in variable names, almost all programs have been coded to match the equations being programmed, thereby making portions of the code more readable. In the first chapter, the use of templates is illustrated so that one can easily create variables with Greek letters and with subscripts. Lastly, since Mathematica is fundamentally different from computer languages usually employed by engineers, the introductory material attempts to make this transition as smooth as possible.

In many of the chapters, tables are used extensively to illustrate families of commands and the effects that different options have on their output. From these tables, the reader can determine at a glance which command and which options can be used to satisfy the current objective. The order of the material is introduced in such a way that the complexity of the examples can be increased as one progresses through the chapters. Thus, the examples range from the ordinary to the challenging. Many of the examples are taken from a wide range of engineering topics. To supplement the material presented in this book, many specific references are made throughout the text to Mathematica's *Documentation Center*, which provide numerous guides and tutorials on topical collections of commands.

The book has two interrelated parts. The first part consists of seven chapters, which introduce the fundamentals of Mathematica's syntax and a subset of commands useful in solving engineering problems. The second part makes extensive use of the material in these seven chapters to show how, in a straightforward manner, one can obtain numerical solutions in a wide range of engineering specialties: vibrations, fluid mechanics and aerodynamics, heat transfer, controls and signal processing, optimization, structures, and engineering statistics. In this part of the book, the vast majority of the solutions are presented as interactive graphics from which one can explore the results parametrically.

In Chapter 1, the basic syntax of Mathematica is introduced and it is shown how to intermingle symbolic and numerical calculations, how to use elementary mathematical functions and constants, and how to create and manipulate complex numbers. Several notational programming constructs are both illustrated and tabulated and examples are given on how to attach physical units to numerical and symbolic quantities. The basic structure of the notebook interface and its customization are presented. In addition, the various templates that can be used to simplify the integration of Greek letters, superscripts and subscripts, and other mathematical symbols into the programming process, and the commands that represent many basic mathematical functions and mathematical constants are illustrated.

In Chapter 2, the commands that can be used to create lists are discussed in detail and their special construction to form vectors and matrices composed of numerical and/or symbolic elements that are commonly employed to obtain solutions engineering applications are introduced. The use of vectors and matrices is discussed in two distinctly different types of applications: to perform operations on an element-by-element basis or to use them as entities in linear algebra operations.

In Chapter 3, ways to create functions, exercise program control, and perform repetitive operations are discussed. The concept of local and global variables is introduced and its implications with respect to programming are illustrated.

In Chapter 4, two types of symbolic manipulations are illustrated. The first is the simplification and manipulation of symbolic expressions to attain a compact form of the result. The second is to perform a mathematical operation on a symbolic expression. The mathematical operations considered are: differentiation, integration, limit, solutions to ordinary and partial differential equations, power series expansion, and the Laplace transform.

In Chapter 5, several Mathematica functions that have a wide range of uses in obtaining numerical solutions to engineering applications are presented: integration, solution to linear and nonlinear ordinary and partial differential equations, solution of equations, determination of the roots of transcendental equations, determining the minimum or maximum of a function, fitting curves and functions to data, and obtaining the discrete Fourier transform.

In Chapter 6, a broad range of 2D and 3D plotting functions are introduced and illustrated using numerous tables and examples from engineering topics. It is shown how to display discrete data values and values obtained from analytical expressions in different ways; that is, by displaying them using logarithmic compression, in polar coordinates, as contours, or as surfaces. The emphasis is on the ways that the basic figure can be modified, enhanced, and individualized to improve its visual impact by using color, inset figures and text, figure titles, axes labels, curve labels, legends, combining figures, filled plot regions, and tooltips.

In Chapter 7, the creation and implementation of interactive graphics and animations are introduced and discussed in detail and illustrated with many examples. The control devices

that are considered are the slider/animator, slider, 2D sliders, radio buttons, setter buttons, popup menus, locators, angular gauges, and horizontal gauges.

In Chapter 8, the response of single and two degree-of-freedom systems and thin elastic beams are determined when they are subject to various loadings, damping, initial conditions, boundary conditions, and nonlinearities.

In Chapter 9, the commands used to determine the mean, median, root mean square, variance, and quartile of discrete data are presented and the display of these data using histograms and whisker plots are illustrated. It is shown how to display the results from a regression analysis using a probability plot, a plot of the residuals, and confidence bands. The ways to perform an analysis of variance (ANOVA) and to setup and analyze factorial designs are introduced with examples.

In Chapter 10, the modeling and analysis of control systems using transfer function models and state-space models are presented. It is shown how to connect system components to form closed-loop systems and to determine their time-domain response. Examples are given to show how to optimize a system's response with a PID controller and any of its special cases using different criteria. The creation and use of different models of high-pass, low-pass, band-pass, and band-stop filters are presented and the effects of different types of windows on the short-time Fourier transform are illustrated. The spectral analyses of sinusoidal signals in the presence of noise are presented using root mean square averaging and using vector averaging.

In Chapter 11, several topics in heat transfer and fluid mechanics are examined numerically and interactive environments are developed to explore the characteristics of the different systems. The general topic areas include: conduction, convection, and radiation heat transfer, and internal and external flows.

Edward B. Magrab  
*Bethesda, MD*  
USA  
*October, 2013*



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# **Part I**

## **Introduction**



# 1

## Mathematica<sup>®</sup> Environment and Basic Syntax

### 1.1 Introduction

Mathematica is a programming language that integrates, through its notebook interface, symbolic and numerical computations, visualization, documentation, and dynamic interactivity. It provides access to a large collection of such diverse and continually updated and expanded data sets as geometric shapes, a searchable dictionary, and individual country attributes. It also permits one to simultaneously program with different programming paradigms, such as procedural, functional, rule-based, and pattern-based. Its interface has a real-time input semantics evaluator that uses styling and coloring to provide immediate visual feedback on such coding aspects as function names, variable selection, and argument structures. Many of the Mathematica functions used for computation and visualization contain a fair amount of high-level automation so that the user has to interact minimally with their inner workings. If desired, many aspects of the automation procedures can be bypassed and specific choices can be selected.

In this book, we shall employ a subset of Mathematica's library of functions and use them to obtain solutions to a variety of engineering applications. It will be found as one becomes more confident with Mathematica that it is most effectively used interactively. In later chapters, emphasis will be placed on displaying the results as dynamically interactive graphical displays so that real-time parametric investigations can be performed.

In this chapter, we shall introduce the fundamental syntax of Mathematica. In Chapters 2 to 7, we shall introduce additional syntax and illustrate its usage. We start by stating that all variables by default are symbols and global in nature, and unless specifically restricted or cleared, are always available in all open notebooks until Mathematica is closed. Also, because Mathematica treats all variables initially as symbolic entities, any undefined symbol appearing in an expression (that is, any variable appearing on the right-hand side of an equal sign) is perfectly acceptable and will not produce an error message. However, depending on how the expression is used, subsequent operations may not perform as expected depending on the intent for this variable.

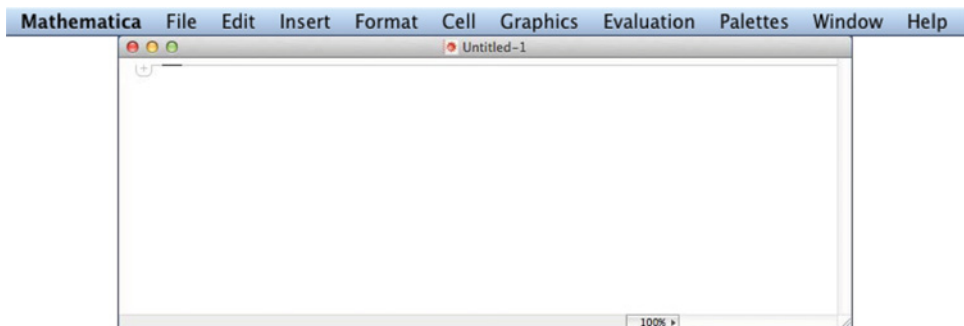
In addition to the functions that are an integral part of Mathematica, each version of Mathematica comes with what are called standard extra packages that provide specific additional functionality. Frequently, the capabilities of these packages become an integral part of Mathematica. What the names of these packages are and a brief description of what they do can be obtained by entering *Standard Extra Packages* into the search area of the *Documentation Center Window*, which is found in the *Help* menu. Each package is loaded by using the **Needs** function. One such case is illustrated in Example 4.11.

## 1.2 Selecting Notebook Characteristics

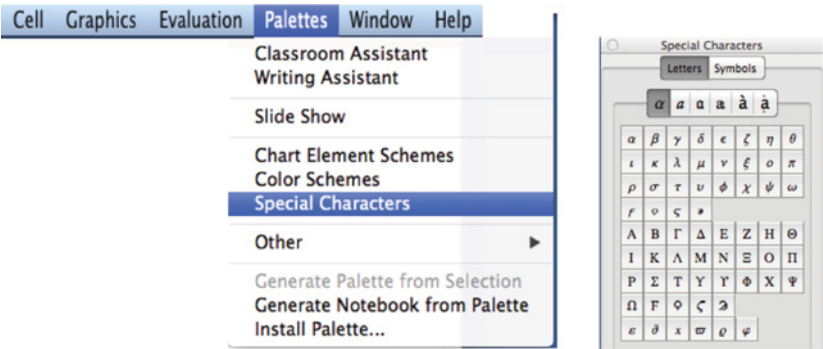
Interaction with Mathematica occurs through its notebook interface. As we shall be concerned primarily with presenting graphically solutions to engineering analyses, our discussion will be directed to one type of use of the notebook: entering, manipulating, and numerically evaluating equations typically encountered in engineering.

Upon opening Mathematica, the window shown in Figure 1.1 appears on the computer screen. Since virtually all types of mathematical symbols can appear in Mathematica expressions, it is beneficial to also have its *Special Characters* palette open. As indicated in Figure 1.2, the letters and symbols are accessed by selecting *Palettes* from the Mathematica menu strip and then choosing *Special Characters*. These operations produce the windows shown in Figure 1.2.

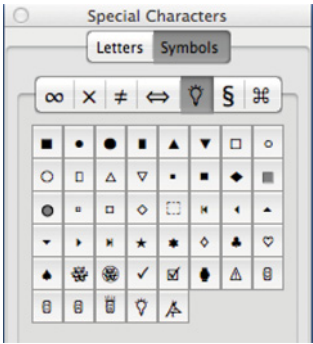
To increase or decrease the font size of the characters displayed in the notebook, *Window* from the Mathematica menu strip is selected, then *Magnification* is chosen, and the amount of magnification (or reduction) is clicked. These operations are illustrated in Figure 1.3. As shall be discussed in what follows, various types of expression delimiters are used in constructing expressions: parentheses, brackets, and braces. When nested expressions are employed and various combinations of these delimiters are used, one frequently needs to verify that these delimiters are grouped as intended. A tool that performs this check by highlighting the region that appears between the delimiter selected and its closing delimiter is accessed from the *Edit* menu and then by clicking on *Check Balance*, as shown in Figure 1.4. In Mathematica 9, the placement of the cursor adjacent to either an opening or closing delimiter will highlight them in green. This is a very valuable editing tool; however, it can be disabled by going to *Preferences* in the *Mathematica* menu strip, selecting *Interface*, and then deselecting *Enable dynamic*



**Figure 1.1** Window appearing upon opening Mathematica

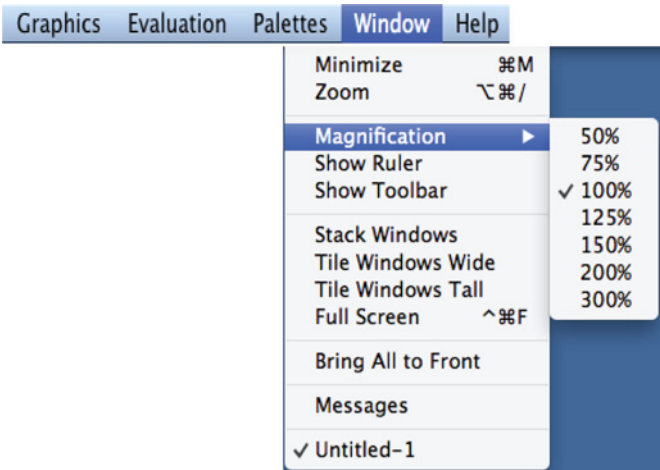


(a)

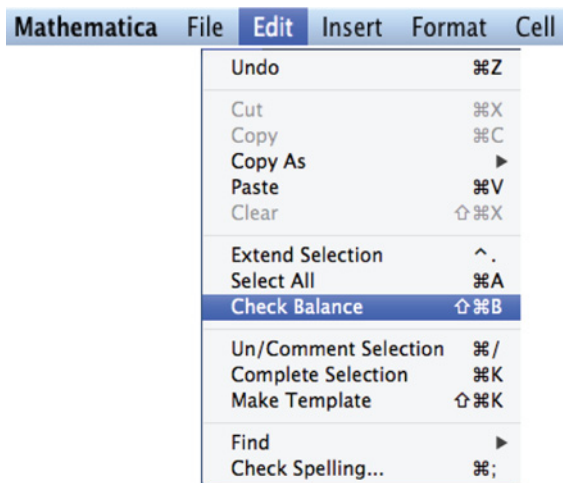


(b)

**Figure 1.2** (a) Opening the *Special Characters* window to select various alphabet symbols; (b) Accessing various types of symbols; shown here are shapes that can be used as plot markers



**Figure 1.3** Setting the notebook font size

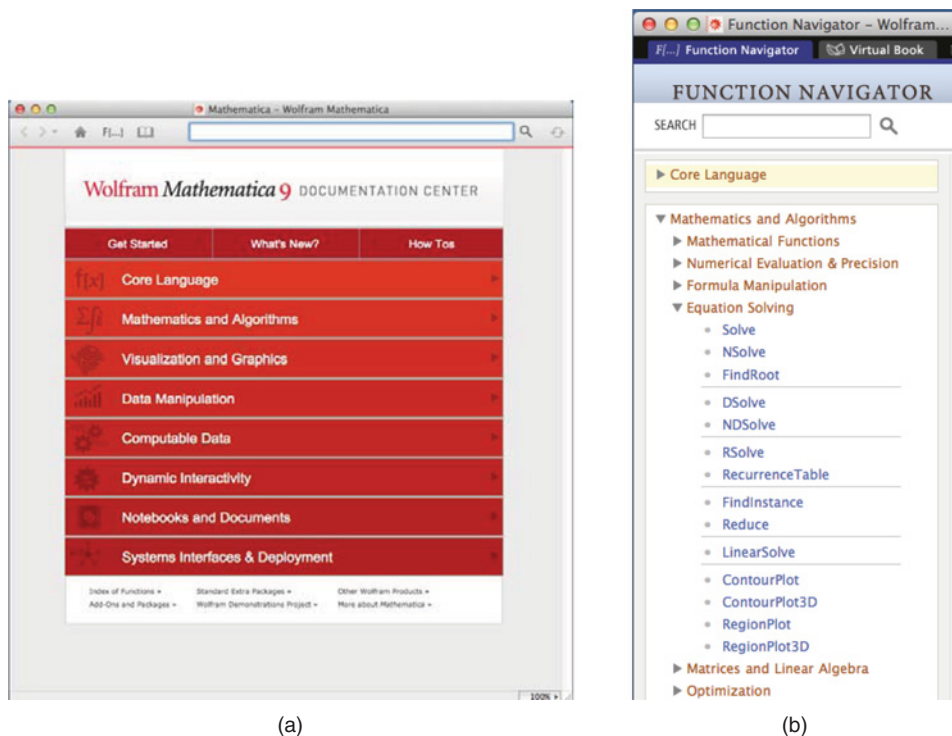


**Figure 1.4** Selecting *Check Balance* for implementation of delimiter region identification for (...), [...], and {...}

*highlighting*. Just below *Check Balance* is another useful tool: *Un/Comment Selection*. This feature comments out text highlighted or removes the comment symbols if the selected text had been commented out. The commenting is produced by the system by placing the highlighted text between the asterisks of the set (\*...\*). (See also Table 1.2.)

Since Mathematica has such a large selection of functions to choose from and since the arguments and their individual form and purpose vary, one should keep the *Documentation Center* window and/or the *Function Navigator* window open for easy access to descriptions of these functions. The *Documentation Center* window is accessed by selecting *Help* from the Mathematica menu strip and then selecting *Documentation Center*. The *Function Navigator* is accessed either by selecting *Function Navigator* from this same menu or by selecting the fourth icon from the left at the top of the *Documentation Center*'s menu strip, which is labeled *F[...]*. Performing these operations, the windows shown in Figure 1.5 are obtained. Entering either the function name or several descriptive words in the *Documentation Center* search entry area will bring up the appropriate information. In the *Function Navigator*, one will see the candidate functions by selecting the appropriate topic. Using the search function in the *Function Navigator* is the same as using the search function in the *Documentation Center* window; that is, the results appear in the *Documentation Center* window.

After some proficiency has been attained with Mathematica, one can also access the types of functions available for certain tasks and what their arguments are from the *Basic Math Assistant*. The *Basic Math Assistant* is accessed from the *Palettes* menu as shown in Figure 1.6. Visiting the region labeled *Basic Commands*, one can find what arguments are required for many commonly used Mathematica functions. The functions are grouped into seven areas as indicated by the seven tabs. The two rightmost tabs refer to plotting commands. There are two other programming aids that have been added in Mathematica 9. They are the *Next Computation Suggestions Bar* and the *Context-Sensitive Input Assistant*; these are discussed in Section 1.3.



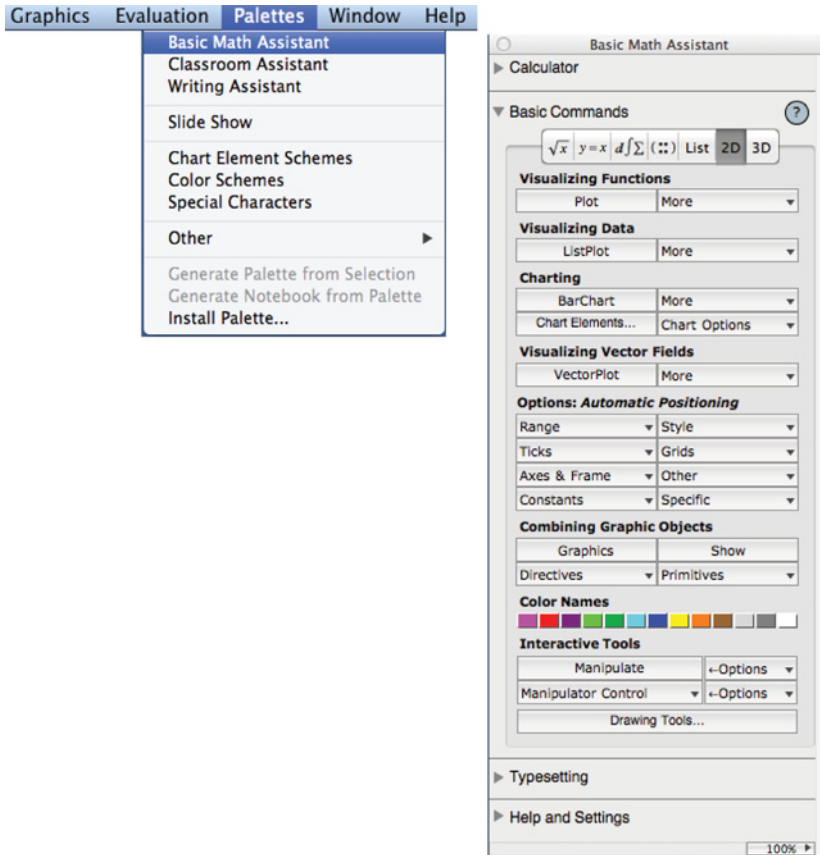
**Figure 1.5** (a) *Documentation Center* window and (b) *Function Navigator* window

The *Documentation Center* window also provides access to tutorials on various topics concerning the usage of classes of functions and also has a page that summarizes a collection of functions that can be applied to solve specific topics. Listed in Table 1.1 are selected search entries that can be used as a starting point in determining what is available in Mathematica for obtaining solutions to a particular topic or class of problems. In addition, entering *tutorial/VirtualBookOverview* in the *Documentation Center* search box provides a table of contents to a “how to” introduction to the Mathematica language and contains a very large number of examples illustrating the options available for a specific function.

Lastly, the appearance of the code and the numerical results displayed in the notebook can be altered by selecting *Preferences* in the *Edit* menu. In the *Preferences* window, the *Appearance* tab is chosen and the appropriate tab is selected. For example, the default value of the number of decimal digits to be displayed is 6. To change this value, one goes to the *Numbers* tab and then to the *Formatting* tab. In the box associated with *Displayed precision*, the desired integer value is entered.

## Creating New Notebooks or Opening Existing Notebooks

To create a new notebook, one clicks on *File* on the Mathematica menu strip and selects *New* and then *Notebook*. A new notebook window will appear. To open an existing notebook, one clicks on *File* on the Mathematica menu strip and selects *Open* or *Open Recent*. Selecting



**Figure 1.6** Opening the *Basic Math Assistant* window to access the 2D palette of plotting commands

*Open* will bring up a file directory window, whereas *Open Recent* will bring up a short list of the most recently used notebooks.

### Saving Notebooks

To save a notebook that was created during a Mathematica session, one clicks on *File* on the Mathematica menu strip and selects *Save As...*. This brings up a file directory from which an appropriate directory is selected and a notebook name is entered. This procedure is also used for renaming an existing notebook. For an existing notebook that has been modified and the existing notebook name is to remain the same, one clicks on *File* on the Mathematica menu strip and selects *Save*.

## 1.3 Notebook Cells

To execute an expression or a series of expressions, one has two ways to do it. To execute each expression separately, one types the expression and then simultaneously depresses *Shift*