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Study Guide for Economic Growth and Development

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Chapter 2
Overlapping-Generations Model of Economic Growth

Reader's Guide

Section 2.1 *Firms and Production*

Purpose: Link the productivity of labor and capital to the income that those inputs generate.

Sticking Points: (i) Where do the conditions for profit maximization, (2.2a) and (2.2b), come from? Economists derive the "best plans" for all economic agents by solving *mathematical optimization problems*. If you are curious about how this works, skim section A.2 of the Technical Appendix at the end of the book

(ii) More importantly, you need to understand the rules for profit-maximization at an intuitive level. It may help to relate the discussion in the text back to your microeconomics principles course. To do this, studying Question 3 and its solution will be helpful.

Take Away: Accumulating physical capital per worker (k) makes workers more productive and increases their wages—the primary source of a household's living standard.

Section 2.2 *Households*

Purpose: Show how household savings provides the physical capital available for firms to use in production.

Sticking Points (i) In actual economies, most households save by buying financial assets such as stocks, bonds, and bank accounts (although in many developing economies these assets do not exist and households are forced to save by purchasing physical capital directly). These financial assets are just claims on physical capital. For simplicity, we bypass the financial middle-men (intermediaries such as banks and brokers) that issue financial assets and assume that households purchase physical capital directly.

(ii) There is a subtle timing issue that has an important implication. Households save and accumulate capital over their working lives and then rent the capital out to firms during retirement to finance their consumption in old age. This means, in any period, the number of capital *owners* (last period's workers or this period's retirees) may be different than the number of workers that *use* the capital in production. If the population is growing fast (n is high), then it becomes difficult for the economy to raise capital per worker (k).

(iii) Where do the optimal consumption and saving equations (2.5a, 2.5b, and 2.5c) come from? Same answer as for the profit maximizing equations of the firm in section 2.1. We assume that households, just as with firms, are trying to do the best they can when making choices. This means we can find the behavioral implications by solving a mathematical optimization problem where households choose their plans to maximize lifetime utility subject to a budget constraint. The details are in the Technical Appendix but you primarily need to grasp the basic intuition as revealed in questions such as Question 7.

Take-Away: The supply of physical capital in an economy is determined by household savings which is a fraction of lifetime wage income.

Section 2.3 Competitive Equilibrium

Purpose: To explore the underlying dynamics of the economy that leads to growth and development.

Sticking Points: (i) To understand economic growth, you need to go beyond the standard one period snapshot of a static equilibrium (see Fig 2.1). We have to dig deeper into the capital market equilibrium condition that says the capital-labor ratio this period was determined by the saving plans of workers last period: $k_t = \frac{\beta}{1+\beta}\frac{w_{t-1}}{n} = \frac{\beta}{1+\beta}\frac{(1-\alpha)Ak_{t-1}^{\alpha}}{n}$. It is the fact that last period's capital-labor ratio determines wages, a portion of which is saved, that makes a dynamic connection: $k_{t-1} \to k_t$ or in words: previous period's capital → worker productivity→ wages → retirement saving → this period's capital.

(ii) The diagram (Fig 2.2) that sketches the dynamic relationship given by the transition equation (2.8) is the foundation of much of the modeling that follows in the course. You need to understand Fig. 2.2 completely.

Take-Away The transition equation exhibited in Fig 2.2 reveals three important implications.

(i) An economy's growth is relatively rapid when it is far from its long-run potential (steady state)

(ii) An economy's growth rate slows as it builds up capital or "industrializes."

(iii) Growth will stop when the economy reaches the highest capital-labor ratio it can maintain.

Sections 2.3.1-2.3.2 Growth Analytics

Purpose: To develop the analytical skills needed to use the transition equation and Fig 2.2, just as you did with the demand and supply models of introductory economics.

Sticking Points Sections 2.3.1 and 2.3.2 are crucial for understanding how the growth process is altered by various events. We have seen how the sketch of the transition equation can depict economic growth through the accumulation of private capital. The sketch can also be used to analyze how various events impact the growth process. There are two general categories of such events: *changes in the fundamental structure* of economies, as captured by the parameters β, A, and n, and *discrete shocks* to the values of K and N due to events not captured by the gradual growth process. It is important to see how each category of events affects economic growth in the short-run and the long-run.

Take-Away The results of the growth analytics can be summarized as follows. Be able to explain the effects of each type of shock.

	Effect on Worker Productivity			
Positive Shock in	*Level*		*Growth rate*	
	Short-run	Long-run	Short-run	Long-run
β	higher (period after shock)	higher	higher	no effect
A	higher	higher	higher	no effect
n	lower (period after shock)	lower	lower	no effect
K	higher	no effect	higher then negative	no effect
N	lower	no effect	lower then positive	no effect

Section 2.3.3 Technical Progress

Purpose: The current version of the model predicts that economic growth will converge to zero. We have had positive economic growth for over two centuries—too long for the prediction to be plausible. *Technical knowledge*—new ideas about production and machine design must be added to allow for long-run steady state growth.

Sticking Points (i) Our modeling of technical progress is based on the concept called the *efficient labor input* (H) which adjusts the actual work force (M) with an index that captures how technology makes a given worker more productive (D).

(ii) With the new concept of labor input, we can proceed just as before if we redefine k as the ratio of physical capital to the effective workforce. This new definition reveals one drawback of technical progress—as new ideas and types of machines make workers more productive, an economy will find it more difficult to supply workers with the needed quantity of capital.

(iii) Technical progress alters how the growth rate in worker productivity evolves. Pay special attention to the last two equations in this section.

Take-Away: We now have two sources of economic growth: a transitory *endogenous* source that we can fully explain (k) and a permanent *exogenous* source that is not explained within the model (D).

Section 2.4 Testing the Model

Purpose: With the addition of technical progress, the model seems to yield reasonable predictions. But just how accurate are the predictions when compared to real world data? And how much growth can be explained by the endogenous component, physical capital accumulation? This section answers these questions by using the model in an attempt to replicate historical growth in the US.

Sticking Points: (i) The model is calibrated to generate the observed growth of worker productivity in the US from 1870 to 1990. So what's the test if the simulation is rigged to capture the total growth over the period? The test is to see if the model predicts the correct *pattern* of growth rates and interest rates over time. Both growth rates and interest rates showed little trend during most of the 20th century.

(ii) Problem 19 gives the details of how the simulation is constructed. Jumping right into Problem 19 might be a bit much. Problem 18 eases you into how the simulation is done.

Take-Away: The model misses the mark on the pattern of growth rates and interest rates—the model predicts that they both should have declined much more than they did over the historical period. The main conclusion is that physical capital accumulation per worker can only explain a relatively small part of historical growth.

Section 2.5 Human Capital

Purpose: The historical simulation revealed that sources other than physical capital must have contributed to economic growth. In this section we assess whether public education could have significantly increased worker productivity, while maintaining consistency with the data.

Sticking Points (i) A new modeling adjustment generalizes the concept of effective labor supply using equation (2.15) which now includes the effects of student time and resources devoted to public education. Notice that if public education investments do not raise worker productivity then $\theta_1 = \theta_2 = 0$ and the effective labor input is solely determined by the technology index as we assumed previously.

(ii) We can rewrite (2.15) as $H_t = D_t M_t h_t$, with $h_t = \left(x_{t-1} / D_t\right)^{\theta_1} e_{t-1}^{\theta_2}$ as the new modeling component that picks up the effect of public education on worker productivity. Equipped with estimates of θ_1 and θ_2, along with the data on public school investments given in Table 2.2, we can redo the historical simulation in the same fashion as before but now with three sources of growth (k_t, h_t, D_t).

Take-Away When we attempt to explain half of the historical growth with the contribution of both k_t and h_t, the predictions of the model match the trendless growth rates and interest rates observed over the 20th century. We now have a good explanation for half of the historical growth in the US. Other growth investments are added in Chapter 3 to make further progress. Later in the text, we identify gains in worker productivity that result from improved efficiency in the allocation of resources.

2.6 Intergenerational Transfers

Purpose: A theory of human capital accumulation naturally begins with parents that have a preference to improve the life-chances of their children by investing in their skills.

Sticking Points: (i) The altruistic motive for investing in children leads to a rather complex formulation of household behavior. It allows one to go from an overlapping generations model to an infinitely-lived agent model, linking the two workhorse models of macroeconomics. The infinitely-lived agent model has advantages (see Appendix B of this chapter) and one must at least be familiar with it to read the macroeconomics literature. However, for our purposes it is not essential.

(ii) The 'warm-glow" approach to modeling a preference for intergenerational transfers is simpler than the altruism approach and in some ways is more empirically accurate. It is the approach we use when modeling transfers.

Take-Away: Intergenerational transfers are important for economic growth and macroeconomics in general. For our purposes, using the warm glow motivation for transfers suffices.

Solutions to Exercises

Questions

1. (a) *Technology* generally refers to disembodied ideas about production. In practice, it has two meanings. First, it refers to the entire production function, such as the Cobb-Douglas production function given in (2.1). The production function is a mathematical representation of how production takes place and what inputs are needed. Second, a narrow use of the word focuses directly on how the productivity of labor, via the technology index (D), changes over time as new ideas are discovered and developed. The productivity index is introduced in section 2.3.

More concretely one can think of technology as a collection of ideas about production methods and organization of the firm, machine design that improves function, exogenous aspects of the skills and health of the workforce, and complementary inputs that may not be explicitly modeled (such as seeds, fertilizer, energy inputs, and public infrastructure)

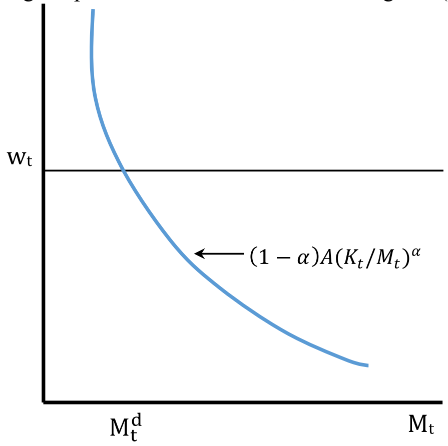
(b), (c), (d) *Capital* refers to the assets used in production. Capital is comprised of physical and human capital. *Physical capital* refers to physical assets such as plant and equipment and, in some applications, land. *Human Capital* refers to the stock of embodied knowledge and skills possessed by the work force that are explicitly modeled. If human capital is not explicitly modeled, then its contribution to production is contained in the productivity index, D.

2. Equations (2.2a) and (2.2b) are conditions that must be satisfied for the firm to be maximizing profit in a competitive market setting where the factor prices are taken as given. The equations say that the marginal benefit of choosing the inputs, represented by the marginal products, is equal to the marginal cost of choosing the inputs, represented by the factor prices.

A tricky feature of these equations is that they seem to give two equations that the capital-labor ratio must satisfy. For given factor prices, the two resulting solutions for the capital-labor ratio will not generally be consistent. So, one cannot think of these equations as being satisfied at the level of the firm for *any* factor prices. Instead, one of the equations must be thought of as determining one of the equilibrium factor prices (w or r). The other factor price will be determined by the condition that the demand for the capital-labor ratio must equal the supply of capital relative to labor supplied by the households. It is intuitive to think of w and k

as determined by (2.2a) and (2.2b), with the price of capital, *r*, determined by a market-clearing condition for the capital market.

3. The marginal product of labor is a downward-sloping function of the employment level, for a given capital stock, due to diminishing marginal productivity. The competitive market wage is not affected by the employment choice of an individual firm. It is represented by a horizontal line. The profit-maximizing employment choice is found where the wage rate and the marginal product curve intersects, indicating that (2.2b) is satisfied.



A larger capital stock shifts the marginal product curve upward. For a given wage rate, this would lead to an increase in the firm's demand for labor, as the profit-maximizing intersection shifts to the right. An increase in *A* does the same. If the market wage increases the quantity of labor demanded decreases (a movement along the labor demand curve).

4. The average product is $\frac{Y_t}{M_t} \equiv y_t = Ak_t^\alpha$ and the marginal product is $\frac{\Delta Y_t}{\Delta M_t} = (1-\alpha)Ak_t^\alpha$.

The two concepts are proportional to each other with the marginal product being smaller by the constant factor $1-\alpha$. The average product is the observable measure of worker productivity. In theory, the marginal product informs the firm about hiring additional workers. As long as the marginal product exceeds the market wage, it makes sense for a firm to hire another worker. For this reason, profit maximization requires that workers be hired until the marginal product is equal to the wage rate.

5. The capital and labor shares are the fractions of total income paid to owners of capital and to workers. With a Cobb-Douglas production function the income shares are the constants α and $1-\alpha$, respectively. Until very recently the evidence supported the prediction of constant income shares with the capital share being approximately 1/3 and the labor share being

approximately 2/3. However, since the turn of the last century, it appears that the capital share has been rising and the labor share has been falling. The changes appear to have ended around 2010, when the shares again remained constant but at new levels. Economists do not yet have a conclusive explanation for this surprising change.

6. The *rental rate* on physical capital is the payment that the firm makes to the capital owner for renting one unit of capital that is used in production. The rental rate is denoted by *r*. The *rate of return* on capital is the rental rate received by the owner minus the depreciation rate, that part of the capital that is lost in production. The rate of return is then $r-\delta$. The interest rate is the rate of return on financial assets. If financial assets exist and are held in equilibrium, the interest rate must equal the return on physical assets. So, the interest rate must also equal $r-\delta$. In most of the analysis of this book, we abstract from any financial assets. Nevertheless, it is common for economists to refer to $r-\delta$ as the "interest rate."

7. (a) A higher wage raises current and future consumption (both are "normal" goods with positive income effects under our assumptions). An increase in future consumption when the wage is higher requires an increase in saving.

(b) In general, the return to capital has an ambiguous effect on current consumption because of conflicting *income* and *substitution* effects. Households rent capital, so a higher return raises lifetime resources, allowing households to afford more consumption in each period of life. However, a higher return also raises the cost of current consumption because every unit of current consumption now means more units of future consumption are forgone. The increased cost causes households to substitute away from the relatively more expensive current consumption in favor of the relatively cheaper future consumption. The strengths of these opposing effects on current consumption are determined by the intertemporal elasticity of substitution, σ (see *Problem* 7). The higher is σ, the greater is the willingness to substitute consumption across time and the more likely it is that the substitution effect dominates the income effect. With $\sigma = 1$, which corresponds to our standard log preferences, these two effects exactly cancel. So there is no effect of the return to capital on current consumption and saving. However, a higher return to capital will increase future consumption because the income and substitution effects are reinforcing in this case.

(c) A higher value of β means households are more patient. Greater patience lowers the value of current consumption relative to future consumption. Thus, current consumption falls in favor of more saving and future consumption.

8. The interest elasticity of saving measures the response of saving to changes in the interest rate. Technically, it is the percentage change in saving divided by the percentage change in the interest rate. Economic theory predicts an ambiguous interest elasticity because of conflicting substitution and income effects—see *Questions* 7 and 9 and *Problems* 6 and 7. For the preferences assumed in the text, the elasticity is zero because the two conceptual effects exactly cancel. This is roughly consistent with the very low empirical measures of the interest elasticity typically estimated in the literature.

9. First, an explanation for the *slope* of the supply of capital. Equation (7) gives the supply of capital that is financed by last period's saving. How saving is related to the interest rate depends on the value of the parameter σ (see *Question* 7 and *Problem* 7). The ambiguous effect of the interest rate on saving is because of two opposing effects. The *substitution effect* says when the interest rate increases, current consumption is relatively more expensive in present value than is future consumption— causing a substitution of less current consumption for more future consumption. This effect causes current consumption to fall and saving to increase with the interest rate. The *income or wealth effect* says that when the interest rate increases all savers (which all young households are in this model) have greater future income or wealth because at any level of saving there is more interest income in the second period. The greater income causes households to increase their demand for current and future consumption, causing saving to fall. Thus, the income effect causes current consumption to increase and saving to fall as the interest rate increases.

The relative strength of the substitution and income effects is determined by the parameter σ. The higher is σ the stronger is the substitution effect and the weaker is the income effect. When σ exceeds one, higher interest rates cause more saving. In this case, the supply of capital has a positive slope. When σ equals one, higher interest rates cause exactly offsetting substitution and income effects, resulting in no change in saving. In this case, the supply of capital matches the sketch of (2.7) in the Figure, a perfectly vertical line because the level of saving is independent of the interest rate. Finally, when σ is less than one, the supply of capital is downward sloping because higher interest rates cause less saving.

The values of *w* and *n* cause the sketch of (2.7) to "shift." An increase in w_{t-1} causes an increase in saving, and next period's supply of capital, for every possible interest rate. This is displayed as a rightward shift in the capital supply curve.