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A Fractional Perspective

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Preface

Cell signaling is, without a doubt, one of the most important parts of contemporary cell biology. All cells, whether they survive as single cells or as part of a multicellular organism, are constantly bombarded by signals in various forms. An organism's ability to sense and react to its environment, or the ability of particular cells within an organism to do so, is essential to its survival.

This text presents specifically calcium signaling, which is a vital and integral part of modern biology. For adaptation to changes in the environment, cells need signals, and signaling requires messengers whose concentration varies with time. Calcium ion (Ca^{2+}) rules cell signaling by playing an essential role in stimulus–response reactions of cells as a second messenger. This is done by keeping cytoplasmic Ca^{2+} concentration low at rest and by mobilizing calcium in response to stimulus, which in turn activates the cellular reaction. The range of cell activities controlled by calcium is wide and varied. Almost every cell type shows the Ca^{2+} signals. In fact, in every single cell, there are colossal signaling pathways that involve Ca^{2+}.

Mathematical models play a crucial role in unfolding the multilayered complex systems of biological systems. Since the intracellular processes are dynamic systems, investigating these by analyzing the mathematical models that depict the behavior of intracellular networks facilitated tremendous advances in this discipline. The dynamic mathematical modeling of biological systems is an abstraction of reality. The intersection of mathematics and biology has resulted in significant breakthroughs in both fields. Mathematics offers a tool for modeling and understanding biological events and biology driving developments in the theory of non-linear differential equations. The ongoing application of mathematics to biology has much potential, and it might be the applied mathematics of the twenty-first century and hence instigated us to write this text.

The journey of this book starts from the study of fractional calculus, which is the calculus of derivatives and integrals of arbitrary order, and later applies it to the study of dynamical modeling in calcium signaling. This text's intended audience is mathematically proficient cell biology students or mathematics students interested in learning about modeling in cell biology. The ideal class would include students from both biology and applied mathematics, who may be encouraged to collaborate on exercises or class projects.

The core objective of the book is to cover the fundamental processes involved in calcium signaling via fractional mathematical modeling. This text will inspire mathematics students to collaborate with experimentalists while also offering the direction needed to explore the modeling literature.

We aim to present the processes involved in calcium signaling. This book consists of four chapters. Chapter 1 (Introduction) that tries to make the reader familiar with the new area of mathematics, i.e., ‘fractional calculus’ along with the operators, methods, and also some more mathematical definitions used as a tool in work presented in upcoming chapters.

To understand the mechanism of calcium signaling, the receptor activation part is a preliminary requisite because the mobilization of intracellular calcium from intracellular stores depends upon the binding of the agonist to the cell surface receptor. Chapter 2 is devoted to understanding the receptor activation by the ligand: Thrombin via the fractional model incorporating the Caputo–Fabrizio fractional operator. Further, this chapter covers the study of the calcium profile in red blood cells (RBCs) with the help of the advection–diffusion equation. After the activation of receptors, the flow of calcium ions occurs from intracellular stores to the cytoplasm that raises the cytosolic calcium concentration. In response, calcium is exchanged from the extracellular space to intracellular reserves, causing the amount of cytosolic calcium to fall and, as a result, the flow of calcium from the extracellular area to the cytosol to resume, as discussed in Chap. 2. Following that, in Chap. 3, we look in depth at the process of buffering, which aids in lowering the calcium concentration in cytosol, which rises due to calcium influx from extracellular space and intracellular storage. The main focus of this chapter is on the variation of cytosolic calcium with various buffers (calcium binding proteins found in the cytosol), which is accomplished by developing a fractional model with the inclusion of a fractional order derivative operator, association and dissociation rates, diffusion coefficients, and buffer concentration.

In Chap. 4, based on the concept of anomalous diffusion, a mathematical model is proposed to characterize the anomalous subdiffusion of cytosolic calcium. Because of the complex structure of cytoplasm (the Golgi apparatus and mitochondria endoplasmic reticulum, sarcoplasmic reticulum imposing a complex reticular network) and cytoskeletal elements such as microtubules and actin filaments, diffusion particles become obstructed. Thus, the traditional diffusion equation based on Fick’s law is incapable of modeling and elaborating the anomalous character of the diffusive mass transport. In this chapter, the subdiffusive behavior of cytosolic calcium is studied in the cardiac myocytes with the help of a fractal derivative model for the anomalous diffusion process. Here, the investigation considers the fractional–fractal formulation of the problem by using two different fractional derivative operators and observing the transient subdiffusive behavior. In biological systems having very few trajectories with a relatively short observation time, the time fractional model is more effective. The primary goal of identifying the transport behavior having a short observation time is achieved by using the fractional model.

The fractional calculus provides a powerful mathematical tool to study dynamical biological systems. In this way, it is a truly interdisciplinary work that we hope will be enjoyed by biology readers interested in theoretical approaches to their topic and mathematicians interested in discovering new areas of application.

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