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Physics of Granular Suspensions

Micro-mechanics of Geophysical Flows

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Marco Mazzuoli · Laurent Lacaze
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Micro-mechanics of Geophysical Flows

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Editors
Marco Mazzuoli
Department of Civil, Chemical
and Environmental Engineering (DICCA)
University of Genoa
Genoa, Italy

Laurent Lacaze
Institut de Mécanique des Fluides de
Toulouse (IMFT)
Université de Toulouse, CNRS
Toulouse, France

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Preface

The book originates from the lectures delivered during a CISM course held in Udine in the Summer of 2023 and is conceived to provide graduate students and young scientists with fundamental knowledge on the mechanics of granular suspensions as well as on the mathematical and numerical techniques that can be adopted to investigate geophysical flows. To this end, three formidably complex problems (sediment transport, flow-like landslide inception and gravity currents) are considered. The reader will find a thorough combination of elements of fluid and solid mechanics, rheology, geotechnics, geomorphology, civil and coastal engineering. The first part of the book is devoted to introducing the problem of granular suspensions from the mathematical viewpoint, focusing on issues that characterise geophysical flows such as turbulence, the effects of inter-particle contacts and strong velocity gradients. In the second part of the book, different models that were successfully used to investigate the mechanics of granular suspension in environmental flows are presented.

Genoa, Italy — Marco Mazzuoli
Toulouse, France — Laurent Lacaze

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Part I
Elements of Granular Suspension Dynamics

Chapter 1
Granular Suspension: From Single Fluid to Two-Phase Particulate Systems

Elisabeth Guazzelli

1.1 What are Granular Suspensions?

Suspensions of particles play an important role in a wide variety of natural phenomena and industrial processes. Familiar examples of suspensions of particles include sediments in rivers or estuaries, raindrops, pastes, biological suspensions (such as blood), paints, ink, and waste waters carrying suspended solids. Suspensions are also present in many technological and industrial processes such as water treatment and filtration, separation in mineral processing, synthesis of composite materials, paper making, to name but a few. The hydrodynamics of suspensions is a relatively old subject, dating back to the middle of the 19th century, with the work of George Gabriel Stokes in particular, where more or less constant activity has been maintained with an increase in the literature on the subject and its applications over the last 20 years or so. The term microhydrodynamics was suggested by George Keith Batchelor around 1970 to define a new field of hydrodynamics for which the characteristic length scale of the flow is between 0.01 and 100 μm, and therefore for which effects that are ignored on larger scales become important. The flow of particles suspended in a viscous fluid is an important part of this subject.

Figure 1.1 shows typical particle sizes L (diameter or length) (this term is taken in a broad sense and includes macromolecules, for example) encountered either in nature or in technological or industrial processes. The figure also shows the sedimentation velocity U, Reynolds number Re, Brownian diffusion coefficient D, and Péclet number Pe, for particles of density 2 $\mathrm{gcm^{-3}}$ sedimenting in water at 20 °C under terrestrial gravity conditions. By an amusing coincidence, a particle of radius 0.5 μm sediments at 0.5 $\mu\mathrm{ms}^{-1}$ and has a Brownian (translational) diffusion coefficient of 0.5 $\mu\mathrm{m}^2\mathrm{s}^{-1}$.

E. Guazzelli (✉)
Université Paris Cité, CNRS, Matiére et Systémes Complexes (MSC), UMR 7057, Paris, France
e-mail: elisabeth.guazzelli@u-paris.fr

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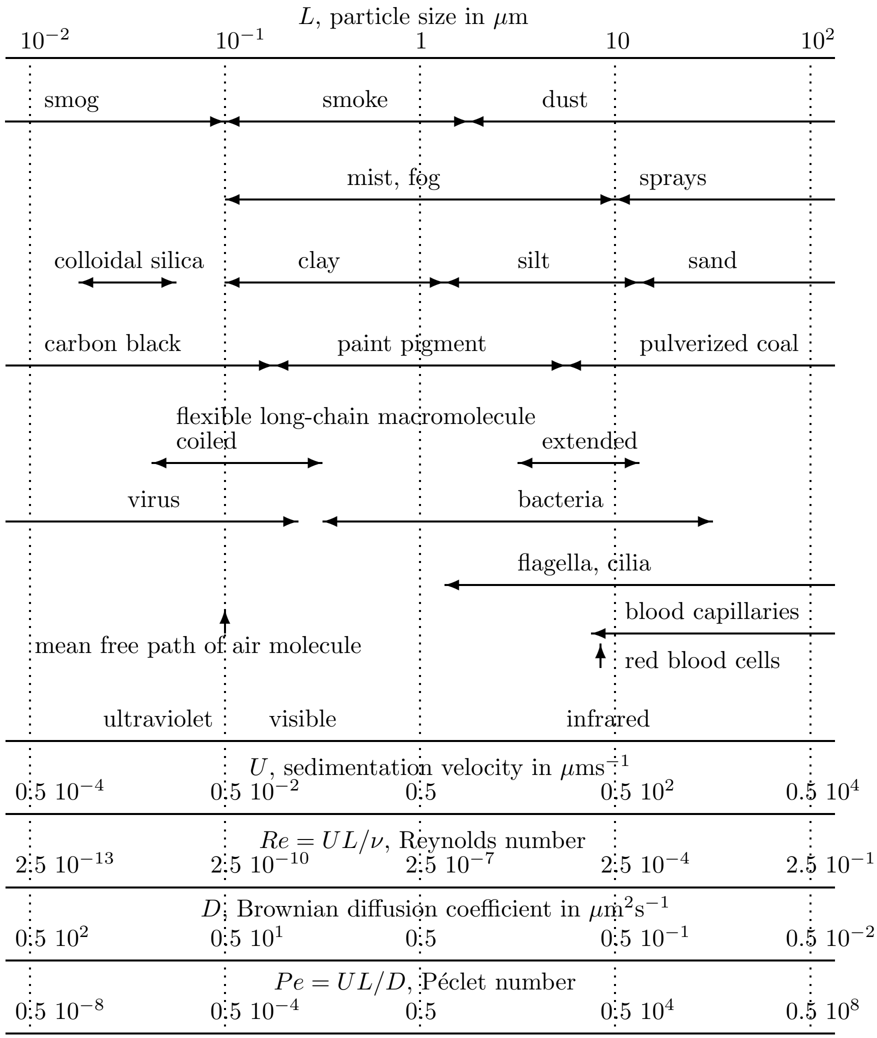

Fig. 1.1 Orders of magnitudes for typical particles after Batchelor (1977)

For particles of size L between 0.01 and 100 μm, the inertial forces are very small compared with the viscous forces, and the Reynolds number $Re = UL/\nu$ (ν is the fluid kinematic viscosity), which gives an estimate of the ratio of these forces, is very small compared with unity. The flows are then governed by the Stokes equations, which have been studied in great detail and which, because of their linearity, have a wider class of solvable problems than the Navier-Stokes equations. For larger particles, inertia may play a significant role and must be accounted for.

When particle size is less than 1 μm, suspensions are called colloidal. Brownian motion caused by thermal agitation of the fluid molecules can then be significant and particle motion is no longer deterministic. The Péclet number that describes the ratio of convective and Brownian motion $Pe = UL/D$ is small in front of unity. Interaction forces between particles, such as attractive van der Waals forces and electrical double-layer repulsive forces, can also be important. This is a direct consequence of the large surface-to-volume ratio for small particles. These inter-particle forces can also be important for larger particles when they are close to each other or to a wall.

For non-Brownian suspensions ($L > 1$ μm), the physics is dominated by the reciprocal effects of driving forces (such as gravity in sedimentation) and hydrodynamics, and fluctuations of thermal origin are negligible. It is no more a state of thermal equilibrium (or a state close to thermal equilibrium) but in fact a state far from equilibrium. The distribution of particle positions (and also of particle orientations in the anisotropic case), i.e. the microstructure of the suspension, is no longer an equilibrium distribution (or close to it) given by the tools of statistical physics, but is determined by the macroscopic flow and also determines it in return, placing questions in the domain of non-equilibrium statistical physics.

In these non-colloidal Stokesian suspensions, it is conventionally assumed that inter-particle interactions are determined solely by the fluid, as the lubrication forces grow strongly with decreasing inter-particle separation to prevent direct contact. However, when the volume fraction of the particles becomes larger (typically above 40%), particle contacts play a significant role when suspensions are submitted to shearing flows. Understanding the contact network interactions becomes thus of fundamental importance in the rheological behaviour of these highly dense suspensions. In dry granular materials, i.e. when the interstitial fluid is a gas which has often unimportant effects, particle contact plays also a dominant role. This dense regime of suspensions has been named 'granular' in reference to the numerous connections that can be made between dense suspensions wherein both hydrodynamics and contact interactions are present and dry dense granular media which are solely controlled by direct contact interactions

This introductory chapter is concerned by granular suspensions consisting of non-Brownian spherical particles suspended in a Newtonian fluid and does not address the problem of colloidal interactions. The basic principles of microhydrodynamics, from the dynamics of a single particle—a prerequisite for the understanding of suspension flows—to hydrodynamic interactions between particles can be found in the first four chapters of Guazzelli and Morris (2012). The present chapter summarize the content of Guazzelli and Pouliquen (2018) and of chapter seven of Guazzelli and Morris (2012) where further detailed information can be found. It is concerned by the rheology of granular suspensions and shows that suspension rheology can be approached in different ways which are complementary and can be chosen depending on the problem to be tackled. The suspension can be seen as an effective fluid but also as a two-phase system comprising a fluid and a particle phase, and even described by a frictional approach inspired by that developed for dense granular flows.

1.2 The Suspension as a Single Effective Fluid

A rigid particle in an ambient flow can have a solid-body motion such that it satisfies the velocity and vorticity conditions of the ambient flow while suspended freely in the flow, i.e. without any external force or torque. A rigid particle, on the other hand, has no mechanism to respond to local deformation motion. So, in an ambient flow, a particle suspended freely without force or torque produces a disturbance that decreases as the square of the distance to its center. This disturbance increases the dissipation of energy and thus adding particles to a fluid produces a higher effective viscosity than that of the pure fluid. Einstein (1906, 1911) uses such dissipation argument to show that the effective viscosity of a dilute suspension of solid particles is $\eta_f(1 + 5\phi/2)$ where η_f is the viscosity of the suspending fluid and ϕ the particle volume fraction. The complete calculation using the volume average of the stress tensor of the whole suspension following Batchelor (1970) can be found in the chapter seven of Guazzelli and Morris (2012).

Restricting the discussion to suspension of non-colloidal, mono-disperse, hard spheres at low Reynolds number, dimensional analysis shows that the viscosity of the suspension relative to that of the suspending fluid, η_s, is independent of the shear-rate $\dot{\gamma}$ and is a sole function of ϕ. The suspension can be seen as a Newtonian fluid with a viscosity increasing with increasing ϕ since adding particles increases dissipation, as seen in Fig. 1.2 which collects some typical rheological measurements performed for non-colloidal hard spheres. The linear dependence given by the Einstein viscosity, $\eta_f(1 + 5\phi/2)$, captures the experimental data only in the very dilute limit (up to $\phi \approx 0.05$). The first effect of particle pair interactions leading to correction of $O(\phi^2)$ (Batchelor & Green, 1972) can predict the semi-dilute limit (up to $\phi \approx 0.10 - 0.15$). For larger ϕ, there are no exact analytic calculations and one must rely on numerical simulations or on phenomenological correlations (Stickel & Powell, 2005) such as the Maron-Pierce correlation represented Fig. 1.2. The difficulty comes from the fact that multi-body hydrodynamic interactions must be computed together with determining the microstructure but also that the particles suffer direct mechanical contact. The contact contribution rapidly increases with increasing ϕ and becomes dominant in the dense regime (above $\phi \approx 0.4$) (Gallier et al., 2014). This is particularly true (with the predominance of the extended network of contacts) close to the jamming transition where the viscosity diverge at ϕ_c. This maximum flowable volume fraction, $\phi_c \approx 0.54 - 0.62$, differs from the random close packing fraction (≈ 0.64) and varies depending on the particle size distribution and surface interactions and more precisely on particle frictional interactions.

The quasi-Newtonian character of the suspension viscosity that we just discussed does not fully capture the suspension rheology as, for non-dilute suspensions, normal stress differences develop, i.e. normal stresses are no longer isotropic. This non-Newtonian behavior is linked to the loss of isotropy in the suspension microstructure under shear when the concentration is increased and is also influenced by frictional particle contacts. The two normal stress differences (N_1 and N_2) describe the non-isotropic nature of the stress tensor. They are linear in the modulus of the shear

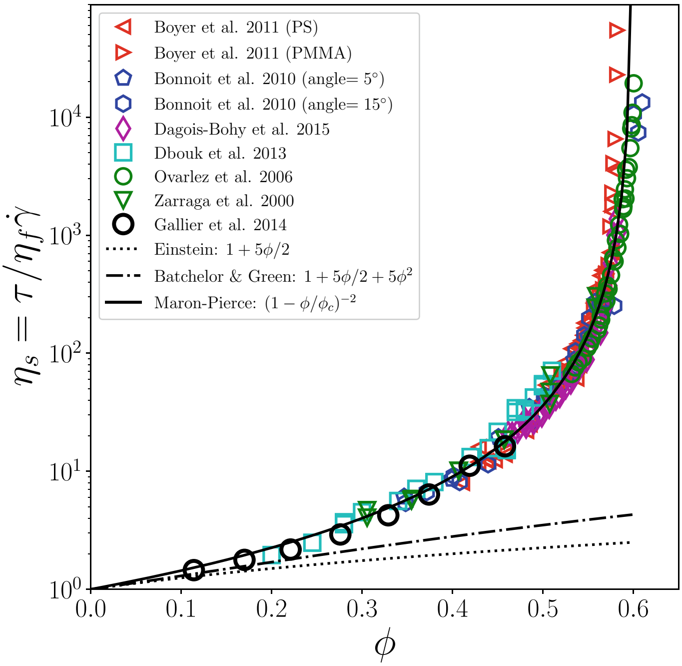

Fig. 1.2 Relative suspension viscosity, η_s, versus volume fraction, ϕ. Experiments of Boyer et al. (2011) using pressure-imposed rheometry with polystyrene (PS) spheres of diameter $d = 580\,\mu\text{m}$ suspended in polyethylene glycol-ran-propylene glycol monobutylether as well as poly(methyl methacrylate) (PMMA) spheres of diameter $d = 1100\,\mu\text{m}$ suspended in a Triton X-100/water/zinc chloride mixture, of Bonnoit et al. (2010) using an inclined plane rheometer tilted at two different angles with polystyrene spheres of diameter $d = 40\,\mu\text{m}$ suspended in silicone oil, of Dagois-Bohy et al. (2015) using pressure-imposed rheometry with polystyrene (PS) spheres of diameter $d = 580\,\mu\text{m}$ suspended in polyethylene glycol-ran-propylene glycol monobutylether, of Dbouk et al. (2013) using a parallel-plate rotational rheometer with polystyrene spheres of diameter $d = 140\,\mu\text{m}$ suspended in a mixture of water, UCON oil, and zinc bromide, of Ovarlez et al. (2006) using MRI technique and a wide-gap Couette geometry with polystyrene spheres of diameter $d = 290\,\mu\text{m}$ suspended in silicone oil, of Zarraga et al. (2000) using a parallel-plate rotational rheometer with glass spheres of diameter $d = 44\,\mu\text{m}$ suspended in 3 different fluids. Numerical simulations of Gallier et al. (2014) with frictional spheres. Viscosity laws of Einstein (1906, 1911), of Batchelor and Green (1972), and simple correlation $(1 - \phi/\phi_c)^{-2}$ (Maron-Pierce) (Stickel & Powell, 2005)

rate as they are independent of the direction of the flow. As most of the repulsive collisions between spheres happen in the plane of shear and fairly equally in the flow and the flow-gradient directions, N_2 is negative with a magnitude increasing with increasing ϕ while N_1 is much smaller with a sign depending on the flow-induced microstructure of the particles (slightly negative in the bulk and positive near a wall).

Another manifestation of the non-Newtonian character of suspension flows concerns particle migration phenomena in concentrated suspensions. For example, in

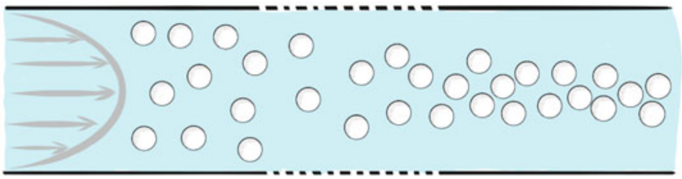

Fig. 1.3 Shear-induced migration of neutrally-buoyant spheres in pressure-driven Poiseuille flow in a tube: the particles migrate irreversibly from the high shear region at the wall towards the low shear region at the centerline

pressure-difference-induced flow, i.e. Poiseuille flow, neutrally-buoyant particles can migrate towards the center of the pipe as sketched in Fig. 1.3. This leads us to the following section as a single-phase approach is not able to capture this phenomenon.

1.3 Two-Phase Flow of Suspensions

The previous single-fluid view is no longer appropriate when the fluid and the particles experience relative motion. This is the case in the irreversible migration of particles observed in a pipe flow that has been already mentioned above but also in the erosion of sedimented bed of particles under the action of shearing flows or the triggering of immersed granular avalanches, both situations being depicted in Fig. 1.4, where the coupling between the granular and fluid phases play a major role.

While the suspension mixture is incompressible, the particle phase is not. It is important to introduce the notion of particle pressure P (or more generally of normal stresses of the particle phase) which drives the motion of the particles, i.e. which is linked to the tendency of particle phase to spread or contract. The idea of a dispersive particle pressure under shear was introduced early by Bagnold (1954b). It can be also considered as an analog to the osmotic pressure (Deboeuf et al., 2009). It can be seen as the non-equilibrium continuation of osmotic pressure and drives the shear-induced migration in pipe flows that is mentioned above. They are difficulty in measuring particle normal stresses because it is not easy to differentiate between particle and fluid pressures as the particle pressure is balanced by an equal and opposite change in liquid pressure. As for the viscosity and the normal stress differences of the whole suspension mentioned in the above Sect. 1.2, particle normal stresses are also found to scale viscously and are linear in the modulus of the shear rate. Considering the particle normal stress along the direction perpendicular to the shearing flow direction, a relative normal viscosity, η_n, can be introduced. It is again a sole function of ϕ and presents the same divergence with ϕ as η_s when approaching the critical volume fraction ϕ_c, as shown in Fig. 1.5.

To be able to model particulate flows on a continuum scale in the flow configurations depicted in Figs. 1.3 and 1.4 requires application of a two-phase approach where the interstitial fluid and the particles are considered as two intertwined continuous

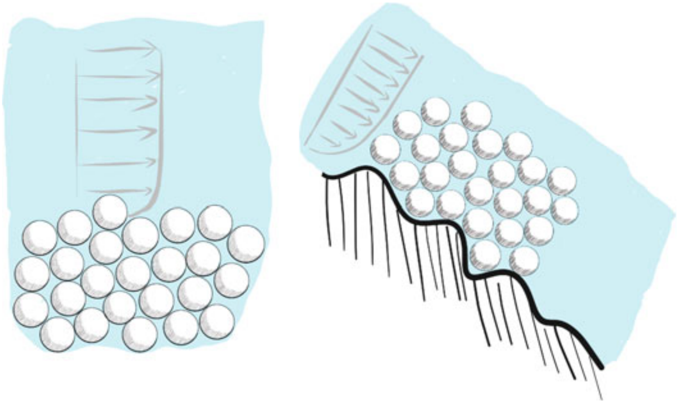

Fig. 1.4 (left) Erosion of sedimented particles under the action of shearing flows and (right) immersed granular avalanches

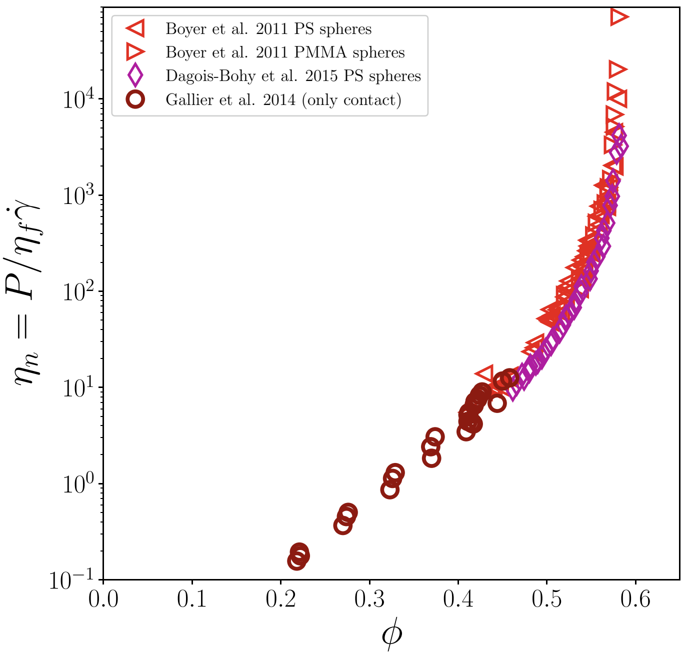

Fig. 1.5 Relative normal viscosity, η_n, versus volume fraction, ϕ. Experiments of Boyer et al. (2011) using pressure-imposed rheometry with polystyrene (PS) spheres of diameter $d = 580\,\mu$m suspended in polyethylene glycol-ran-propylene glycol monobutylether as well as poly(methyl methacrylate) (PMMA) spheres of diameter $d = 1100\,\mu$m suspended in a Triton X-100/water/zinc chloride mixture and of Dagois-Bohy et al. (2015) using pressure-imposed rheometry with polystyrene (PS) spheres of diameter $d = 580\,\mu$m suspended in polyethylene glycol-ran-propylene glycol monobutylether. Numerical simulations of Gallier et al. (2014) with frictional spheres

phases. The strategy consists in deriving the governing equations that describe the system in an average sense for each phase. There are different ways of performing the averaging process, using space or ensemble averaging, which should led to essentially the same results if properly done. It is not the purpose here to give the detailed derivation which can be found in Jackson (1997), Lhuillier (2009), Nott et al. (2011) and which is discussed in other chapters of the present book. It is just important to point the interest of a two-phase approach to tackle these flow configurations. For instance, applying the two-phase approach to describe shear-induced migration leads to relate the migration flux to the divergence of the normal stress of the particle phase (Nott & Brady, 1994; Morris & Boulay, 1999). A physical understanding of the migration process can be easily given in the simplest two-dimensional case. The fully-developed particle-phase momentum balance in the shear direction 2 yields $\partial P/\partial x_2 = \partial(\eta_n\dot{\gamma})/\partial x_2 = 0$, meaning that the particle pressure is constant across the channel. Where the shear rate is low, the concentration is high and vice versa and the particles must have migrated to the center of the channel.

1.4 An Alternative Approach: The Frictional Rheology of Suspensions

In the preceding Sects. 1.2 and 1.3, the rheological laws have been expressed in term of a single control parameter ϕ. However, in the situation of sediment transport such as those depicted in Fig. 1.4, the volume fraction is a free adjustable parameter and the driving force is gravity which controls the level of stress experienced by the particle phase. This rheological situation has been termed ‘pressure imposed’. It has been shown that a description in terms of a frictional rheology inspired by that describing dense dry granular flow (Forterre & Pouliquen, 2008) can be applied to viscous suspensions (Boyer et al., 2011). In the inertial case of a dry granular material sheared at a shear rate $\dot{\gamma}$ under an imposed granular pressure P, the shear stress τ is proportional to P with an effective friction coefficient μ and volume fraction ϕ being sole functions of the inertial number $I^2 = \varrho_p d^2\dot{\gamma}^2/P$ where d and ϱ_p are respectively the diameter and density of the particles. A similar frictional formalism can be applied to viscous suspensions of non-Brownian spheres but with a viscous number $J = \eta_f\dot{\gamma}/P$ in place of the inertial number I^2. This frictional formulation is equivalent to the more classical presentation presented in the preceding Sects. 1.2 and 1.3 using viscosities being sole function of ϕ using $\eta_s = \tau/\eta_f\dot{\gamma} = \mu/J$ and $\eta_n = P//\eta_f\dot{\gamma} = 1/J$. Recent work have aimed at establishing a unified theoretical framework across the viscous to inertial flowing regimes by using superposed inertial and viscous stresses of the form $J + \alpha I^2$ where α is the inverse of the Stokes number at the transition (Trulsson et al., 2012; Tapia et al., 2022).

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Chapter 2
Mathematical Modelling of Particulate Flows

Julien Chauchat and Laurent Lacaze

Abstract In this chapter, the equations governing the dynamics of particulate flows are presented and discussed. We focus here on the notion of 'particle resolution scale', in terms of whether individual particle dynamics are resolved or not. The concept of particle resolution scale is fundamental for obtaining insights into mechanisms ranging from the particle scale processes up to the geophysical flow scales. Since it is not feasible to simultaneously resolve all of these scales, we presently discuss micro- and meso-scale models of relevance for macro-geophysical applications. In this chapter, the particle-resolved methods are shown first, which are based on the Eulerian description of the carrier flow and the Lagrangian description of the motion of individual particles. In order to investigate the meso-scale processes of geophysical flows, it is necessary to work with equations averaged over scales much larger than the particle scale, enabling Eulerian description of both an equivalent fluid phase and an equivalent particle phase (Euler-Euler). The Euler-Euler approach requires closures, as part of the dynamics and mechanics are not resolved, which include fluid-particle interaction forces, subgrid turbulence and granular rheology. Such problem closures are discussed, though not exhaustively, along the book where necessary. There are other approaches that focus on resolution of different scales that may be found in the literature and some being also used in other chapters of this book, and will be briefly explained throughout the work as needed. Some of them result from simplifications of the physical processes involved in the fluid-particle system. As for instance, depending on the size, the relative density and the solid volume fraction, the dynamics of the particle phase can be either coupled or uncoupled (one-way coupling approach) from that of the fluid. This can lead to single-phase methods aug-

J. Chauchat
Université Grenoble Alpes, CNRS, Grenoble INP, LEGI, 38000 Grenoble, France

L. Lacaze (✉)
Institut de Mécanique des Fluides de Toulouse (IMFT), Université de Toulouse, CNRS, Toulouse, France
e-mail: laurent.lacaze@imft.fr

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mented with a particle concentration transport equation, useful to describe systems that extend over very large scales (kilometers and more).

2.1 Equations of Motion: A Resolved Approach

2.1.1 Eulerian Description of Fluid Motion and Resolution Method

We discuss here the governing equations for the fluid phase. In the case of granular suspension, the fluid domain is interspersed with solid boundaries. This obviously indicates that the resolution of the fluid motion is to be performed on a complex geometry with moving solid boundaries. Note that no averaging operators are introduced so far, which could lead to a homogeneous description of the fluid phase over the entire domain; this is the subject of Sect. 2.3.

Governing equations of a fluid flow satisfying mass and momentum conservations for an incompressible and viscous fluid are

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{2.1}$$

$$\varrho_f \left[\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} \right] = \frac{\partial \sigma_{ij}^{(f)}}{\partial x_j} + \varrho_f g_i, \tag{2.2}$$

where Einstein summation convention is used here. The components of the Cauchy stress tensor for the fluid phase are $\sigma_{ij}^{(f)} = -p^{(f)}\delta_{ij} + \tau_{ij}^{(f)}$ with $p^{(f)}$ the fluid pressure and $\tau_{ij}^{(f)}$ its deviatoric contribution. For a viscous Newtonian and incompressible fluid, one can write $\tau_{ij}^{(f)} = \eta_f(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$. u_i are the components of the fluid velocity $\boldsymbol{u}$. ϱ_f and η_f are the density and the dynamic viscosity of the fluid, respectively. g_i stands for the components of the gravity acceleration. Equations (2.1) and (2.2) for the fluid phase are subjected to boundary conditions at the surface S_p of each solid particle. In the case of inert solid particles in a viscous fluid as consider along this book, the condition is a no-slip boundary condition as

$$\boldsymbol{u} = \boldsymbol{u}_S^{(p)} \quad \text{on} \quad S_p, \tag{2.3}$$

with $\boldsymbol{u}_S^{(p)}$ is the local particle velocity at its surface. Due to condition (2.3), even the resolution of a simple shear flow configuration remains complex, providing extra-dissipation due to the fluid-particle interaction (as discussed in Chap. 1). Then, solving more complex geometry or flow condition as encountered in many applications as in geophysical flows, remain challenging.

The use of numerical solver is often required to resolve fluid flow around moving particles. Unfortunately, body-fitted grid methods to conform the grid to the boundary of the fluid domain (see Thompson et al., 1985; Liseikin, 1999 for grid generation techniques) lead to a substantial computational cost when particles move. These methods can moreover be not usable for specific situation, particularly when two particles approach each other and enter into solid contact. To circumvent this issue, fix-grid methods became popular to solve granular suspension configurations. Here, fix-grid methods refers to solver for which the meshgrid over which the fluid phase is numerically resolved remain fix in time. This obviously requires to account for the presence of solid object onto the mesh. Several approaches exist for that purpose (Bigot et al., 2014 and references there in).

Among other methods, one of the popular one for granular suspension, and used in the present book when referring to resolved numerical approach, is the Immersed Boundary Method (IBM). Details of this method can be found in Peskin (2002), Uhlmann (2005), Mittal and Iaccarino (2005), Bigot et al. (2014). Even if the numerical procedure and related algorithms can differ from case to case, the main idea of such method is always the same. For rigid particles, momentum conservation (2.2) is enforced by an extra volume force $\boldsymbol{f}^{IBM}$ accounting for the presence of the particles, as

$$\varrho_f \left[\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} \right] = \frac{\partial \sigma_{ij}^{(f)}}{\partial x_j} + \varrho_f g_i + f_i^{IBM}, \tag{2.4}$$

with $\boldsymbol{f}^{IBM}$ being applied over the particles volume and zero outside (Bigot et al., 2014; Mittal & Iaccarino, 2005; Uhlmann, 2005). Note that the force $\boldsymbol{f}^{IBM}$ used in the present approach explicitly depends on the particle velocity $\boldsymbol{u}^{(p)}$, as it aims at enforcing the flow field (at the surface and/or in the volume) to be the one of the rigid motion of the particle.

Solving (2.1) and (2.4) then leads to the flow field outside the particles, satisfying no-slip condition at S_p, and a force term leading to the fluid stress on S_p applied by the fluid flow onto the particle. Note that force and torque applied on the particle by the fluid flow can be directly obtained by integrating (2.4) over volume of the particle $\mathcal{V}_p$ (Bigot et al., 2014; Uhlmann & Chouippe, 2017 and as explained below).

2.1.2 *Lagrangian Description of Particle Motion in a Fluid*

We start by describing the granular medium through a Lagrangian approach. This means that each individual grain is followed in time. The Lagrangian description of particle dynamics is common to many applications and situations from fluidized particulate flows towards dense granular flows. However, the dominant and relevant mechanisms to be modelled can strongly differ, due in particular to the solid fraction ϕ and therefore to the occurrence of solid contacts.

When particles are suspended in a fluid and their volume fraction ϕ is small, it is assumed that the most important contribution of the forces applied to each grain is the one exerted by the fluid. In this case, even if binary contacts are sometimes considered, we often neglect solid contact between particles. Then simulations do not account for solid contact and the particles do not "see" each other. This allows to save computational time as will be discussed later. Of course, when clustering occurs, the description become nonphysical with particles converging all towards a single spatial point for instance.

When increasing the solid fraction ϕ, solid contact cannot be disregarded anymore as it can play a major role on dissipation in the system. This is even more true for dense granular configuration, typically $\phi > 0.5$, for which solid contact strongly participates to the dynamics of the granular medium, usually referred to as dry granular flows. Of course, simulating solid contact also allows to prevent from singular clustering mentioned previously.

If the surrounding fluid phase can be disregarded, i.e. dealing with a single phase problem for the granular medium, usually referred to as dry granular flows, the dynamics of each solid particle is controlled by solid contact force induced by collisions with other particles and any body forces induced by an external field. For the latter, we only consider gravity in the following according to the aim of the courses. If the surrounding fluid can not be disregarded, for instance as silice beads moving in water, an extra force associated with the fluid-particle interaction has to be implemented to obtain the dynamics of each solid particles. Then, assuming that the shape of the grains can be approximated by spheres, the motion of each individual particle p, with $p \in [1, N_p]$ (N_p being the number of particles) is obtained by integrating Newton's equations for linear and angular momentum of a solid sphere of mass $m_p = \varrho_p \mathcal{V}_p$, with ϱ_p and $\mathcal{V}_p$ the density and volume of particle p respectively. They read in our case

$$m_p \frac{d\boldsymbol{u}_p^{(p)}}{dt} = m_p \boldsymbol{g} + \sum_{q \neq p} \boldsymbol{F}^{qp} + \int_{S_p} \boldsymbol{\sigma}^{(f)} \cdot \boldsymbol{n} \, dS, \tag{2.5}$$

$$\frac{m_p {d_p}^2}{10} \frac{d\boldsymbol{\Omega}_p^{(p)}}{dt} = \sum_{q \neq p} \boldsymbol{T}^{qp} + \int_{S_p} \boldsymbol{r} \times \boldsymbol{\sigma}^{(f)} \cdot \boldsymbol{n} \, dS, \tag{2.6}$$

where $\boldsymbol{u}_p^{(p)}$ and $\boldsymbol{\Omega}_p^{(p)}$ correspond to the linear velocity and the angular velocity respectively. Index q labels any particle in solid contact with p, and then $\boldsymbol{F}^{qq}$ and $\boldsymbol{T}^{qq}$ are the solid contact force and torque, respectively, exerted by each particle q on p. Finally, $\boldsymbol{F}^{h|(p)} = \int_{S_p} \boldsymbol{\sigma}^{(f)} \cdot \boldsymbol{n} \, dS$ and $\boldsymbol{T}^{h|(p)} = \int_{S_p} \boldsymbol{r} \times \boldsymbol{\sigma}^{(f)} \cdot \boldsymbol{n} \, dS$ are the fluid force and fluid torque, respectively, exerted by the fluid phase onto the surface S_p of particle p, with $\boldsymbol{n}$ its unit normal vector.