Systems & Control: Foundations & Applications

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Stochastic Teams, Games, and Control under Information Constraints

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To
Adem, Helin, Kate, Sait, and Saibe Yüksel
(S.Y.)
and
Altan, Koray, Eren, Zara, Alya, and Reya
(T.B.)

Preface

The progress in the field of stochastic decision making since the publication of our 2013 book *Stochastic Networked Control Systems: Stabilization and Optimization Under Information Constraints* in the same Series, and expansion and evolution of the topical areas the field encompasses, has given us the impetus to expand on our earlier coverage, to go also beyond stochastic teams to include stochastic games as well.

Given our initial motivation, this book may first be viewed as an addendum to our 2013 book. However, the progress in the field and the encouraging response we have received from the community have led us to make such substantial changes in the coverage (as noted below) that the current book can no longer be viewed as a *2nd edition*. Accordingly, some of the material in our 2013 book have not been included in this book and most of the new material here cannot be found anywhere else in such a complete and unified manner.

Our goal here has been to provide a comprehensive, mathematically rigorous, and accessible treatment of the interaction between information, decision, and control in multi-agent systems. We provide a comprehensive theory and coverage on information structures for stochastic teams as well as stochastic games.

The fields of decentralized stochastic control and networked control have reached a high level of maturity. On decentralization, there has been significant progress on existence, approximations, and optimality analyses, and many outstanding issues have been resolved, even though some still remain open. On networked control, there have been major findings on solution methods for both stabilization and optimization under various information constraints for both linear and non-linear models, and it is perhaps fair to say that the field is slowly converging to a phase where the rate of progress does not have the bursts of initial stages of discovery. Notably, non-linear systems have also found their place toward a broad treatment of information design in control theory and the theory of multi-agent decision making. Importantly, the fields noted above are now important, closely related sub-disciplines of their own, and will likely keep generations of students immersed in the problems involving the interaction of information and control, and decision making in general.

Additionally, we have included in the book a comprehensive treatment of information structures as they arise in stochastic game theory. Game theoretic framework brings in a multitude of additional intricacies with regard to the value and shaping of information, and even the most elementary arguments on the role of information in stochastic teams do not find universal applicability in game theory due to the competitive nature of equilibrium concepts, and fragility of information structure dependent properties, which we cover in detail in the book. We present foundational material regarding the interaction of information and decisions, toward a comprehensive theory of information structures in stochastic teams, games, and (networked) control.

As in the 2013 book, we take information structures both as passively provided (as, for example, in the theory of partially observed Markov decision processes, decentralized stochastic control as well as dynamic game theory) and as actively designed (via encoding and decoding policies when a physical information channel is present, as in communication theoretic problems or signaling game problems). For the latter case, in the context of networked control and signaling games, we may view encoders and decoders also as agents.

Finally, we provide in the book a comprehensive discussion on reinforcement learning theory in the context of the topics presented in the book. Reinforcement learning algorithms use feedback information to improve performance in a control task. The field of reinforcement learning has advanced significantly both in terms of fundamental theoretical contributions and applications. However, it is important to also understand the limitations of learning theory in the presence of model uncertainty and limited information. To this end, we found it appropriate to present several fundamental results toward generating a more complete book. Accordingly, a systematic discussion on reinforcement learning theory under a variety of information structures is presented.

To keep the length of the book within a manageable size, proofs of certain results with particularly tedious derivations have not been included, with relevant references provided as appropriate. Also, we have kept the analysis throughout the book primarily in the discrete-time domain, while we recognize that much research has been done in the continuous-time setup as well. Bibliographic remarks provide some references in this domain.

The book draws and utilizes a diverse set of tools (of both conceptual and analytical nature) from various disciplines, including stochastic control, team theory, game theory, information theory, probability theory, and dynamical systems, and amalgamates them into a unified, coherent, and applicable theory. It could be used as a textbook or as an accompanying text in a graduate-level course on the topic.

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As in the 2013 book, the work here would not have been possible had we not have the opportunity of interacting with several outstanding scholars and extremely resourceful students.

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Contents

Acronyms and Notations

$\mathrm{trace}(A)$	Trace of a square matrix A
$\det(A)$	Determinant of a square matrix A
1_E	Indicator function for event E
$\mathbb{X}$	A generic vector space
$\langle x, y \rangle$	Inner product between x and y on a Hilbert space
$\mathcal{B}(\mathbb{X})$	The Borel σ-field on $\mathbb{X}$
$\sigma(y)$	σ-field generated by a random variable y
$\mathcal{P}(\mathbb{X})$	Set of probability measures on $\mathcal{B}(\mathbb{X})$
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^n$	Vector space of n-dimensional real vectors
$\mathbb{Z}$	Set of integers
$\mathbb{Z}_+$	Set of non-negative integers
$\mathbb{N}$	Set of positive integers
$\mathcal{Q}$	A space of quantizers
Q	Quantizer or channel depending on context
$\Pi^{comp,i}$	Composite quantization policy for encoder i
Q_t^i	Quantizer used by Agent i at time t
DM*i*	Decision Maker i
A*i*	Agent i
$\underline{\gamma}^i$	Policy of DM i, that is $\{\gamma_t^i, t \geq 0\}$
$\underline{\gamma}$	Ensemble of policies for all decision makers, that is $\{\underline{\gamma}^i\}$
I_t^i or $\mathcal{I}_t^i$	Information variable at Agent i at time t
$\underline{\eta}$	Information structure inducing map $\{\eta^1, \ldots, \eta^N\}$
$E_P^{\underline{\gamma}}\{\cdot\}$	Expectation under policy $\underline{\gamma}$, with initial condition measure P
E_x	Expectation conditioned on an initial condition realization x, or with respect to a random variable x, depending on the context
$Df(x)$ or $J(f(x))$	Jacobian of f at x
$H(\cdot)$	Discrete entropy
$h(\cdot)$	Differential entropy

$I(\cdot;\cdot)$	Mutual information
$D(P_1\|\|P_2)$	Kullback-Leibler Divergence between P_1 and P_2
$\|x\|$	Euclidean norm of a finite-dimensional real-valued vector x
A' or A^T	transpose of matrix A
$\|S\|$	Cardinality of a set S
$A \setminus B$	Ordered set difference: $\{x : x \in A, x \notin B\}$
$A \triangle B$	$(A \setminus B) \cup (B \setminus A)$
$\ln(x)$ or $\log(x)$	Natural logarithm of positive real x
$\underline{o}$	Zero vector
$\mathcal{T}$	Time/stage index set, $\{1, 2, \ldots, T\}$ or $\{0, 1, \ldots, T-1\}$
$\mathcal{N}$ ($\mathcal{L}$)	Decision maker (DM) index set, $\{1, 2, \ldots, N\}$ ($\{1, 2, \ldots, L\}$)
$u_{[k,s]}$	action (decision) variables from $t = k$ to $t = s$ for $s > k$, $\{u_k, u_{k+1}, \cdots, u_s\}$
$\mathbf{u}$	Collection of actions in $\mathcal{N}$ (or $\mathcal{L}$): $\{u^1, u^2, \ldots, u^N\}$