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# Singularly Perturbed Jump Systems

## Stability, Synchronization and Control

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# Contents

# Symbols

| | |
|---|---|
| $\mathbb{R}^n$ | $n$ dimensional Euclidean space |
| $\mathbb{R}^{n\times m}$ | Set of all $n \times m$ real matrices |
| $\mathbb{Z}$ | The set of all the integers |
| $\mathbb{Z}_{\geq n}$ | The set of integers greater than $n$ |
| $\mathbb{Z}_{[n,m]}$ | The set of integers greater than $n$ and less than $m$ |
| $I_n$ | $n \times n$ dimensional identity matrix |
| $0_n$ | $n \times n$ dimensional zero matrix |
| $0_{n\times m}$ | $n \times m$ dimensional null matrix |
| $Y^\top$ | Transpose of matrix $Y$ |
| $Y^{-1}$ | Inverse of a matrix $Y$ |
| $\mathrm{sym}(Y)$ | $Y + Y^\top$ |
| $\lambda_{\min}(Y)$ | Minimum eigenvalue of matrix $Y$ |
| $\lambda_{\max}(Y)$ | Maximum eigenvalue of matrix $Y$ |
| sup | Supremum |
| inf | Infimum |
| $\mathbb{E}$ | Mathematical expectation |
| $\otimes$ | Kronecker product |
| $\in$ | Belongs to |
| $\forall$ | For all |
| $\left[\pi_{ij}\right]_{n\times n}$ | $\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} \end{bmatrix}$ |
| $\Pr(A)$ | Occurrence probability of the event $A$ |
| $Y > 0$ | Real symmetric positive definite matrix $Y$ |
| $Y \geq 0$ | Real symmetric semi-positive definite matrix $Y$ |
| $Y < 0$ | Real symmetric negative definite matrix $Y$ |
| $Y \leq 0$ | Real symmetric semi-negative definite matrix $Y$ |
| diag{...} | A block-diagonal matrix |

| | |
|---|---|
| $\mathrm{diag}_n\{Y\}$ | $\mathrm{diag}\Bigg\{\underbrace{Y, Y, \ldots, Y}_{n}\Bigg\}$ |
| $\Vert\omega(k)\Vert_2$ | $\Vert\omega(k)\Vert_2 \triangleq \sqrt{\sum_{k=0}^{\infty}\omega^{\top}(k)\omega(k)}$ |

# Chapter 1
# Introduction and Preview

## 1.1 Motivation and Background

Many practical engineering systems have the phenomenon of dynamic coexistence of multi-temporal scales [38, 39, 60]. For example, in the aircraft system, there are position coordinate variables that change relatively slowly and speed and direction angle variables that change quickly [67]; in the power systems, electromagnetic transient changes are extremely rapid, while mechanical motion dynamic changes are relatively slow [116]; in production and application processes such as chemical industry and nuclear reactors, there are extremely fast-changing chemical reactions or atomic fission, and there are also relatively slow-changing machine movements. Generally, the phenomenon of the coexistence of multiple-time-scales dynamic exists in almost all large-scale engineering systems [47]. Since this type of system has dynamics at different time scales at the same time, the analysis and control problems of this type of system become complicated. From the perspective of system modeling, such systems with multi-time-scale dynamics can be well described by singularly perturbed systems (SPSs). The main feature of SPSs is that the "fast" state and the "slow" state coexist, and the degree of separation of the them is described by a small parameter (called singular perturbation parameter (SPP)). In past decades, the theory of SPSs has attracted the attention of many scholars, and some meaningful research results have been published [4, 6–11, 32, 34, 37, 40–42, 53–59, 62, 68, 77, 79, 80, 94–96, 119, 121, 122, 128].

On the other hand, system structure and parameters may encounter sudden changes because of sudden external noise disturbances, failure of connections of components within the system, and failure of connections between subsystems, which may deteriorates system performance and even cause the system to become unstable [13, 14]. To describe this phenomenon, the sudden change characteristics of the system are usually described by stochastic processes. At present, the stochastic processes commonly used by scholars include Bernoulli process [25, 28, 113, 129, 130] and



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Markov process [13, 14, 76, 126]. Compared with the Bernoulli process, which can only simulate the transition of two working modes, the Markov process can describe the random switching of the system between multiple modes [23, 24, 30, 36, 114, 127].

It is worth pointing out that there are certain limitations in using the Markov process to model the stochastic switching phenomenon of actual system structure/parameters. On the one hand, the Markov process requires that the sojourn-time of the stochastic switching phenomenon must follow a specific distribution, that is, the exponential distribution for continuous-time domain or the Geometric distribution for discrete-time domain [23, 24, 30, 36, 114, 127]. This limitation makes it difficult to apply the current research results to the analysis of some actual systems. Unlike the Markov process, semi-Markov process has no specific constraints on the sojourn-time [33, 35, 43, 66, 85, 98, 112, 123–125, 131]. Most of the current results on semi-Markov jump systems presume that the transition probability information of systems can be accurately obtained [33, 98, 112, 123–125]. However, in some complex engineering scenarios, it is usually hard to directly acquire all system transition probability information [63, 64, 84, 85, 99]. Although there are some results to deal with partially unknown probability information problem of semi-Markov jump systems [63, 64, 84, 85, 99], they cannot be directly applied to semi-Markov jump singularly perturbed systems (SMJSPSs).

On the other hand, how to deal with the phenomenon of limited acquisition of system mode information is one of the key problems in studying Markov jump systems [90]. Generally, there are three approaches to deal with the limited acquisition of system mode information problem: mode-independent design approach [12, 26, 65, 82, 86, 100, 111], clustering approach [16, 27], and the filtering technique [29, 91–93]. Recently, a hidden Markov model has been proposed to deal with the phenomenon of limited acquisition of modal information in Markov jump systems [19, 90]. Based on this, how to extend its ideas to the problem of limited acquisition of jump information in singularly perturbed jump systems is one of the hot and difficult issues in the research of singularly perturbed systems (SPSs).

Base on the above discussion, this book gives various singularly perturbed jump system models in continuous-time domain or discrete-time domain, such as Markov jump singularly perturbed systems (MJSPSs), semi-Markov jump singularly perturbed systems (SMJSPSs), hidden Markov jump singularly perturbed systems (HMJSPSs), and singularly perturbed jump complex network model. Some control synthesis problems of them have been considered for. Also, some partial probability information cases are taken into account when addressed those control synthesis problems.

## 1.2 Mathematical Descriptions and Basic Concepts

### *1.2.1 Continuous-Time Singularly Perturbed Jump Systems*

In mathematical, there are some type singularly perturbed jump linear systems

$$\begin{cases} \dot{x}_s(t) = A^{11}_{\sigma(t)} x_s(t) + A^{12}_{\sigma(t)} x_f(t) + B^1_{\sigma(t)} u(t) \\ \varepsilon \dot{x}_f(t) = A^{21}_{\sigma(t)} x_s(t) + A^{22}_{\sigma(t)} x_f(t) + B^2_{\sigma(t)} u(t) \end{cases} \tag{1.1}$$

where $x_s(t) \in \mathbb{R}^{n_s}$ and $x_f(t) \in \mathbb{R}^{n_f}$ are the slow state and fast state, respectively; $\varepsilon$ is the SPP; $u(t) \in \mathbb{R}^{n_u}$ means the control input; $\{\sigma(t), t \geq 0\}$ is the jump process of systems, which takes values in a finite set $\mathbb{S} = \{1, 2, \ldots, s\}$; $A^{11}_{\sigma(t)} \in \mathbb{R}^{n_s \times n_s}$, $A^{12}_{\sigma(t)} \in \mathbb{R}^{n_s \times n_f}$, $A^{21}_{\sigma(t)} \in \mathbb{R}^{n_f \times n_s}$, $A^{22}_{\sigma(t)} \in \mathbb{R}^{n_f \times n_f}$, $B^1_{\sigma(t)} \in \mathbb{R}^{n_s \times n_u}$ and $B^2_{\sigma(t)} \in \mathbb{R}^{n_f \times n_u}$ are matrices.

When the the jump process is subject to a Markov chain, the system (1.1) is called as MJSPSs, and when the the jump process is subject to a semi-Markov chain, the system can be called as SMJSPSs.

Generally, for continuous-time case, $\{\sigma(t), t \geq 0\}$ is a Markov process and the mode transition rates of it can be described as follows

$$\Pr\{\sigma(t+\Delta) = j | \sigma(t) = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta) & \text{if } i \neq j \\ 1 + \lambda_{ii}\Delta + o(\Delta) & \text{if } i = j \end{cases} \tag{1.2}$$

where

$$\Delta > 0, \lim_{\Delta \to 0} \left( \frac{o(\Delta)}{\Delta} \right) = 0$$

and $\lambda_{ij} \geq 0$ ($\forall i, j \in \mathbb{S}, i \neq j$) means the transition rate from $\sigma(t) = i$ at time $t$ to $\sigma(t+\Delta) = j$ at time $t + \Delta$, and $\lambda_{ii} = -\sum_{j \in \mathbb{S}\backslash\{j=i\}} \lambda_{ij}$ for all $i \in \mathbb{S}$.

For continuous-time case, $\{\sigma(t), t \geq 0\}$ is a homogeneous semi-Markov process and the mode transition rates of it can be described as follows

$$\Pr\{\sigma(t+\Delta) = j | \sigma(t) = i\} = \begin{cases} \lambda_{ij}(\Delta)\Delta + o(\Delta) & \text{if } i \neq j \\ 1 + \lambda_{ii}(\Delta)\Delta + o(\Delta) & \text{if } i = j \end{cases} \tag{1.3}$$

where

$$\Delta > 0, \lim_{\Delta \to 0} \left( \frac{o(\Delta)}{\Delta} \right) = 0$$

and $\lambda_{ij}(\Delta) \geq 0$ ($\forall i, j \in \mathbb{S}, i \neq j$) means the transition rate from $\sigma(t) = i$ at time $t$ to $\sigma(t+\Delta) = j$ at time $t + \Delta$, and $\lambda_{ii}(\Delta) = -\sum_{j \in \mathbb{S}\backslash\{j=i\}} \lambda_{ij}(\Delta)$ for all $i \in \mathbb{S}$.

### 1.2.2 Discrete-Time Singularly Perturbed Jump Systems

In mathematical, there are some type singularly perturbed jump linear systems

$$\begin{cases} x_s(k+1) = A^{11}_{\sigma(k)} x_s(k) + A^{12}_{\sigma(k)} x_f(k) + B^1_{\sigma(k)} u(k) \\ x_f(k+1) = \varepsilon A^{21}_{\sigma(k)} x_s(k) + \varepsilon A^{22}_{\sigma(k)} x_f(k) + \varepsilon B^2_{\sigma(k)} u(k) \end{cases} \tag{1.4}$$

$$\begin{cases} x_s(k+1) = A^{11}_{\sigma(k)} x_s(k) + \varepsilon A^{12}_{\sigma(k)} x_f(k) + B^1_{\sigma(k)} u(k) \\ x_f(k+1) = A^{21}_{\sigma(k)} x_s(k) + \varepsilon A^{22}_{\sigma(k)} x_f(k) + B^2_{\sigma(k)} u(k) \end{cases} \tag{1.5}$$

$$\begin{cases} x_s(k+1) = \left(I + \varepsilon A^{11}_{\sigma(k)}\right) x_s(k) + \varepsilon A^{12}_{\sigma(k)} x_f(k) + \varepsilon B^1_{\sigma(k)} u(k) \\ x_f(k+1) = A^{21}_{\sigma(k)} x_s(k) + A^{22}_{\sigma(k)} x_f(k) + B^2_{\sigma(k)} u(k) \end{cases} \tag{1.6}$$

where $x_s(k) \in \mathbb{R}^{n_s}$ and $x_f(k) \in \mathbb{R}^{n_f}$ are the solw state and fast state, respectively; $u(k) \in \mathbb{R}^{n_u}$ is the control input; $\varepsilon$ is the SPP; $\{\sigma(k), k \geq 0\}$ is the jump process of systems, which taks values in a finite set $\mathbb{S} = \{1, 2, \ldots, s\}$; $A^{11}_{\sigma(t)} \in \mathbb{R}^{n_s \times n_s}$, $A^{12}_{\sigma(t)} \in \mathbb{R}^{n_s \times n_f}$, $A^{21}_{\sigma(t)} \in \mathbb{R}^{n_f \times n_s}$, $A^{22}_{\sigma(t)} \in \mathbb{R}^{n_f \times n_f}$, $B^1_{\sigma(t)} \in \mathbb{R}^{n_s \times n_u}$ and $B^2_{\sigma(t)} \in \mathbb{R}^{n_f \times n_u}$ are matrices. When the the jump process is subject to a Markov chain, the system can be called as MJSPSs, and when the the jump process is subject to a semi-Markov chain, the system can be called as SMJSPSs.

Generally, for discrete-time case, $\{\sigma(k), k \geq 0\}$ is a homogeneous Markov process and the mode transition probabilities of it can be described as follows

$$\Pr\{\sigma(k+1) = j \mid \sigma(k) = i\} = \pi_{ij} \tag{1.7}$$

where

$$\pi_{ij} \geq 0, \forall i, j \in \mathbb{S}$$
$$\sum_{j \in \mathbb{S}} \pi_{ij} = 1, \forall i \in \mathbb{S}.$$

The transition probability matrix $\Pi$ is defined as $\Pi \triangleq \left[\pi_{ij}\right]_{s \times s}$.

For discrete-time case, $\{\sigma(k), k \geq 0\}$ is a homogeneous semi-Markov process, and the mode transition probability of $\{\sigma(k), k \geq 0\}$ is generated by a semi-Markov kernel (SMK) $\Pi(\tau) \triangleq \left[\pi_{ij}(\tau)\right]_{s \times s}$, $\forall i, j \in \mathbb{S}$ with

$$\begin{aligned} \pi_{ij}(\tau) &\triangleq \Pr\{R_{m+1} = j, T_{m+1} = \tau \mid R_m = i\} \\ &= \frac{\Pr\{R_{m+1} = j, R_m = i\}}{\Pr\{R_m = i\}} \frac{\Pr\{R_{m+1} = j, T_{m+1} = \tau, R_m = i\}}{\Pr\{R_{m+1} = j, R_m = i\}} \\ &= \vartheta_{ij} \varpi_{ij}(\tau). \end{aligned} \tag{1.8}$$

where $m$ denotes the time when system at $m$th jump; $R_m$ is system mode at the $m$th jump; $T_{m+1}$ denotes the sojourn time between system at $m$th jump and the $m+1$th jump; $\vartheta_{ij} \triangleq \Pr\{R_{m+1}=j \mid R_m=i\}$ with $\vartheta_{ii}=0$ and $\varpi_{ij}(\tau) \triangleq \Pr\{T_{m+1}=\tau \mid R_{m+1}=j, R_m=i\}$. The cumulative density function (CDF) of sojourn time (ST) for mode $i$ is defined as

$$F(i,\tau) = \Pr\left\{T_{m+1} \le \tau \mid \eta\left(\bar{k}\right)=i\right\} = \sum_{l=1}^{\tau}\sum_{j\in\mathbb{S}} \pi_{ij}(l)$$

with $F(i,0)=\varpi_{ij}(0)=0$.

### 1.2.3 Lemmas

**Lemma 1.1** ([120]) *For a scalar $\bar{\varepsilon}>0$ and symmetric matrices $U_1$, $U_2$ with appropriate dimensions, if*

$$U_1 \ge 0 \tag{1.9}$$

$$U_1 + \bar{\varepsilon}U_2 > 0 \tag{1.10}$$

*then*

$$U_1 + \varepsilon U_2 > 0 \tag{1.11}$$

*holds for all $\varepsilon \in (0,\bar{\varepsilon}]$.*

**Lemma 1.2** ([120]) *For a scalar $\bar{\varepsilon}>0$ and symmetric matrices $U_1$, $U_2$ and $U_3$ with appropriate dimensions, if (1.9), (1.10) and*

$$U_1 + \bar{\varepsilon}U_2 + \bar{\varepsilon}^2U_3 > 0 \tag{1.12}$$

*then*

$$U_1 + \varepsilon U_2 + \varepsilon^2U_3 > 0 \tag{1.13}$$

*holds for $\forall\varepsilon \in (0,\bar{\varepsilon}]$.*

**Lemma 1.3** ([45]) *For a scalar $\bar{\varepsilon}>0$ and symmetric matrices $U_l$ $(l=1,2,\ldots,n)$ with appropriate dimensions, if (1.9) and*

$$U_1 + \sum_{r=1}^{l-1}\bar{\varepsilon}^r U_{r+1} > 0, l=2,3,\ldots,n \tag{1.14}$$

*then*

$$U_1 + \varepsilon U_2 + \varepsilon^2 U_3 + \cdots + \varepsilon^{n-1}U_n > 0 \tag{1.15}$$

*holds for $\forall\varepsilon \in (0,\bar{\varepsilon}]$.*

**Lemma 1.4** ([17]) *For a scalar* $\bar{\varepsilon} > 0$ *and matrices* $U_1$, $U_2$ *and* $U_3$ *with appropriate dimensions, if*

$$U_1 < 0 \tag{1.16}$$

$$U_3 \geq 0 \tag{1.17}$$

$$U_1 + \bar{\varepsilon} U_2 + \bar{\varepsilon}^2 U_3 < 0 \tag{1.18}$$

*then*

$$U_1 + \varepsilon U_2 + \varepsilon^2 U_3 < 0 \tag{1.19}$$

*holds for* $\forall \varepsilon \in (0, \bar{\varepsilon}]$.

**Remark 1.1** The above four lemmas are useful for obtaining the $\varepsilon$-independent stability analysis conditions and controller design methods for SPSs, and they are also significant for constructing $\varepsilon$-dependent Lyapunov function for SPSs.

**Lemma 1.5** ([83]) *Considering the parameterized linear matrix inequality*

$$\sum_{\alpha=1}^{r} \sum_{\beta=1}^{r} h_\alpha(z) h_\beta(z) \Delta_{\alpha\beta} < 0. \tag{1.20}$$

*with* $\Delta_{\alpha\beta} = \Delta_{\alpha\beta}^\top$, *if*

$$\Delta_{\alpha\alpha} < 0 \tag{1.21}$$

$$\Delta_{\alpha\beta} + \Delta_{\beta\alpha} < 0 \tag{1.22}$$

*hold for* $1 \leq \alpha < \beta \leq r$, *then (1.20) hold.*

**Lemma 1.6** ([89]) *Considering the parameterized linear matrix inequality (1.20) with* $\Delta_{\alpha\beta} = \Delta_{\alpha\beta}^\top$, *if*

$$\Delta_{\alpha\alpha} < 0 \tag{1.23}$$

$$\frac{1}{s-1} \Delta_{\alpha\alpha} + \frac{1}{2} \left( \Delta_{\alpha\beta} + \Delta_{\beta\alpha} \right) < 0 \tag{1.24}$$

*hold for* $1 \leq \alpha < \beta \leq r$, *then (1.20) hold.*

**Remark 1.2** Lemmas 1.5 and 1.6 are available for controller design problem of T-S fuzzy systems. Compared with Lemma 1.5, Lemma 1.6 is less conservative without introducing additional decision variables. More details can be found in [89].

**Lemma 1.7** ([115]) *For given matrices* $\Xi_1$ $\Xi_2$ *and* $\Xi_3$ *with appropriate dimensions, and* $O(k)$ *satisfying* $O(k) O^\top(k) \leq I$, *then*

$$\Xi_1 + \Xi_2 O(k) \Xi_3 + \Xi_3^\top O^\top(k) \Xi_2^\top < 0 \tag{1.25}$$

*holds, if exists a positive scalar* $\varepsilon > 0$ *such that*

$$\Xi_1 + \varepsilon^{-1}\Xi_2\Xi_2^\top + \varepsilon\Xi_3^\top\Xi_3 < 0. \tag{1.26}$$

**Lemma 1.8** ([3], Schur Complement) *Given matrices* $\Xi_{11} = \Xi_{11}^\top$, $\Xi_{12}$ $\Xi_{22} = \Xi_{22}^\top$, *the following three conditions are equivalent:*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^\top & \Xi_{22} \end{bmatrix} < 0 \tag{1.27}$$

$$\Xi_{11} < 0,\ \Xi_{22} - \Xi_{12}^\top\Xi_{11}^{-1}\Xi_{12} < 0 \tag{1.28}$$

$$\Xi_{22} < 0,\ \Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^\top < 0 \tag{1.29}$$

## 1.3 Literature Review

For singularly perturbed jump systems, up to now, the research results of the MJSPSs are much richer than those of the SMJSPSs and the HMJSPSs. This is mainly because that the SMJSPSs and HMJSPSs are more complex and challenging to analyze than the Markov jump ones. This section provides a brief review of the results related to singularly perturbed jump systems.

### *1.3.1 Stability and Stabilization*

MJSPSs possess the properties of both MJSs and SPSs. From the point of view of MJSs, the availability of transition probabilities or transition rates information for MJSPSs has an impact on their stability analysis. Therefore, when analyzing the stability of MJSPSs, scholars mainly consider the two aspects of whether the system is accessible or not. Under the consideration that the transition probabilities and transition rates information in MJSPSs is completely accessible, the mean-square exponential stability criteria based on the method of fast-slow decomposition for the stochastic MJSPSs were gave in [20, 78]. Under the consideration that the transition probabilities are time-varying in MJSPSs, [87, 88] gave the stochastic stability criteria for the reduced-order subsystems, respectively, and on the basis of these, the stochastic stability conditions for the whole system were gave. Under the consideration that the time-varying transition probabilities in MJSPSs belong to polyspore uncertainty, [50] investigated the problem of stochastic asymptotic stability analysis of MJSPSs under robust control. Reference [102] studied the stochastic stability analysis problem of closed-loop MJSPSs under the cases that the transition probability information is fully accessible, the transition probability information is partially accessible, and the system mode information is partially observable.

For SMJSPSs, the $L_2 - L_\infty$ stochastic synchronization analysis problem of semi-Markov jump singularly perturbed complex networks (SPCNs) in continuous-time domain was discussed in [48]. The stabilization problem for the slow sampling systems was addressed by employing a slow state feedback strategy in [71] and a design approach of the slow state feedback stabilization controller was established in [71]. By utilizing the T-S fuzzy model, the above results obtained in [71] was further extended to nonlinear systems in [69] and the gain fluctuation of the controller was considered.

### *1.3.2 Robust Control*

For continuous-time MJSPSs, the $H_\infty$ control problem was considered in [22], in which the controller can be obtained by solved some Riccati equations. The linear matrix inequalities (LMIs) form $H_\infty$ performance criterion was gave in [51], in which the $H_\infty$ state controller design methodology and the $H_\infty$ slow state feedback controllers design methodology were established. In [52], in order to overcome the difficulty of solving the Riccati equation or nonlinear matrix inequalities, a sufficient condition based on LMIs was established for solving the $H_\infty$ control problem of MJSPSs. Reference [50] further considered the robust control problem for continuous-time MJSPSs with uncertain parameters, where the system transition probability matrix is considered in polyspore form, and the results are available for both standard and nonstandard continuous-time MJSPSs. By employing the T-S fuzzy model, the $H_\infty$ state feedback control problem and $H_\infty$ output feedback control problems for a class of nonlinear continuous-time MJSPSs were researched in [61], and the mode-dependent $H_\infty$ state feedback controller and mode-dependent $H_\infty$ output feedback controller design methods were established.

Considering that the time delay widely exists in practical systems, the passive control problem of a class of continuous-time MJSPSs with uncertain parameters and time-varying delay was investigated in [108], and the time delay-dependent passive performance analysis criterion was gave and the mode information related passive controller design method was obtained. The sliding mode control problem for a class of nonlinear MJSPSs with constant time delay was studied in [110], where the constructed sliding mode surface and the designed controller are mode-dependent. For stochastic MJSPSs, [102] investigated the stability bound problem, where the case that the system mode information is partially observable was considered. The optimal control problem for stochastic MJSPSs was explored in [21], where optimal controllers are designed that depend on both the SPP and the mode information.

Considering the possible uncertainties in the system parameters, the sliding mode control problem for a class of MJSPSs with uncertain parameters was investigated in [46], case of obtaining information about the transition rate is considered, that is, the transition rate is partially unknown but the system mode information is considered to be directly obtained and used for the sliding mode surface and the controller design. Considering that the transition rate matrix (TRM) may be time-varying, the $H_\infty$

control and filtering problem for a class of nonlinear MJSPSs with the help of the T-S fuzzy model was investigated in [106], where the TRM is considered to be time-varying, and its time-varying property obeys a higher-order Markov process. From the point of view of mode information acquisition, the case considered in [106] is that both the system mode information and the higher-order Markov state information are directly accessible. Therefore, the controller and filter designed in [106] rely on both system mode and higher-order Markov state information. Moreover, in practice, the dynamics of SPSs may also contain algebraic equations, and such systems can be modeled as singular SPSs. When the Markov jump parameters are taken into account in the singular SPSs, they constitute singular MJSPSs. The study for this class of systems is more complicated due to the fact that this class of systems has the properties of MJSs, SPSs and singular system, simultaneously [103]. The sliding mode control problem for continuous time singular MJSPSs was studied in [107], where the time-varying TRM subject to a higher-order Markov processes.

For fast-sampling discrete-time MJSPSs, the nonfragile $H_\infty$ control problem was addressed in [31] for nolinear MJSPSs by employing the T-S fuzzy model, where a controller is independent on the Markov parameters. It is worth noting that the results in [31] were obtained based on the inequality derivation in [18] did not consider the Markov jump parameter. Therefore, although the derivation established therein can deal with the control problem of fast-sampling discrete-time MJSPSs, using it to address the control problem of the fast-sampling discrete-time MJSPSs inevitably brings a certain degree of conservatism due to the fact that the matrices in the derivation do not sufficiently take into account the Markov jump property. Reference [75] proposes an improved lemma which introduces the transition probability into the improved lemma. Since the improved lemma takes into account the information of the transition probability, the results obtained by using it for fast-sampling discrete-time MJSPSs have a lower conservatism than the results obtained by using the lemma in literature [18]. Based on this lemma, literature [75] considers its non-fragile extended dissipative control problems for nonlinear fast-sampling discrete-time MJSPSs based on the T-S fuzzy model, and unlike literature [31], the non-fragile controllers designed in [75] is mode-dependent, i.e., the Markov state information is fully accessible and used in the design of the controllers.

For slow-sampling discrete-time MJSPSs, the dissipative fault-tolerant control problems is investigated in [104], in which a slow state feedback control strategy is adopted because the fast state information in the SPSs may be limited to be acquired, and a mode-dependent slow state feedback fault-tolerant controller design method was established. Reference [118] further considered the fault-tolerant control problem of slow-sampling discrete-time MJSPSs by using a sliding-mode control strategy, in which the sliding-mode surface was chosen to be mode-independent and the designed controller was accomplished by considering that the fast state information can be directly acquired. It is worth pointing out that the mode-independent sliding mode surface has some conservatism in some cases. For this reason, [105] improved the mode-independent sliding mode surface by adding a discrete-time integral sliding mode surface to it, which was more flexible and effective in dealing with vibration.

For SMJSPSs, the [72] considered the quantization phenomenon in the network and provided a quantization controller design method. The extended dissipative synchronicity control problem of a singularly perturbed neural network with semi-Markov jump parameters was discussed in [109], in which the extended dissipative synchronicity controller design method was provided. A new system framework for solving the asynchronous sliding mode control problem of SMJSPSs was gave in [81], which could effectively eliminate the problem of unavailable system modes caused by sliding mode controllers in existing work.

### *1.3.3 Filtering, State Estimation and Synchronization*

In some practical situations, some of the state variables of a system are inherently non-measurable, or the state information of the system cannot be known precisely due to the influence of external noise. In such cases, it is necessary to design corresponding state estimators or filters to reconstruct the system state variables, and then make them further severing the system analysis and control problems. Some meaningful results have also been reported on the study of the filtering/state estimation problem for MJSPSs. The $H_\infty$ filtering problem for nonlinear uncertain MJSPSs was addressed in [2], and the linear matrix inequality conditions for the singular perturbation parameter-dependent fuzzy filter gain solution were presented. The $H_\infty$ filtering problem for discrete-time MJSPSs with partial transition probability information was investigated in [101], in which the SPP was dependent on the Markov parameters. The multi-objective filtering problem for nonlinear MJSPSs was investigated in [15], in which the SPP-dependent filter solving method was gave. The sampling-based $H_\infty$ filtering problem with considering the measurement output loss problem was investigated in [117]. In [97], the finite-time state estimation problem for Markov jump SPCNs was addressed and a hybrid event-based state estimation strategy was proposed.

For SMJSPSs, the state estimation problem for a class of semi-Markov jump SPCNs was addressed in [74] with considering mixed passive and $H_\infty$ performance. The synchronization problem of a class of nonlinear SPCNs with impulse effects and semi-Markov jump topology was studied in [49], in which a method was proposed to solve the upper bound of SPP with different coupling strengths.

## 1.4 Organization of the Book

This book is organized as follows.

Part I: Markov Jump Singularly Perturbed Systems

Chapter 2 addresses the stochastic stability analysis and stabilization problem for MJSPSs. For the stochastic stability analysis, some criteria are given, and the complete probability information case, partial probability information case and general probability information case are fully considered. For stabilization problem, the mode-dependent stabilizing controller and mode-independent one design methods are presented for three probability information cases.

Chapter 3 discusses the $H_\infty$ control problem for linear MJSPSs. Both the mode-dependent $H_\infty$ controller and mode-independent one design methods are presented for complete probability information case, partial probability information case and general probability information case.

Chapter 4 focuses on the robust control problems for nonlinear MJSPSs, in which the T-S fuzzy model is introduced to address the nonlinear systems. The nonfragile $H_\infty$ control problem, nonfragile passive control problem, and the $H_\infty$ fault-tolerant control problem are discussed for Markov jump singularly perturbed nonlinear systems based on the T-S fuzzy model. The stochastic stability analysis criteria are given and the controller design methods are established.

Chapter 5 addresses the $H_\infty$ synchronization of Markov jump SPCNs.

Part II Semi-Markov jump singularly perturbed systems

Chapter 6 gives the discussions for the stabilization problems of semi-Markov jump singularly perturbed linear systems, in which the complete probability information case and the partial probability information case are considered. Moreover, a unified controller form is presented and one can choose the type of controller according to the availability of fast states.

Chapter 7 considers the stabilization problems of semi-Markov jump singularly perturbed nonlinear systems based on the T-S fuzzy model. The complete semi-Markov kernel information case and the all the elements in the partial SMK information case are discussed. The stochastic stability analysis criteria for the two cases are presented and the design approaches of mode-dependent stabilizing controller for two cases are established.

Chapter 8 focuses on the synchronization of singularly perturbed complex networks subject to the semi-Markov chain. The $H_\infty$ synchronization criteria are given for the semi-Markov jump SPCNs.

Part III Hidden Markov jump singularly perturbed systems

Chapter 9 considers the $H_\infty$ control problem for linear HMJSPSs. The complete probability information case, partial probability information case and general probability information case are considered. For complete probability information case, A hidden Markov model, the partial probabilities HMM and the general probabilities HMM are introduced in this chapter to describe the phenomenon that the system mode information cannot be obtained directly with/without partial known probabilities or general probabilities. The HMM-based controller design approach for three probability information cases are established.

Chapter 10 expands the results in Chap. 9 to the nonlinear systems case by employing the T-S fuzzy model. The HMM-based fuzzy controller design approach for three probability information cases are established.

Chapter 11 considers the finite-time control problem for hidden Markov jump singularly perturbed linear systems. The finite-time boundedness analysis criteria are give and the passivity-based finite-time controller design approach for three probability information cases are established the complete probability information case, partial probability information case and general probability information case.

# Part I
# Markov Jump Singularly Perturbed Systems

# Chapter 2
# Stochastic Stability Analysis and Stabilization

## 2.1 Problem Formulation

Let consider the following discrete-time MJSPSs

$$\begin{cases} x_s(k+1) = A^{11}_{\sigma(k)} x_s(k) + A^{12}_{\sigma(k)} x_f(k) + B^1_{\sigma(k)} u(k) \\ x_f(k+1) = \varepsilon A^{21}_{\sigma(k)} x_s(k) + \varepsilon A^{22}_{\sigma(k)} x_f(k) + \varepsilon B^2_{\sigma(k)} u(k) \end{cases} \tag{2.1}$$

where $x_s(k) \in \mathbb{R}^{n_s}$ and $x_f(k) \in \mathbb{R}^{n_f}$ are the slow state and fast state, respectively; $\varepsilon$ is the singular perturbation parameter (SPP); $u(k) \in \mathbb{R}^{n_u}$ is the control input; $\{\sigma(k), k \geq 0\}$ is the jump process of systems (2.1), which satisfies a homogeneous Markov chain taking values in the finite set $\mathbb{S} = \{1, 2, \ldots, s\}$. The mode transition probabilities of $\{\sigma(k), k \geq 0\}$ are as follows

$$\Pr\{\sigma(k+1) = j | \sigma(k) = i\} = \pi_{ij} \tag{2.2}$$

where $\pi_{ij} \geq 0$ $(i, j \in \mathbb{S})$ and $\sum_{j \in \mathbb{S}} \pi_{ij} = 1$ for $\forall i \in \mathbb{S}$. The transition probability matrix (TPM) $\Pi$ is defined as $\Pi \triangleq [\pi_{ij}]_{s \times s}$.

Let

$$x(k) = \begin{bmatrix} x_f(k) \\ x_s(k) \end{bmatrix}, A_{\sigma(k)} \triangleq \begin{bmatrix} A^{11}_{\sigma(k)} & A^{12}_{\sigma(k)} \\ A^{21}_{\sigma(k)} & A^{22}_{\sigma(k)} \end{bmatrix}$$

$$B_{\sigma(k)} \triangleq \begin{bmatrix} B^1_{\sigma(k)} \\ B^2_{\sigma(k)} \end{bmatrix}, E_\varepsilon \triangleq \text{diag}\{I_{n_s}, \varepsilon I_{n_f}\}$$

the system (2.1) can be rewritten as

$$x(k+1) = E_\varepsilon A_{\sigma(k)} x(k) + E_\varepsilon B_{\sigma(k)} u(k). \tag{2.3}$$

To simplify the notation, $A_i \triangleq A_{\sigma(t)}$, $B_i \triangleq B_{\sigma(t)}$ for $\forall \sigma(t) = i \in \mathbb{S}$.



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