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Deepak Kumar
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Advances in Applied Mechanics

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ISSN 2195-4356 ISSN 2195-4364 (electronic)
Lecture Notes in Mechanical Engineering
ISBN 978-981-97-0471-2 ISBN 978-981-97-0472-9 (eBook)
https://doi.org/10.1007/978-981-97-0472-9

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
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Preface

The fifth Indian Conference on Applied Mechanics (INCAM 2022) was organized by the Department of Mechanical Engineering, NIT Jamshedpur, jointly with Indian Society for Applied Mechanics (ISAM). INCAM is the culmination of efforts made by Applied Mechanics departments in the country, to provide a technical platform for discussion among researchers in the broad area of Applied Mechanics. Continuing from the previous successful conferences INCAM 2013, held at IIT Madras, INCAM 2015 held at IIT Delhi, INCAM 2017 held at MNNIT, Allahabad, and INCAM 2019, held at IISc Bengaluru, INCAM 2022 has shown sufficient enthusiasm and paved way for many more such technical enclaves in the future. The Conference was an effort to bring together the scientists/engineers/academia working in the subject areas of biomechanics, design engineering, fluid mechanics, materials engineering, and solid mechanics to a common platform at the national level. More than 187 papers (out of total 200 submissions) were selected for oral presentation and publication in conference proceedings based on different themes of the conference, out of which 147 papers were presented along with 124 papers from IISc/IITs/NITs and reputed research organizations like DRDO, ISRO, etc. All the papers were reviewed by the two independent reviewers.

We specially thank our Ph.D. Scholars Mr. Swaroop Kumar Mandal, Mr. Rahul Kumar, Miss. Apoorva Verma, and many others who have contributed to the publication of these proceedings whom we are unable to mention individually.

Jamshedpur, India	Deepak Kumar
Jamshedpur, India	Vineet Sahoo
Jamshedpur, India	Ashok Kumar Mandal
Prayagraj, India	Karunesh Kumar Shukla

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About the Editors

Dr. Deepak Kumar is an Assistant Professor in the Mechanical Engineering Department, National Institute of Technology, Jamshedpur, India (an Institute of National Importance, Government of India). He is working in the field of damage mechanics of composite materials, nanocomposites, smart materials, product design, and development of biomaterials. He received a bachelor's degree in mechanical engineering from MAHE, India, Master's degree in Computer Aided Analysis and Design from BITS, Mesra, India and Ph.D. in Aerospace Engineering from Gyeongsang National University, South Korea. His Ph.D. thesis was about a multiscale damage model for composite structures, which considered the onset and evolution of intra-laminar matrix cracking and interlaminar delamination in various ply-configurations considering the mixed mode failure. Dr. Kumar also gained valuable experimental expertise in the areas of automated fiber placement technologies and composite bolted-joint mechanics. His latest research interests are the development of nanocomposites for development of super capacitor, energy devices and artificial tissues.

Dr. Vineet Sahoo is an Assistant Professor in Department of Mechanical Engineering in NIT Jamshedpur, India. His research focuses on design and development of special drives such as strain wave gears, LSHT orbit motors etc., which are used in precision power transmission and automation. He is particularly interested in understanding stress-strain behavior such devices in static and dynamic conditions. He has a very good hand on experimentation of such precision devices. Recently his work is focused on unraveling the design methodology of strain wave gears such that it can be developed indigenously. Further, Dr. Sahoo and his associates at NIT Jamshedpur seek to bring design evolutions in various mechanical components like bearings, gears and other power transmitting devices. He has more than 08 years of teaching experiences. He has published 22 research papers in reputed SCI and Scopus indexed journals and international conferences.

Dr. Ashok Kumar Mandal is an Assistant Professor in the Mechanical Engineering Department of National Institute of Technology Jamshedpur, India. His research is in the field of nonlinear vibrations, control, acoustics and hyperelasticity. He received his master's degree in Machine Design from IIEST, Shibpur and Ph.D. in Mechanical Engineering from IIT Kanpur. His Ph.D. thesis was on vibration characteristics of strings in the presence of unilateral smooth finite boundary obstacle. Dr. Mandal has more than seven years of teaching experience in the Departments of Mechanical Engineering of BIT Mesra and NIT Jamshedpur. He has guided four M.Tech. and one Ph.D. thesis. Dr. Mandal is currently working on three sponsored research projects which are funded by Government of India. His research interest also includes mechanics of Indian musical instruments and technology for rural development.

Prof. Karunesh Kumar Shukla is currently the Director of MNIT Bhopal. He earned his B.E. degree from M.M.M.E.C. Gorakhpur (Gorakhpur University) in 1986 and his M.E. from M.N.R.E.C. Allahabad (Allahabad University) in 1988. He completed his Ph.D. from Indian Institute of Technology, Delhi in 2001. He was a Visiting Research Fellow, Mechanical and Computer Aided Engineering Department, Feng Chia University, Taichung, Taiwan during November 2002–March 2003 and June 2004. He was also a Research Scholar, Applied Mechanics Department at I.I.T. Delhi, India from July 1997–September 2000. Prof. Shukla specializes in areas such as composite plates and shells, smart structures, retrofitting and strengthening of RCC structures, computational mechanics, stability and dynamics of structures, multi-scale composite. He has authored the book *An Introduction to Strength of Materials.*

Dynamics of Thin-Walled Metamaterial Beam with Local Resonators

Sunny and Senthil Murugan

Abstract A thin-walled beam with local resonators metamaterial for vibration wave attenuation is studied using a numerical method. The main objective is to analyse the dynamic behaviours of the thin-walled metamaterial beam and to design a local resonator for attenuating the vibration waves. The unit cell of the metamaterial beam is studied to predict the vibration waves in an infinite beam. It is investigated using Bloch's theorem and finite element method. The related result shows two bandgaps in which flexural and torsional waves can both be attenuated at the same time. It is obtained near the 1700 and 2200 Hz frequencies. It is validated by studying a finite metamaterial beam using transmissibility analysis. The six-unit cells are considered to investigate the flexural, torsional, and coupled flexural and torsional waves in a beam. Vibration transmission is reduced in the same frequency range as the band gap in the single cell analysis. The finite metamaterial is simulated with a different number of single cells to observe the effect on vibration transmissibility. The impact of the resonator mass and stiffness on the band gap is also examined for coupled flexural and torsional vibration waves. It will help to identify the best-suited design of a local resonator for the particular application. The metamaterial characteristics can suppress the vibration waves in the thin-walled beam by utilizing the local resonators.

Keywords Thin-walled beam · Metamaterial · Vibration suppression · Local resonators

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D. Kumar et al. (eds.), *Advances in Applied Mechanics*, Lecture Notes in Mechanical Engineering, https://doi.org/10.1007/978-981-97-0472-9_1

1 Introduction

Due to their high stiffness-to-mass ratio, thin-walled beams have found extensive use in a variety of engineering applications. In particular, closed cross-section thin-walled beams are crucial structural elements for achieving high torsional and bending rigidities. This is exemplified by the fact that thin-walled closed beams comprise the majority of load-bearing members in both automobiles and aeroplanes [1]. The vibration characteristics of these beams must be carefully taken into account when designing thin-walled closed structures. Recent years have shown much interest in vibration attenuation studies in periodic structures [2].

Mead [3] presented a broad wave propagation theory in periodic systems. The property of the periodic structure is to filter or attenuate the vibration waves within specific frequency ranges. These frequency ranges are band gaps (BG), in which the structure is not affected by vibration waves. It can be achieved by the Bragg's mechanism known as the Bragg band gap. It appears near frequencies where the wavelength is similar or smaller in size compared with the structure or gaps in the system.

Consequently, the Bragg's mechanism hardly satisfies the need for low-frequency vibration control because it is challenging and expensive to design periodic structures with large sizes in practical applications. In contrast to conventional periodic systems, wave propagation in engineered periodic structures, commonly referred to as Phononic materials or Phononic crystals (PCs), has recently attracted interest. They are typically described as artificial materials composed of repeating scattering patterns interwoven in a host medium.

Low-frequency band gaps are typically preferred for the system's vibration suppression. Acoustic metamaterial has been proposed by Liu et al. [4] that attenuates the vibration waves at low-frequency ranges. It creates local resonance BG along with the Bragg's stop band. It is formed due to the resonance of the local resonators (LR) attached to the host structure at a certain periodic length. It occurs near to the resonator's natural frequency. The behaviour of LR BG is also slightly influenced by the spatial periodicity of the structure.

To control the different vibration waves, the researcher has been developed distinct configurations of local resonators. Sun et al. [5] investigated the uniform isotropic beam with damped vibrating systems. It was connected throughout the beam to serve as vibration dampers. The objective was to attenuate the flexural vibrations. In another study, the beam with Bernoulli assumptions was studied with attached lateral local resonators for suppression of flexural waves by Wang et al. [2].

Similarly, the beam with Euler assumption attached to perpendicular beam resonators was investigated by Banerjee et al. [6]. They evaluated the effect of torsional vibration on the flexural bandgap. Mondal et al. [7] analysed the coupled flexural and torsional waves in the beam with locally resonant metamaterials. Until now, no work has been done on the thin-walled rectangular beam using metamaterial characteristics for vibration suppression. In the present work, a thin-walled beam is studied using LR metamaterial. The research is to perform a dynamic analysis of the

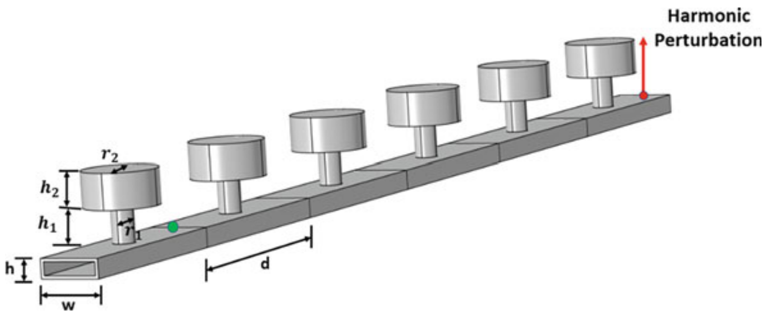

Fig. 1 Finite LR metamaterial beam with six-unit cells

Table 1 Structure dimension of thin-walled beam and local resonator

Structure parameters	d	w	h	t	h_1	r_1	h_2	r_2
Dimensions (mm)	300	80	26	4	47	15	47	50

beam with thin-walled and design the local resonator. The impact of LR weight and stiffness on the band gaps of the thin-walled beam is also investigated.

2 LR Metamaterial Structure

The metamaterial beam is made of a periodic arrangement of local resonators on the top of the thin-walled beam. The LR is designed by considering the column resonator with cylindrical tip mass. From the physical model aspect, the column resonator acts as a spring and the cylindrical tip mass as a lumped mass. The spacing between two adjacent LR is d, as shown in Fig. 1. The width and height of the outer boundary of the beam are w and h, and its uniform thickness is t. The LR dimension of the column is $h_1 \times r_1$, and the cylindrical tip mass is $h_2 \times r_2$. Aluminium is used for thin-walled beams and resonator columns, whereas structural steel is for the cylindrical tip mass. All dimensions are given in Table 1.

3 Dynamic Analysis

3.1 Dispersion Analysis of an Infinite Structure

The vibration wave behaviour in the infinite metamaterial beam can be analysed using the unit cell approach. This approach examines a single repeating unit cell rather than the entire structure. By utilizing Bloch theory, the dispersion study of the unit cells is carried out. The Bloch defines the vibration wave field in the periodic medium. It shows the displacement (q) and force (f) relation between the left-hand and right-hand limits of the single cell and is given below [8]:

$$q_R = e^{-ikl} q_L, \quad f_R = e^{-ikl} f_L, \tag{1}$$

where k represents the wave vector and l denotes the length of the unit cell. The subscripts are on the right-hand (R) and left-hand (L) sides of the single cell, respectively.

The finite element technique is implemented on the unit cell using the COMSOL software 6.0. The dynamic equation is modelled for the free vibration waves of the thin-walled beam. The displacement field is then supposed to have a harmonic solution. The equilibrium equation is transformed into a frequency region where it is a function of the wave vector. The governing dynamic equation of the unit cell at frequency $\omega(k)$ is

$$(K - \omega(k)^2 M)q = 0, \tag{2}$$

where K and M are the stiffness and mass matrices of the cell, respectively [8]. The eigenfrequencies for a parametric sweep along the condensed wave vector are solved to produce the dispersion curves.

3.2 *Transmissibility Analysis of a Finite Resonator Beam*

The wave transmissibility of the finite metamaterial beam is examined using a finite element approach via COMSOL 6.0. The finite beam comprises six-unit cells [8]. The transmissibility is determined under the different perturbations, such as flexural, torsional, and coupled, to predict the distinct vibration waves in the beam. The input displacement (u_i) is applied on the boundary of the beam, shown in Fig. 1, by a red dot. The output displacement (u_o) is calculated at the green dot. The transmissibility (T_m) is calculated as [9]:

$$T_m = 10 * \log \frac{u_\mathrm{o}^2}{u_\mathrm{i}^2}. \tag{3}$$

4 Research Outcomes and Analysis

The findings of a unit cell and finite metamaterial beam investigation are presented in this section. The bandgaps structure, transmissibility of vibration waves, and parametric study of the resonator are discussed. It is divided into three subsections.

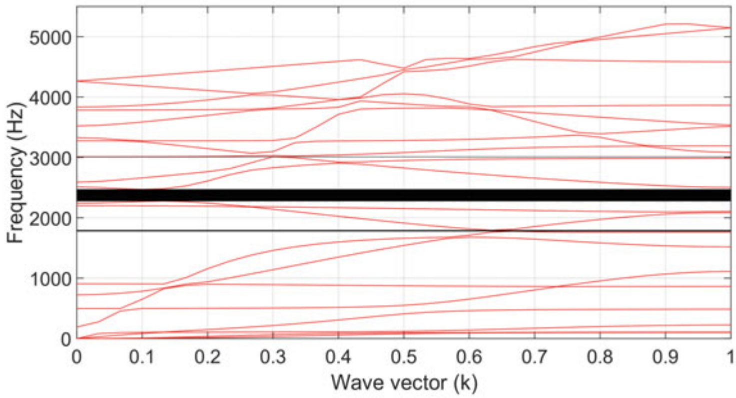

Fig. 2 Band gap of thin-walled metamaterial beam

4.1 Band Gap of Unit Cell

The dispersion curves are used to investigate the bandgaps of the vibrational waves in the unit cell, shown in Fig. 2. It is observed that the two bandgap regions are obtained. They are denoted by black colour stripes. The first BG is seen in the frequency range of 2275–2470 Hz, and the second BG is around 1777–1793 Hz. These bandgaps show that all vibration waves are attenuated, such as flexural, torsional, and coupled torsional and flexural waves (coupled waves). In these gaps, the vibration waves propagate near zero group velocity. Therefore, no vibration waves are propagated in the thin-walled beam.

4.2 Transmissibility of Vibration Waves

To validate the band gap, the transmissibility of vibration waves in the finite metamaterial beam is investigated in Fig. 3. It is calculated for flexural, torsional, and coupled waves. The solid blue and black dashed line indicates the transmissibility of the LR metamaterial beam and simple beam without LR, respectively. The blue regions represent the transmission gaps where waves are successfully suppressed in Fig. 3a. It demonstrates the precision of the stop bands. Likewise, torsional and coupled vibration waves are also attenuated in the same frequency range in Fig. 3b and c, where complete band gaps exist.

The six-unit cell is significant for comparing the transmissibility of vibration waves with the band gap [8]. But, the frequency region of vibration wave suppression is difficult to observe directly from the transmissibility plot. The influence of the quantity of single cells on the vibration transmissibility of the finite metamaterial beam is studied. The results are shown in Fig. 4 for coupled vibration waves. As the

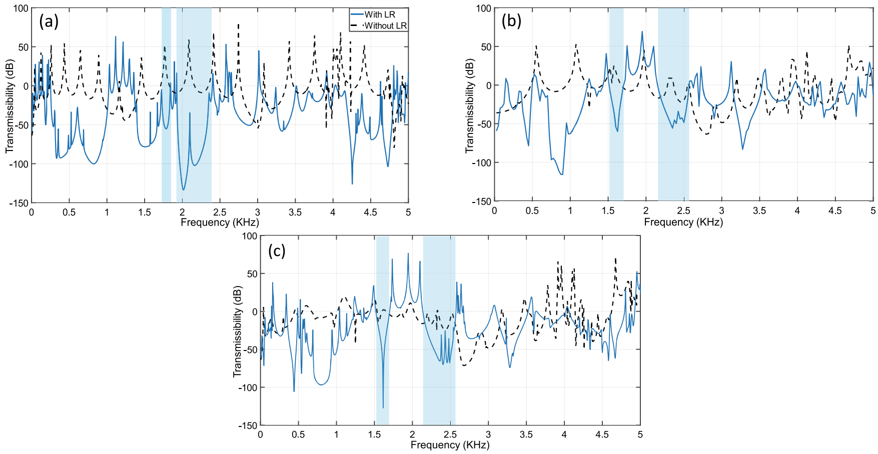

Fig. 3 Transmissibility of metamaterial beam for **a** flexural, **b** torsional, and **c** coupled vibration waves

quantity of single cells increases, the vibration waves are attenuated precisely in the frequency region where band gaps occur, as shown in Fig. 4d. This study suggests that a maximum number of unit cells should be taken to precisely identify the band structures from the transmissibility plot.

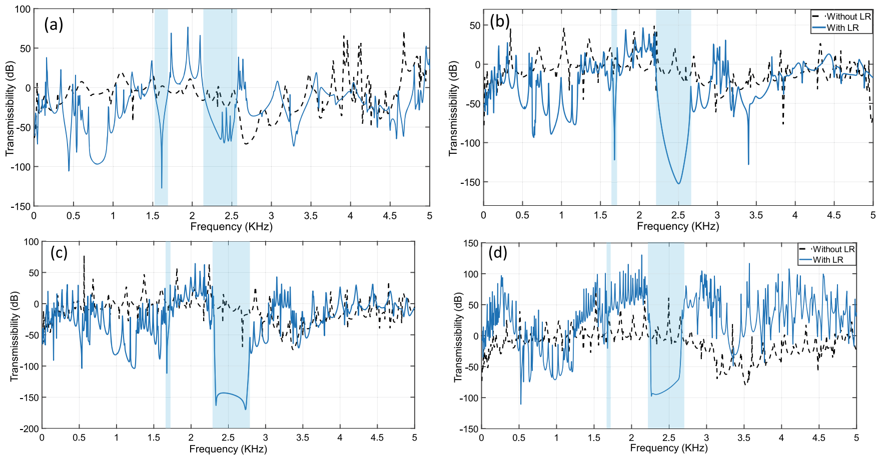

Fig. 4 Effect of number of unit cells **a** 6, **b** 10, **c** 15, and **d** 20 on the band gap

4.3 Resonator Parametric Study

The variation in stiffness (K_1) of the local resonator column and tip mass (m_2) on the width and position of the bandgap is investigated. This study is done with 20-unit cells of metamaterial beam for coupled vibration waves. The response of the metamaterial beam from the transmissibility curves is shown in Figs. 5 and 6.

The stiffness influence on the band gap is determined by an increase of 67%, 25%, 0% and decrease of 20% in stiffness, as shown in Fig. 5. The results show that the BG location shifts to a higher frequency range with an increase in stiffness and is indicated by a black arrow in Fig. 5. The bandwidth is also increased, as given in Table 2. The maximum performance of transmissibility reduction occurs in case of a 25% increase in stiffness.

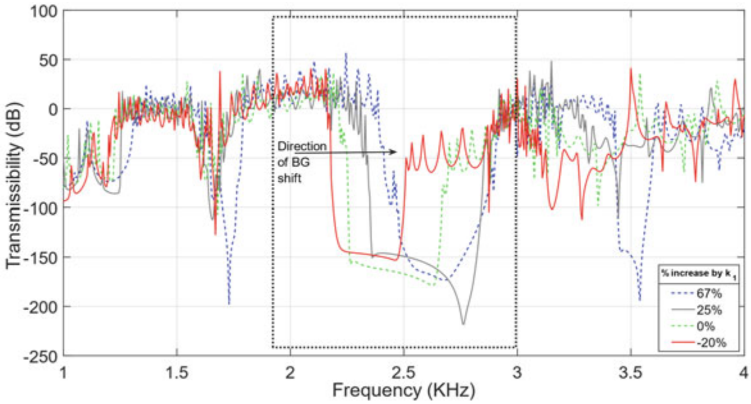

Fig. 5 Effect of resonator column stiffness on the bandgap behaviour

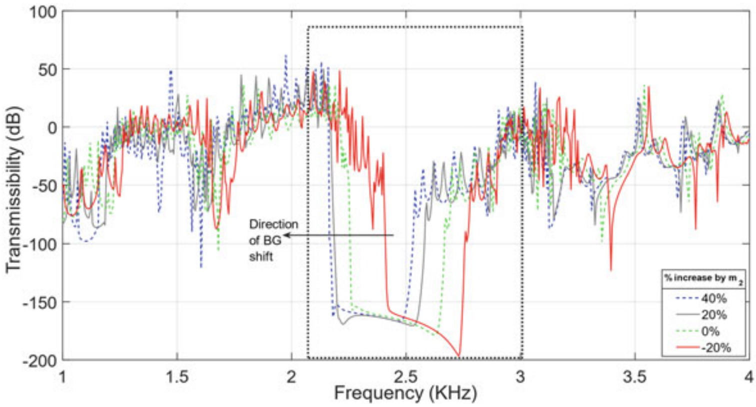

Fig. 6 Effect of resonator tip mass on the bandgap behaviour

Table 2 Bandwidth for various column resonator stiffness

% increase by k_1	Bandwidth (KHz)
− 20	0.33
0	0.48
25	0.51
67	0.62

Table 3 Bandwidth for various resonator tip mass

% increase by m_2	Bandwidth (KHz)
− 20	0.39
0	0.48
20	0.42
40	0.38

The tip mass resonator influence on the bandgap of the metamaterial beam is studied. It is performed by variation in mass by 40%, 20%, 0, and − 20%, as shown in Fig. 6. It is observed from the transmissibility curves that the location of the stop band is moved towards the low-frequency region when the amount of mass rises in Fig. 6. The performance of transmissibility reduction increases with a decrease in mass. The maximum bandwidth occurs for a base amount of mass, given in Table 3.

5 Conclusions

The flexural, torsional, and coupled waves are analysed in the thin-walled metamaterial beam using local resonators. Using Bloch's theorem, the band gap of the beam with local resonators is investigated by studying the single cell. To verify the band gap, the transmissibility analysis of the finite beam with local resonators is examined using the six-unit cells. The band gap and transmissibility are determined by the FE approach using COMSOL Multiphysics software 6.0. The results are concluded as follows:

1. Two band gaps appear around 1777–1793 and 2275–2470 Hz frequency ranges in which all wave types are attenuated.
2. The band gap results are well-matched with the transmissibility analysis of a metamaterial beam.
3. The metamaterial beam with 10-, 15-, and 20-unit cells are studied. The reduction in vibration transmission is accurately examined when a large quantity of single cells are taken.
4. The effect of resonator parameters like column stiffness and tip mass significantly impacts on the band gap behaviour.

5. The location of the bandgap shifts to a higher frequency value, and bandwidth increases when the stiffness value is increased.
6. In case of mass variation, the band gap location changes to a lower frequency region as increases in resonator mass. The reduction in vibration transmission increases with decreases in mass.
7. This work depicts that local resonator metamaterial can efficiently attenuate the vibration waves in the thin-walled beam.

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Modeling of Indentation on Wooden Surface in Drop Tower Impact

Sanketh Tonannavar and N. D. Shivakumar

Abstract Wood is a lightweight material that finds applications in sports equipment and structural members despite the advent of many advanced synthetic composite structures. This paper investigates the inelastic percussion mechanics of wood in low-speed impact events like drop-tower impact experimentally, theoretically, and numerically. A 22 kg tup, mounted with a hemispherical punch, is dropped onto wooden plates from a specific height. Indentation followed by secondary cracking observed on the surface of the wood is analyzed from the Hunt-Crossley approach in formulating contact force model, phase diagrams and impact parameters. A simple elastoplastic numerical model aids in understanding indentation and damping, despite wood being an orthotropic material. The phase diagrams show that the damping effect due to indentation renders a coefficient of restitution (COR) less than or equal to 0.4. Additionally, the results reveal the vibromechanics effect (damping and kinematic restitution coefficient) on interaction forces between the tup and plate.

Keywords Wood · Indentation · Low-speed impact · Vibromechanics · Damping

1 Introduction

Wood is an energy-absorbing material used for millennia in tools, weaponry, and construction fields. It is also used in sports applications like cricket bats (high impulse) to bowling floors (low impulse). The wood used for such purposes is almost homogeneous and designed to withstand high stresses through various seasoning methods. On the other hand, natural wood used in low-end applications like carpentry artifacts [1] is often subjected to sharp/blunt nail and hammer impact. Such wood is inevitably heterogeneous and is vulnerable to damage and fracture under different

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D. Kumar et al. (eds.), *Advances in Applied Mechanics*, Lecture Notes in Mechanical Engineering, https://doi.org/10.1007/978-981-97-0472-9_2

loading directions. For example, when an untreated wooden plate is loaded statically or dynamically perpendicular to the fibers with a spherical punch of specific geometry, it leads to an indentation or plastic deformation in the vicinity of the contact area. In this context, McKittrick et al. [2] and Tonannavar et al. [3] have investigated the indentation properties of wood in drop- tower impact experiments inflicted by a hard spherical tup. In formulating an indentation model from Hertzian impact theory, plastic indentation was estimated to be 60% [3] more than the elastic indentation lending low COR (≤ 0.4). Therefore, it is essential to model the impact response of such soft targets under low-speed impact considering the damping effect.

Low-speed impact occurs between objects of several combinations of geometries and materials for which the COR values are moderate or low. Typically, when two spherical bodies collide, their relative velocity changes due to interpenetration at the contact point. Researchers have worked on the classical problems of contact-impact mechanics since the nineteenth century. The foundation theory for this problem was first given by Hertz in 1880, which deals with the quasi-elastic collision of two spheres. Later, Timoshenko developed expressions for impact force, normal approach, and impact time based on Hertzian mechanics. On the other hand, the model described by Kelvin-Voigt includes viscous damping by hypothesizing the contact region with a spring-mass damper system. This linear model shows the presence of hysteresis during the impact; however, the force equation becomes negative during the restitution phase indicating the tensile forces between the parting bodies. All these theoretical frameworks proposed are valid for only perfectly elastic collisions. Later, Hunt and Crossley [4] proposed a more realistic and nonlinear model which expresses damping force being proportional to the power of deformation. Lankarni and Nikravesh [5], based on the same grounds, develop a direct relationship between contact force and the COR. Recently, Flores et al. [6] and Alaci et al. [7] deduced expressions of viscous damping coefficient for the impact of soft spheres with COR between 0 and 0.4. Additionally, Flores' flexible mathematical model to determine contact forces between soft and rigid bodies explains the importance of damping in impact.

Taking interesting experimental findings and theoretical analyses of the impact event by previous researchers as the motivation, this work aims to develop a mathematical model to describe the impact of a rigid steel impactor colliding with a relatively soft target like a wooden plate. As observed from the experiments, the indentation in wood reduces rebound COR for a threshold drop impact velocity. It is imperative to model phenomena involving dissipative processes pertaining to impact events, such as indentation and fracture. Therefore, we initially begin the analysis by hypothesizing that *damping effects significantly contribute to reducing the COR primarily through indentation.* The main objective of this paper is to propose an analytical model for indentation from the Hunt-Crossley method to account for damping by transforming the governing equation with suitable modifications. Additional support from experimental results and numerical simulations reasonably validates the impact model.

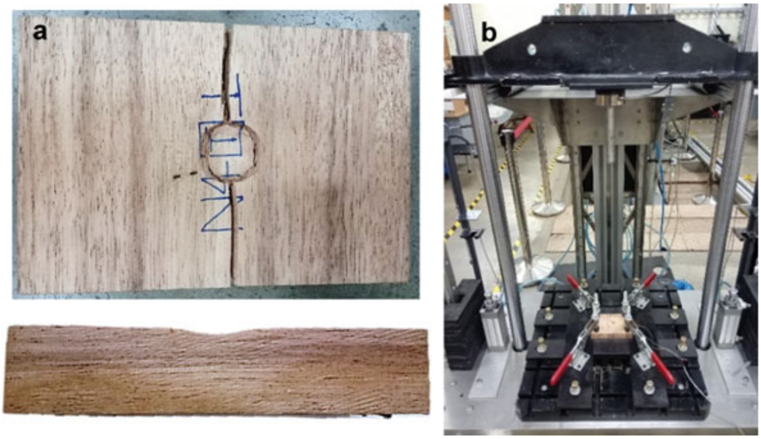


Fig. 1 Materials and methods [3]. **a** Indentation after impact showing top view (*upper figure*) and sectional view (*lower figure*). **b** Drop- tower impact test setup

2 Experimental Procedure

Figure 1a shows the indented and fractured wood specimen and Fig. 1b shows the drop tower impact setup designed as per the ASTM standard D7136 [8]. The experiments were conducted on dried and untreated natural wooden plates with dimensions of $150 \times 100 \times 25$ mm, placed over a rigid foundation, and firmly mounted by toggle clamps. A 22 kg tup mass with a 25 mm diameter indenter was released pneumatically via guide rails up to a drop height of 0.6 m, and an arrangement of stop blocks ensured a single impact onto the specimen. More details regarding experiments can be found in the authors' preceding paper [3]

3 Impact Model

In formulating the impact model, we postulate that an elastic indentation is perceived for very small drop heights, resulting in higher values of COR (0.9, say). Also, the effect of material damping is more prominent in indentation than vibrational damping. Figure 2 shows the free body diagram of the impact system representing tup with a spherical indenter of radius R dropped from a fixed height H. When the tup contacts the wooden plate, it is excited harmonically, whose response is given by $Y(t)$. Consequently, the relative penetration is given by [6]

$$y = H - [R + Y(t)]. \tag{1}$$

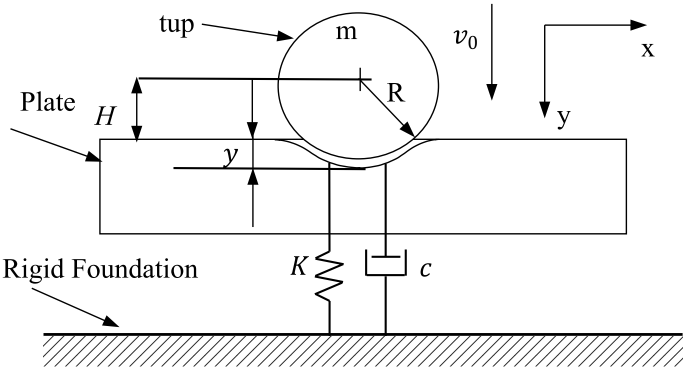


Fig. 2 Equivalent system of the impacting bodies: The model represents a non-conformal contact between a sphere and plane, which is a simplified depiction of impact of elastic spheres

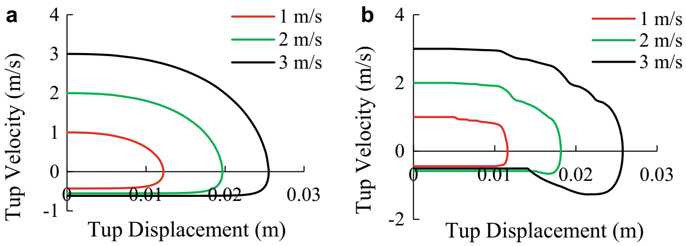
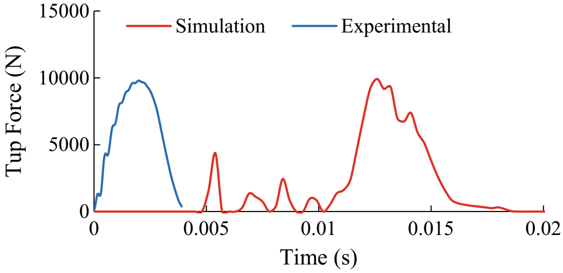


Fig. 3 Phase diagrams showing tup impact velocity and tup rebound velocity as a function of indentation or tup displacement. **a** Exponential model. **b** Simulations

Fig. 4 Force–time response plots from experiments [3] and simulations



For $H > R$, implies that there is no contact; $H = R$: contact is established and $y = Y(t)$ given by the free vibration of the contact spring and when $H < R$, contact is continued, and the indentation begins.

The damping coefficient depends on the COR and effective stiffness of the vibrating system. The inherent heterogeneity of the wood renders variable restitution coefficients, and corresponding damping factors can be calculated. Therefore, owing to the postulates made earlier, the analysis starts by modifying the hysteresis damping function given by Hunt and Crossley [4] using an exponential function, to be valid for COR $\rightarrow$ 0; in other words, the governing equation becomes feasible for soft impacts with $0 < \text{COR} \leq 0.4$. However, the original Hunt-Crossley equation can be reverted in the limiting case for COR $\rightarrow$ 1. Therefore, the damping function and corresponding equation of motion can be written as

$$f(c) = ce^{(\dot{y}/v_0)^{-n}}, \tag{2}$$

where $c = \lambda(y)^p(\dot{y})^q$, such that $\lim_{y \to 0} f(c) \rightarrow c$ to retain the original form and $n = 3/2$ for the contact between a sphere and a half-space. The values of λ, p, and q are determined suitably from boundary conditions [4].

$$m\ddot{y} + f(c)\dot{y}y^n + Ky^n = 0. \tag{3}$$

The above equation can be further simplified as

$$\ddot{y} + \alpha e^{(\dot{y}/v_0)^{-n}}\dot{y}y^n + \beta y^n = 0, \tag{4}$$

where $\alpha = c/m$ and $\beta = K/m$. Equation (4) does not yield a closed-form solution, so we solve it numerically. Let $y = z_1$ and $\dot{y} = \dot{z}_1 = z_2$. After simplifications and substitutions, Eq. (4) can be written as

$$\dot{z}_2 = -\alpha e^{(z_2/v_0)^{\frac{3}{2}}} z_2 z_1^{\frac{3}{2}} - \beta z_1^{\frac{3}{2}}. \tag{5}$$

The three coefficients m, c, and K must be known in order to solve Eq. (5) The generalized stiffness constant, K, can be calculated from the nonlinear Hertzian force relation given by $F = Ky^n$ where K is given as

$$K = \frac{4}{3}\left(\frac{1}{\sigma_1 + \sigma_2}\right)\sqrt{\frac{R_1 R_2}{R_1 + R_2}}, \tag{6}$$

where $\sigma_{1,2} = \frac{1-\nu_{1,2}^2}{E_{1,2}}$. E, ν, and R are Young's modulus, Poisson's ratio, and radius of the contacting bodies, and the suffixes 1 and 2 represent the tup and plate properties, respectively. For the present case, $R_2 = \infty$ and Eq. (6) simplifies to $K = \frac{4}{3}\left(\frac{\sqrt{R_1}}{\sigma_1+\sigma_2}\right)$. Similarly, the COR approaches zero for a higher degree of damping in the impact system. Therefore, the coefficient α is given by [6]

$$\alpha = \frac{3(1 - c_r)}{5c_r}\frac{K}{v_0}, \tag{7}$$

where c_{r} is COR and v_0 is the initial impact velocity. We primarily calculate the damping coefficient and the effective stiffness assuming elastic impact for a drop weight of 22 kg and a tup radius equal to 12.5 mm, falling with a velocity of 1 m/s. Accordingly, α and β turn out to be 7×10^4 $Ns\sqrt{m}$ and 6×10^4 $Nm^{\frac{3}{2}}$, respectively.

4 Results and Discussion

Figure 3 shows the phase diagrams outlined from the theoretical model and simulations for three impact velocities. From simulations, it was found that an impact velocity of 1 m/s causes indentation allowing the tup to rebound with a COR of 0.4. Further, if the impact velocity is increased to 2 m/s, then COR reduces to 0.25, and finally, the COR reduces to 0.1 for the impact velocity equal to 3 m/s. In experiments, impacting with still higher velocity triggers several cracks accompanied by indentation, further decreasing COR rapidly to zero. In this context, the COR, determined by Stronge [9], decays exponentially as a function of relative impact speed due to nonlinear spring and dashpot in the system. Along the same lines, wood under low-speed impact acts as a soft (nonlinear) spring dissipating the impact energy spontaneously through indentation, vibrations, and fracture.

The simulation results are corroborated and contradicted at the same time by Shivaswamy [10], who showed that only damping plays a significant role in energy dissipation under low-speed impact. In this paper, however, we discover that the plastic indentation, in conjunction with the harmonic response of the plate, plays a chief role in inducing internal damping at a critical impact speed in a soft target like wood. The results agree well with the works by Alaci [7] and Flores [6].

With this idea, therefore, the impact indentation is theoretically modeled by introducing an exponentially decaying damping function that demonstrates overdamping of the system (damping ratio, $\zeta > 1$). Accordingly, the analytical model describes the indentation damping response of the plate under low-speed impact through phase diagrams and impact variables. Unlike elastic impact, whose phase paths are nearly semi-circular, the phase paths of plastic impact become semi-elliptical with increasing velocity, which is evident from Fig. 3a, b. Nonetheless, part of the tup energy, in addition to damping energy, could have been distributed to the rigid foundation, which needs further investigation.

In the force–time plot shown in Fig. 4, the fluctuations in the compression phase of the force show the internal vibrations in the plate due to impact excitation by the tup. The number of oscillations is 6 in the compression phase for both experimental and numerical results. However, damped oscillations can be observed in the restitution phase, as displayed in Fig. 4. It should also be noted that the compression force regime decreases due to higher contact duration resulting in indentation induced damping, which instantly diminishes the restitution phase. Similarly, the time interval for the transient force is merely 3 ms, indicating a sudden drop in the contact force. This can be understood by revisiting Eq. (5) in which the damping force incorporates the exponential term as a function of impact velocity, with the nonlinear Hertzian deformation. Hence, the experiments, simulations, and theoretical models highlight the importance of exponential damping due to indentation and harmonic vibrations of the plate.

5 Conclusion

The present paper aims to investigate the impact response of wooden plates in a drop-tower test from a mechanics perspective: the phase plane representations from finite element simulations and the proposed model characterize the internal damping disseminated in the wood under low-speed impact. On the other hand, the impact force–time variation investigated here emphasizes the plate reaction to harmonic excitation during impact. Hence, this article highlights the influence of viscous damping in natural materials like wood, used as soft targets, as the combined vibromechanics effect. The research provides valuable data and quantitative guidelines for designing woodworking products subjected to low-speed impact.

Acknowledgements We greatly acknowledge Prof. K. R. Y. Simha, Department of Mechanical Engineering, IISc, for his valuable inputs and guidance throughout this project. The authors are thankful to Prof. D. Roy Mahapatra and Prof. D. Sivakumar, Department of Aerospace Engineering, IISc, for their support in availing experimental and high-speed photography setup.

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