

Yifei Yu · Chaoqun Liu

# Mechanism of Hairpin Vortex Formation by Liutex

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## Preface

“Turbulence” is still covered by a mystical veil and remains a mystery of nature after over a century of intensive study. The following comments are cited by Wikipedia web page at <http://en.wikipedia.org/wiki/Turbulence>. Nobel Laureate Richard Feynman described turbulence as “the most important unsolved problem of classical physics” (USA Today 2006). According to an apocryphal story, Werner Heisenberg was asked what he would ask God, given the opportunity. His reply was: “When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first.” (Marshak, 2005). Horace Lamb was quoted as saying in a speech to the British Association for the Advancement of Science, “I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic” (Mullin 1989; Davidson 2004). Note that both Heisenberg and Lamb were not optimistic for the turbulence study. Anyway, the mechanism of turbulence formation and sustenance is still a very tough question for research.

As widely observed by experiment and confirmed by direct numerical simulation (DNS), the hairpin vortex plays a critical role in flow transition from laminar state to a turbulent state at least for the type of the natural boundary layer transition (see Kleiser and Zang, 1991). Apparently, the mechanism of the hairpin vortex formation is the priority of research on flow transition and turbulence. It has been discussed for several decades that where and how the  $\Lambda$ -shaped vortex and hairpin vortex are generated, but there still has not been a convincing answer yet. Hama et al. (1963) described the process of formation of ‘ $\Lambda$ -shaped vortex’ in the vicinity of the Lambda vortex tip. They found that ‘...a simplified numerical analysis indicates that the hyperbolic vortex filament deforms by its own induction into a milk-bottle shape (the ‘Lambda vortex’) and lifts up its tip...’ It was found by Knapp and Roache (1968) that the ring-like vortex rotates by about  $90^\circ$  to an upright position and then dissipates. Hama and Nutant (1963) suggested that ‘...the true cause... (of randomization) ...is in the complicated tangling at the neck of the  $\Lambda$ -shaped vortex loop which might have resulted from the higher-order deformations of a curved vortex filament by its own induction interacted upon by the high-shear layer.’ The same idea was confirmed later by Moin et al. (1986) in a direct

numerical simulation. They concluded “We have demonstrated by numerical experiments that a curved filament of concentrated vorticity evolves into a vortex ring as a result of self-induction effects.” In the same paper, a hairpin vortex “pinch-off” was found and the vortex ring moves away rapidly after the vortex pinch-off, which indicates turbulence was produced by “vortex breakdown”. It looks like that the mechanism of the  $\Lambda$ -shaped vortex and the hairpin vortex formation was discovered. These conclusions have dominated the turbulence community for several decades and cited by countless research papers.

However, if these conclusions were carefully checked, it would not be difficult to find that they have many self-contradictions. First, the target here is the mechanism of the  $\Lambda$ -shaped vortex and the hairpin vortex formation, but the mechanism given by Moin et al. (1986) is about vorticity. As Liu et al. (2014, 2018) pointed out that vorticity is a vector and vortex is another vector, but they are two totally different vectors with very weak correlation in lower boundary layer (Robison 1991). Liu et al. further clearly state that vortex is not “vorticity tube or vorticity filament.” The self-deformation mechanism of vorticity has nothing to do with the mechanism of the hairpin vortex formation, which is formed under a completely different mechanism. Second, in the paper by Moin et al. (1986), vorticity line development was simulated, but the divergence of vorticity is zero, which means vorticity filament or vorticity tube (they mistakenly called vortex filaments) can never be “pinch-off” or “breakdown”, which is directly against the law of zero vorticity divergence everywhere.

Later it was shown by Borodulin and Kachanov (1995) that the spikes (associated with the ring-like vortices) represent very conservative, stable structures, which do not breakdown for a long distance downstream, including stages when the flow becomes strongly randomized in the near-wall region. The Lambda vortex legs turned out to be rather stable despite their shapes were distorted with some spiral non-uniformities in several quantitative experimental visualizations by Borodulin et al. (2003).

The goal of this book is trying to give a clear picture to reveal the mechanism of the Lambda vortex legs and hairpin vortex formation based on high-order DNS for flow transition and turbulence by using the Liutex theory which is the state-of-the-art tool of vortex definition and identification.

The key issue is still the conclusion made by Liu (2014) that “shear instability is the mother of turbulence” or in other words, how shear or vorticity becomes flow rotation or Liutex. This book will explain how hairpin vortex and Liutex structure are created in a boundary layer due to the noise environmental surroundings, step by step.

In writing this book, Yu would like to thank Dr. Chaoqun Liu’s guidance and help during his Ph.D. student life. He also wants to thank his parents, Xiaofeng Yu, Qinghua Hou and girlfriend Yue Yang for their understanding and unconditional support. Liu appreciates the efforts made by the UTA Team including Zhining Liu, Li Jiang, Hua Shang, Ping Lu, Yonghua Yan, Yiqian Wang, Yisheng Gao, Xiangrui Dong, Jianming Liu. Liu

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The authors welcome readers to provide feedback including criticisms and corrections.

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## Nomenclature

$M_\infty$	Mach number
$Re$	Reynolds number
$\delta_{in}$	Inflow displacement thickness
$T_w$	Wall temperature
$T_\infty$	Free stream temperature
$Lz_{in}$	Height at inflow boundary
$Lz_{out}$	Height at outflow boundary
$Lx$	Length of computational domain along x direction
$Ly$	Length of computational domain along y direction
$x_{in}$	Distance between leading edge of flat plate and upstream boundary of computational domain
$A_{2d}$	Amplitude of 2D inlet disturbance
$A_{3d}$	Amplitude of 3D inlet disturbance
$\omega$	Frequency of inlet disturbance
$\alpha_{2d}, \alpha_{3d}$	Two and three dimensional streamwise wave number of inlet disturbance
$\beta$	Spanwise wave number of inlet disturbance
$R$	Ideal gas constant
$\gamma$	Ratio of specific heats
$\mu_\infty$	Viscosity
$x, y, z$	Streamwise, spanwise, normal directions
$\vec{R}$	Liutex vector
$R_x, R_y, R_z$	Components of Liutex vector in the x, y, z directions

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Turbulence remains an unsolved scientific puzzle, bearing immense importance not only in the realm of science but also in numerous engineering applications such as aerospace engineering, mechanical engineering, hydrodynamics, meteorology, and bioengineering. Despite its widespread relevance, the fundamental physics behind turbulence generation and maintenance continues to elude researchers, offering an ongoing and intriguing avenue for exploration.

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## 1.1 Classical Theory on Turbulence Generation Revisit

### 1.1.1 Richardson Vortex Cascade Revisit

Classical turbulence theory, notably Richardson's work in 1924 (see Fig. 1.1a, b), introduced the concept of vortex chains. Richardson's famous poem states, "Big whirls have little whirls, which feed on their velocity; And little whirls have lesser whirls, and so on to viscosity in the molecular sense." However, despite this theoretical framework, the vortex chain generated by large vortex breakdown has never been observed in practice. Recent research, such as Direct Numerical Simulation (DNS) conducted by Liu et al. in 2014, has revealed that turbulence comprises vortices of various sizes, ranging from large to small. Remarkably, all these vortices are generated due to shear layer instability of the Kelvin–Helmholtz type, and no instances of vortex breakdown have been observed. In fact, neither experimental nor computational evidence of the so-called "eddy cascade" has been documented to date.