International Perspectives on the Teaching and Learning of Mathematical Modelling

Hans-Stefan Siller
Vince Geiger
Gabriele Kaiser *Editors*

Researching Mathematical Modelling Education in Disruptive Times

Springer

International Perspectives on the Teaching and Learning of Mathematical Modelling

This book series will publish various books from different theoretical perspectives around the world focusing on Teaching and Learning of Mathematical Modelling at Secondary and Tertiary level. The proceedings of the biennial conference called ICTMA, organised by the ICMI affiliated Study Group ICTMA (International Community of Teachers of Mathematical Modelling and Applications) will also be published in this series. These proceedings display the worldwide state-of-the-art in this field and will be of interest for a wider audience than the conference participants. ICTMA is a worldwide unique group, in which not only mathematics educators aiming for education at school level are included but also applied mathematicians interested in teaching and learning modelling at tertiary level are represented. ICTMA discusses all aspects related to Teaching and Learning of Mathematical Modelling at Secondary and Tertiary Level, e.g. usage of technology in modelling, psychological aspects of modelling and its teaching, modelling competencies, modelling examples and courses, teacher education and teacher education courses.

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Editors

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Editors
Hans-Stefan Siller
Institut für Mathematik
Universität Würzburg
Würzburg, Germany

Gabriele Kaiser
Didaktik der Mathematik
Universität Hamburg
Hamburg, Germany

Vince Geiger
Institute for Learning Sciences and Teacher Education
Australian Catholic University
Brisbane, QLD, Australia

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Series Preface

Applications and modelling and their learning and teaching in school and university have become a prominent topic for many decades now in view of the growing worldwide relevance of the usage of mathematics in science, technology and everyday life. There is consensus that modelling should play an important role in mathematics education, and the situation in schools and university is slowly changing to include real-world aspects, frequently with modelling as real world problem solving, in several educational jurisdictions. Given the worldwide continuing shortage of students who are interested in mathematics and science, it is essential to discuss changes of mathematics education in school and tertiary education towards the inclusion of real world examples and the competencies to use mathematics to solve real world problems.

This innovative book series *International Perspectives on the Teaching and Learning of Mathematical Modelling*, aims at promoting academic discussion on the teaching and learning of mathematical modelling at various educational levels all over the world. The series will publish books from different theoretical perspectives from around the world dealing with Teaching and Learning of Mathematical Modelling at school level and at tertiary level. This series will also enable the *International Community of Teachers of Mathematical Modelling and Applications* (ICTMA), an International Commission on Mathematical Instruction affiliated Study Group, to publish books arising from its biennial conference series. ICTMA is a unique worldwide educational research group where not only mathematics educators dealing with education at school level are included but also applied mathematicians interested in teaching and learning modelling at tertiary level are represented as well. Six of these books published by Springer have already appeared.

The books display the worldwide state-of-the-art in this field, most recent educational research results and new theoretical developments and are of interest for a wide audience. Themes dealt with in the books focus on the teaching and learning of mathematical modelling in schooling from the early years and at tertiary level including the usage of technology in modelling, psychological, social, historical and cultural aspects of modelling and its teaching, learning and assessment, modelling competencies, curricular aspects, teacher education and teacher education courses.

The book series aims to support the discussion on mathematical modelling and its teaching internationally and will promote the teaching and learning of mathematical modelling and research of this field all over the world in schools and universities.

The series is supported by an editorial board of internationally well-known scholars, who bring in their long experience in the field as well as their expertise to this series. The members of the editorial board are: Maria Salett Biembengut (Brazil), Werner Blum (Germany), Helen Doerr (USA), Peter Galbraith (Australia), Toshikazu Ikeda (Japan), Mogens Niss (Denmark), and Jinxing Xie (China).

We hope this book series will inspire readers in the present and the future to promote the teaching and learning of mathematical modelling all over the world.

Series Editors

Hamburg, Germany — Gabriele Kaiser

Ballarat, Australia — Gloria Ann Stillman

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Part I
Introduction

Chapter 1
Researching Mathematical Modelling Education in Disruptive Times—An Introduction

Hans-Stefan Siller, Vince Geiger, and Gabriele Kaiser

Abstract Mathematical modelling is now a fundamental activity within general education. Its importance is evident from recent global events, which will impact on the lives of future generations. Events and phenomena, such as pandemics or the intensive debate before reaching consensus on the Paris climate agreement, show the need to make informed evidence-based decisions, which are generally rely on mathematical models. Thus, an understanding of mathematical modelling and models needs to be promoted across schooling and in other institutions devoted to education. Recent events and phenomena have also demonstrated the key role of mathematics in forming judgements and making decisions within societal contexts. This volume, which is the result of a conference entitled *Researching Mathematical Modelling Education in Disruptive Times*, reflects the response of mathematical modelling education researchers to the consequences of the COVID-19 pandemic. The 55 papers, organised into 6 sections, included in this volume provide insight into research on mathematical modelling in current disruptive times.

Keywords Mathematical modelling · Disruptive times · Informed citizenship · Critical analysis · Interdisciplinary approaches · Learning and teaching of mathematical modelling

Judgements and decisions made on a day-to-day basis and in planning for the future are increasingly informed by advice, claims, opinions and arguments that are based on mathematics and statistics. This has been brought into stark relief by impactful and

H.-S. Siller (✉)
Julius-Maximilians-Universität Würzburg, Würzburg, Germany
e-mail: hans-stefan.siller@uni-wuerzburg.de

V. Geiger
Australian Catholic University—Brisbane, Banyo, QLD, Australia
e-mail: vincent.geiger@acu.edu.au

G. Kaiser
University of Hamburg, Hamburg, Germany

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far-reaching events—disruptive phenomena. Such phenomena include the COVID-19 pandemic, climate change, food insecurity due to current armed conflicts, or increasingly high levels of poverty due to global economic disruption. The role of mathematics in judgement formation and decision making is particularly evident in the use of mathematical models to understand such phenomena. Mathematical modelling is key in addressing issues related to sustainability, an area of increasing global concern, through its predictive power (Geiger, 2024).

An understanding of mathematical models and mathematical modelling should now be considered a key capability for active participation in an increasingly changing world characterized by rapid technological, economic and social change. Reports of and claims about the state of disruptive phenomena, in the media and those made by government agencies or commercial entities, regularly rely on the use of mathematical models and the outcomes of a modelling process. Such reports and claims are often used to argue for necessary changes to the behaviour of citizens and, in some cases, the restriction of personal freedoms. Capability with mathematical models and modelling can enable the critical analysis of such claims and the evaluation of associated advice—informing decisions made by citizens for themselves, their families, and society at large (Vos, 2024).

There are now many aspects of society in which modelling underpins processes and activities—often without the knowledge of those involved. Models in the form of algorithms, for example, are used to direct targeted advertising via social media and to automate other activities such as debt collection and the screening of job applications. A sound understanding of the role mathematical modelling plays in such activities is essential when considering the importance and type of legislation required to ensure reasonable, inclusive and ethical conduct when using models in a way that impact on the life of citizens. Such capability is even more important in the context of rapidly developing artificial intelligence tools and other digital resources.

Understanding models and developing capabilities in mathematical modelling has, therefore, significant implications for education and educational research when considering how to equip both young people and adults for current and future challenges. The research studies presented by authors in this book are aimed at generating new theoretical insight into the nature of mathematical modelling, its role in society and for critical citizenship and how it is taught and learned. This research is presented under the following sections: theoretical approached to mathematical modelling; teaching of mathematical modelling; learning of mathematical modelling; and interdisciplinary approaches to mathematical modelling.

- **Theoretical approaches to mathematical modelling**

There have been significant theoretical developments in mathematical modelling related to understanding how to best provide opportunities for learners to develop the capabilities needed to address real-world problems encountered within both professional and everyday life. More recently, there has been an explicit research focus on the challenges associated with solving real-world problems in terms of interdisciplinary thinking. This means that an understanding of how extra-mathematical knowledge is connected to different types of interdisciplinary knowledge is needed for the development of effective approaches to the solution of a problem. Such an understanding is also essential for a modeller to develop strategies necessary to address a problem (Siller et al., 2023a).

There have also been theoretical developments associated with the use of digital resources within mathematical modelling. While both mathematical and extra-mathematical knowledge is essential for engaging in modelling, the introduction of digital resources into the process of modelling can shape the development of a model in dramatically different ways (e.g. Geiger et al., 2018). Digital resources can also play a role in enhancing the mathematical capabilities of a modeller and/or provide greater access to an understanding of extra-mathematical contexts (Siller et al., 2023b).

The contributions to this book that shed light on mathematical modelling from a theoretical perspective will provide a focus for further discussion in the community. These contributions adopt a range of perspectives, such as those related to cultural, country-specific, and curricular influences on mathematical modelling. A key contribution to these theoretical approaches can be found in the chapter by Vorhölter entitled *Structure, role, and promotion of metacognitive modelling strategies: theoretical derivations and empirical findings* (Vorhölter, 2024). A summary of authors and their chapter title which are related to this theme is listed below:

- Cárcamo Mansilla, N.; Aravena Díaz, M. D.; Berres, S.-Metacognitive Strategies in Mathematical Modelling with Groups of Engineering Students
- de Almeida, L. M. W.; de Castro, E. M. V.-Providing characteristics of metacognition in mathematical modelling activities
- Frejd, P.; Bergman Ärlebäck, J.-Mathematical modelling in vocational mathematics education-a literature review focusing on vertical and horizontal mathematising
- Galbraith, P.-Modelling, Teaching, Reflecting: What more I have learned?
- Jakobsen, A.; Mhakure, D.-A Systematic Review of Research on the Use of Mathematical Modelling in the South African Education

- **Teaching mathematical modelling**

The teaching of mathematical modelling is now a firmly established field of research internationally. Aligned with this theme has been significant research into modelling competences and sub-competences.

A range of definitions exists for the construct of modelling competence, using either a top-down or a bottom-up approach (Niss & Blum, 2020). In various empirical studies, sub-competencies of mathematical modelling are distinguished (e.g. Gerber et al., 2024; Siller et al., 2023c), according to different phases of the modelling cycle. At the same time, other studies have had a broader focus, for example, on metacognitive and social skills (for a comprehensive overview see Cevikbas et al., 2022). The chapter by Lu and Ma (2024) on *The Possibility of Modelling as a Creativity-Demanding Activity: The Perspective of Teachers' Professional Development* provides insight into this area of research.

Practicing teachers have contributed to a number of chapters listed below—such collaborations between practitioners and researchers provide original insight into the theory practice gaps that exist in the field of modelling.

These contributions are listed below:

- Blum, W.; Niss, M.-Origin and development of the notion of mathematical modelling competency/competencies
- Durandt, R.; Blum, W.; Lindl, A.-The Effects of an Independence-Oriented Teaching Design on the Development of Tertiary Students' Modelling Competency
- Lode, B.-A tool for authentic assessment? Preservice teachers' initial learning to teach mathematical modelling in a school setting

In addition, the following contributions highlight developments in the diverse range of approaches for the education of future teachers and those already in-service in the context of mathematical modelling:

- Barquero, B.; Bosch, M.; Wozniak, F.-Moving beyond *mute* modelling praxeologies in pre-service elementary teacher education
- Cevikbas, M.-Utilizing Explanatory Videos in Flipped Mathematical Modelling Classrooms
- Dalto, J. O.; Borssoi, A. H.; da Silva, K. A. P.-Teachers in Continuing Education: planning of a Mathematical Modelling activity
- Gerber, S.; Quarder, J.; Greefrath, G.; Siller, H.-S.-Pre-service teachers' self-efficacy for teaching simulations and mathematical modelling with digital tools
- Hansen, R.-Authenticity in preservice teacher-designed modelling tasks

These contributions are complemented by the following chapters that focus on the principles that support learners in effective engagement with modelling tasks:

- Aroeira, A. J.; Carreira, S.; da Ponte, J. P.-Teacher strategic interventions to support students in constructing the model of the situation in a modelling task
- Brown, J. P.; Stillman, G. A.-Teaching Tactics to Manage Mathematisation during Mathematical Modelling
- Jung, H.; Brady, C.-Teachers Posing Model-Eliciting Activities to Connect Socially-Relevant Perspective with Closely Connected Mathematical Ideas
- Spooner, K.-Investigating Independent Student Work and Instructor Guidance for Tertiary Mathematical Modelling Activities
- Wiehe, K.; Schukaljow, S.; Krawitz, J.; Rakoczy, K.-The OModA Project: Designing a Teaching Method to Help Students Dealing with Openness in Modelling Problems

Knowledge of and attitudes towards mathematical modelling of future and in-service teachers have a significant influence on the effectiveness of instruction. The impact of attitudes and beliefs is highlighted in the following chapters:

- Buchholtz, N.-Error Diagnostic Competencies of Pre-service Teachers in Relation to Students' Activities on Mathtrails

- Ekol, G.-Investigating Beliefs Regarding Mathematical Modelling in Pre-Service Mathematics Teacher Education
- Helder, P.-Norwegian primary teachers' perceptions of mathematical modelling
- Ikeda, T.; Stephens, M.-Pre-service Teachers' Perceptions of Modelling Education
- Ledezma, C.; Font, V.; Sala-Sebastià, G.-What Modelling Tasks do Prospective Teachers Propose? Some Cases in Mathematics Teacher Education
- Saeki, A.; Yata, K.; Kaneko, M.; Kawakami, T.-Advancement of In-Service Teacher's Pedagogical Content Knowledge When Teaching Mathematical Modelling: Through the Interpretation and Design of Modelling Lessons

The research associated with these contributions and their associated research findings indicate that further professional learning opportunities in modelling are needed. The chapters listed in this section provide insight into which areas of the field require the greatest attention.

- **Learning mathematical modelling**

Studies with a focus on the learning of mathematical modelling are also often based on the notions of competences and sub-competencies. Chapters that adopt this perspective include:

- Ferrando, I.; Segura, C.; Castillo, J.-Regular and talented students' behaviour when solving modelling tasks: are there differences?
- Ozturk, A.-Examining Translanguaging Practices in Mathematical Modelling
- Zhou, S.; Steffensen, L.; Hansen, R.-Discourse structures when modelling a dream bag of candies

At the same time, the potential of modelling tasks to promote interdisciplinary capabilities that are needed in everyday life is receiving greater attention in research—both in modelling and more broadly in mathematics education. Maaß (2006) has argued that motivational and attitudinal factors are manifest in "willingness, abilities and skills in actions" (p. 24). Reasoning (Roan & Czocher, 2024) is also a key competence in mathematical modelling, required to justify claims and arguments and to understand and critically evaluate alternative opinions. To support the development of this type of reasoning, attention is needed to the development of principals of design of effective modelling tasks. This can be seen in the contributions of:

- Barlovits, S.; Ludwig, M.; Theiß, L.-Mathematics in the Open: Outdoor Modelling Supported by the Smartphone App MathCityMap
- Kämmerer, M.-Working on modelling tasks with much or little interest in the real-world context of the task—a comparison
- Kularajan, S.; Czocher, J. A.-How is Reasoning with Quantities Limited in Mathematical Modelling?
- Pla-Castells, M.; Garcia-Marques, M.-E.; Melchor, C.-The effect of heuristic training when solving a Fermi problem at primary school
- Roan, E.; Czocher, J.-Reasoning students employed when mathematizing during a predator–prey modelling task.

- Sevinc, S.; Ferrando, I.-Pictures in Fermi problems: How do prospective teachers use them in the modelling process?

- **Interdisciplinary approaches to mathematical modelling**

Science, Technology, Engineering and Mathematics (STEM) have been linked to careers that are needed for the maintenance of national prosperity and are seen as an underpinning of an informed and critical citizenry particularly in the context of rapid technological, industrial and social change (Geiger et al., 2023). It is widely recognized that while M is the last letter in the acronym, it is an underpinning of all other constituent disciplines. This can be seen, on the one hand, because mathematics is the basis for communication across the other disciplines (language) and, on the other, through the close connection between STEM and mathematical modelling. This connection lies in the interdisciplinary nature of STEM and the links between mathematical and extra-mathematical within mathematical modelling. Both approaches require the development of critical and creative thinking in applying interdisciplinary knowledge (mathematical/extra-mathematical) to real-world problems.

While it is widely recognized that mathematics is the key underpinning element of all STEM disciplines, the underrepresentation of mathematics within school teaching and learning practices in STEM has been identified by a number of authors (e.g. English, 2016, Mass et al., 2019). Mathematical modelling provides one avenue to address this issue via its potential to explore complex, interdisciplinary approaches within the context of school mathematics (e.g. Fisher & Galbraith, 2024). Further, while the nature of STEM is interdisciplinary, its connection to mathematical modelling provides opportunity to highlight the role of mathematics in addressing current global challenges (Siller et al., 2023a). The interdisciplinarity nature of modelling is discussed by Rosa (2024) in his chapter entitled *Ethnomodelling as the Glocalization of Mathematical Practices* situating ethnomodelling into ethnomathematics. The issue of interdisciplinarity is also addressed in the following chapters which have been organized around additional sub-themes:

Biology

- Ricks, T. E.—A Cross-Disciplinary Comparison of Mathematical Modelling
- Rogovchenko, S.; Rogovchenko, Y.—Boundary Crossing in Mathematical Modelling Activities with Biology Undergraduates

Education for Sustainable Development

- Adiguzel, C.; Cetinkaya, B.; Erbas, A. K.—Developing Conscious Citizens through Mathematical Modelling
- Wiegand, S.; Borromeo Ferri, R.—Teaching and learning of education for sustainable development through modelling activities with an integrative teaching approach

Engineering

- Rendón-Mesa, P. A.; Castrillón-Yepes, Al; Villa-Ochoa, A.—Situated Mathematical Modelling for Learning Product Design Engineering

Cultural Aspects

- Bergman Ärlebäck, J.; Albarracín, L.; Orey, D.; Rosa, M.; Sevinc, S.—Exploring the potential of using Fermi problems to elicit and develop cultural aspects in modelling processes
- Cordero, F.; Rosa, M.; Orey, D; Carranza, P.—Modelling in the Life of People: An Alternative Program for Teaching and Learning of Mathematics
- de Andrade Aguilar Delfiol, T.; Rosa, M.; Orey, D. C.—Humanizing the Prophets of Aleijadinho: A Qualitative Study of their Proportions through Ethnomodelling
- Orey, D. C.; Dutra, E. D. R.; Rosa, M.—Proposing an Ethnomodelling Pedagogical Action Contextualized on a Coffee Culture

General Interdisciplinary Approaches

- Araujo, J. de L.; Mirson, B. de P. M.—Interdisciplinary Aspects of Modelling in Mathematics Education Activities Conducted in South America
- Huincahue, J.; Gaete-Peralta, C.—Mathematical modeling in interdisciplinary academic scenarios: components for task construction
- Aravena-Díaz, M. D.; Rodríguez Gallardo, M.; Sanhueza Henríquez, S.; Cárcamo Mansilla, N.—Mathematical modeling to reduce gender gaps in STEM: characterization of STEM skills in high school students
- Fisher, D. M.; Galbraith, P.—Approaching Complex Systems Problems: Carrying Capacity of Earth
- Kawakami, T.; Saeki, A.—Roles of Mathematical and Statistical Models in Data-Driven Modelling: A Prescriptive Modelling Perspective

Physics

- Castrillón-Yepes, A.; Gonzáöez-Grisales, A. C.; Arango, S. M.; Rendón-Mesa, P. A.; Villa-Ochoa, J. A.—Interdisciplinary relationships between mathematics and physics through experimentation and mathematical modelling

Geometry

- Montejo-Gámez, J.; López-Centella, E.; Fernández-Ahumada, E.—Solving Estimation Tasks: Novel Features of the Emerging Models When Three-dimensional Geometry Becomes Relevant

Mathematical modelling is now an essential capability in attempts to address the challenges of disruptive phenomena, and for informed and participatory citizenship. Geiger takes up the former issue in his chapter entitled *The need for a critical orientation to mathematical modelling in times of disruption* (Geiger, 2024)**,** which is complemented by Vos' chapter, *The Public Understanding of Mathematical Modelling (PUMM)*, which addresses the later issue (Vos, 2024). These chapters signify the power of mathematical modelling in relation to promoting an understanding of the world and in developing responses to current challenges.

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Part II
Keynotes

Chapter 2
The Need for a Critical Orientation to Mathematical Modelling in Times of Disruption

Vince Geiger

Abstract In this chapter, the role of mathematical modelling in supporting citizens capacity to understand and evaluate reports related to disruptive phenomena, in the media and other public forums, is outlined and discussed. In pursuing this discussion, reference is made to the use of mathematical modelling in media items to evidence claims about the status of disruptive phenomena and their likely progress. The outputs of modelling are similarly used to evidence changes in public policy which impact on the way citizens live their lives. These roles have implications for the topics that should be explored during instruction and highlights the need for great focus on critical aspects of modelling.

Keywords Mathematical modelling · Disruptive phenomena · Critical thinking · Media

2.1 Introduction

Mathematical modelling is an essential capability for understanding and making predictions about natural and human generated phenomena, generating solutions to real-world problems, and providing the basis (evidence) for forming judgments and making decisions. This capability is increasingly recognised as key for informed and responsible citizenship and for the STEM capable workforce needed to address a growing number of challenges globally (e.g. Maaß et al., 2019). This recognition has seen the inclusion of mathematical modelling in of curriculum frameworks and documents internationally.

V. Geiger (✉)
Institute for Learning Science and Teacher Education, Australian Catholic University, Brisbane, QLD, Australia
e-mail: vincent.geiger@acu.edu.au

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The connections between mathematical modelling, citizenship, and work-life were highlighted during the recent lived experience of the COVID-19 pandemic, when models and modelling were used by the media to inform the public about the status of the crisis, report on predictions about its progress, and justify changes to public policy, including restrictions on citizens and their workplaces. While COVID-19 underscored the role of modelling during a time when most aspects of life were disrupted, it should be noted that the pandemic is only one type of disruption faced by society which are modelled on an ongoing basis. Other disruptions, while long term, have also come into stark relief in recent times, for example, global warming, economic decline, and sustainability. These forms of disruption are also described, analysed, and reported on using models and modelling.

The prevalence of models in public forums—used by governments, commercial entities, expert, and non-expert commentators to support propositions and claims—raises questions about the capabilities citizens require to interpret and evaluate information, claims, and arguments based on the results of mathematical/statistical modelling. These capabilities would appear to include an understanding of the nature of mathematical modelling and its processes as well as critical capabilities related to the evaluation of evidence. How these capabilities are related and how they need to be linked is the focus of this chapter.

In exploring the links between modelling and the criticality needed to evaluate evidence about the progress of disruptive phenomena and arguments for specific responses, I first provide a broad outline of the status of research and practice in mathematical modelling education internationally. Second, I present an argument for why a critical orientation is important when evaluating the use of mathematical modelling to report on and justify responses to disruptive phenomena in public forums. Third, the prevalence of mathematical modelling in the media and the nature of its use is discussed. Finally, provide commentary about the current alignment of research into mathematical modelling and the need for citizens to develop critical capabilities including those related to evidence literary.

2.2 Mathematical Modelling in Research and Practice

In this section, I argue that research in mathematical modelling is expanding range the scope of research topics while also making impact on practice through its inclusion in curriculum documents internationally. At the same time, it is important to question if current work in both research and practice is aligned with the capabilities required by young people and adults to address the significant challenges they face in personal, work, and civic life.

Research has provided a broad range of insights into the acquisition of mathematical modelling capability: modelling competency and competency as a modeller (e.g. Maaß, 2010; Treilibs, 1979); metacognitive processes (e.g. Stillman & Galbraith, 1998); teachers' and students' personal knowledge (e.g. Blum, 2011); dispositions and beliefs (e.g. Jankvist & Niss, 2019); blockages between phase transition (e.g. Galbraith & Stillman, 2006); implemented anticipation (Niss, 2010); task design and implementation (Czocher, 2017; Geiger et al., 2022); and use of digital tools (e.g. Geiger, 2011; Siller et al., 2022). Studies of mathematical modelling have also been considered from a variety of perspectives (Kaiser & Sriraman, 2006), for example, atomistic and holistic (Blomhøj & Jensen, 2003; Niss, Blum, & Galbraith, 2007); cognitive (Borromeo Ferri & Blum, 2010); ethno-mathematical (D'Ambrosio, 1999; Rosa & Orey, 2013); and semiotics (Geiger, 2013). The scope and depth of this research indicates the high level of interest in what is a relatively young field. While this represents a growing list of areas of research endeavour, there appear to be relatively few studies that focus on the development of the critical capabilities needed to evaluate claims and arguments, made in public forums, and especially those associated with disruptive phenomena (see Sect. 2.3).

Parallel to the expansion of research in the field, mathematical modelling has developed an increasing presence in curricula internationally, for instance, the US Core Standards (2012):

> **Model with mathematics**
>
> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flow charts, and formulas. They can analyse those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

In Germany, national standards for mathematics were introduced after disappointing Programme for International Student Assessment (PISA) results in 2000. These included a dimension devoted to mathematical modelling. The standards were adopted by all sixteen federal states and were integrated into their curricula. This initiative has resulted in mathematical modelling being compulsory at all educational levels, from primary to upper secondary. Further, since 2015, mathematical modelling has been a compulsory element of the final examination (Abitur).

The Brazilian Common Curriculum Base-BNCC (Brasil, 2017) for Mathematics positions Processes of Problem-Solving, Investigation, Project Development, and Mathematical Modelling as privileged forms of mathematical activity. As such, it is an objective and strategy for learning throughout elementary, middle, and high school. In addition, the development of these mathematical processes is seen as underpinning fundamental skills of mathematical literacy (reasoning, representation, communication, and argumentation), that is, to develop the ability of students to act mathematically in the most diverse situations, inside and outside of the school. The objective is for the student to have more autonomy to make decisions.

Similarly in Singapore, mathematical modelling has been an element of the national Mathematics Framework, as set-out in Fig. 2.1.

Within the Australian curriculum context, mathematical modelling is included as a mandatory 'process' with which all students must engage, as described below:

> Students develop an understanding of mathematical modelling when they use mathematics to make decisions, predict outcomes and understand relationships in authentic scenarios. They recognise, connect, and apply mathematical structures and use mathematical approaches to manipulate, analyse, generalise, solve, interpret, and communicate within the context of the modelling situation. Mathematical modelling is fundamental to the contemporary discipline of mathematics and the practical application of mathematics (ACARA, 2022).

The need for all students to develop the capacity to solve problems in the real world is also reinforced in the Australian Curriculum through a General Capability related to numeracy:

> Mathematics has a more fundamental role in the development of numeracy compared to other learning areas. The mathematics curriculum provides opportunities to apply mathematical understanding and skills in other learning areas and to real-world contexts. An important context for the application of number, algebra, measurement, and probability is financial mathematics. In measurement and space, there is also an opportunity to apply understanding to design and construction. Today's world is information driven; through statistics and probability, students can interpret and critically analyse data, and make informed judgements about events involving uncertainty (ACARA, 2022).

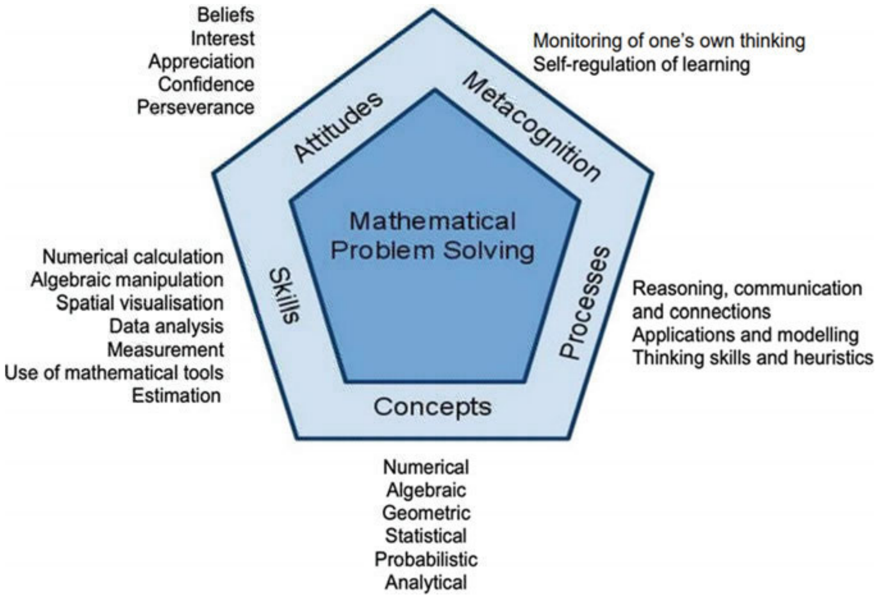

Fig. 2.1 National mathematics framework of Singapore

The inclusion of mathematical modelling in curriculum documents, as illustrated above, indicates an acceptance that it is essential for young people to develop the capacity to use mathematics confidently when addressing real-world problems. At the same time, the direction provided by curriculum documents is broad, and fail to provide advice about the types of substantive problems that should be the focus of teaching and learning—those that matter to citizens and society at large. Of the above-mentioned documents, it would appear that only the Brazilian curriculum provides attention to socio-critical issues, and especially on how evidence based on mathematical modelling can be critically evaluated. In the next sections, I extend the argument that there are significant topics that should be the focus of mathematical modelling education and ask if these are receiving limited attention.

2.3 Mathematical Modelling in and Disruptive Phenomena

The disruptions we are currently experiencing affect nearly all aspects of lives across the planet. These disruptions include interrelated challenges to health (e.g. COVID-19), the environment (e.g. global warming sustainability), food and energy security, poverty, and national security (e.g. Geiger, 2019; Maass et al., 2022). The scale and severity of these disruptions have resulted in responses from influential global organisations such as the United Nations (United Nations General Assembly, 2015) (and the OECD (https://www.oecd.org/dac/sustainable-development-goals.htm). As a focus of its response, the United Nations developed 17 Goals for Sustainable Development to be achieved by 2030 (Table 2.1).

While global organisations are responding to the far-reaching impact of these phenomena, it raises the question of how citizens should be best prepared to meet associated challenges for themselves, their families, and society at large. Given the focus of this chapter, I ask the question—is research in mathematical modelling in tune with this need?

Table 2.1 United Nations 17 goals for sustainable development

Goal 1 No poverty	Goal 2 Zero hunger	Goal 3 Good health and well-being	Goal 4 Quality education	Goal 5 Gender equality	Goal 6 Clean water and sanitation
Goal 7 Affordable and clean energy	Goal 8 Decent work and economic growth	Goal 9 Industry, innovation and infrastructure	Goal 10 Reduced inequality	Goal 11 Sustainable cities and communities	Goal 12 Responsible consumption and production
Goal 13 Climate action	Goal 14 Life below water	Goal 15 Life on land	Goal 16 Peace, justice and strong institutions	Goal 17 Partnerships for the goals	

While much discussion about the need for young people to develop knowledge of, and capability with, mathematical modelling has centred on the skills needed for the modern workforce (e.g. STEM, economics, health), an important attribute for informed and active citizenship is the capacity to critically evaluate and claims that are evidenced by reference to models, modelling, and predictions in public forums, for example, via the mass and social media. These claims, by experts and non-experts, can be inconclusive, misleading, or contradictory. Such an evaluative capability aligns with critical aspects of mathematics education (e.g. Ernest, 2002; Skovsmose, 2021) and connects with Vos' (2023, this volume) discussion about the Public Understanding of Mathematical Modelling (PUMM). To provide key background to this observation, in the next section, I provide a brief outline of recent initiatives related to critical thinking and its connection to mathematics education.

2.4 Critical Thinking as an Educational Goal in Mathematics Education

The development of students' critical thinking has been a long held educational goal. This is evident in frameworks aimed at identifying key capabilities that equip young people to live satisfying lives in which they contribute productively to their societies. Two examples of such frameworks, the Partnership for Twenty-First Century Skills Project (2002) and the Assessment and Teaching of Twenty-First Century Skills Project (2015) are presented in Table 2.2. In both frameworks, critical thinking for problem-solving is a key element.

The increased focus on critical thinking skills, or criticality, is also reflected in mathematics education. An important recent example is the Programme for the International Assessment of Adult Competencies (PIAAC), which is conducted in over 40 countries/economies and measures the key cognitive and workplace skills needed

Table 2.2 Key capability frameworks

Partnership for Twenty-First Century Skills Project (2002)	Assessment and Teaching of Twenty-First Century Skills Project (2015)
Learning and Innovation skills: Critical thinking and problem-solving; Communication and Collaboration; Creativity and Innovation Information, Media and Technology skills: Information Literacy, Media Literacy, ICT (Information, Communications, and Technology) Literacy Life and Career Skills: Flexibility and Adaptability; Initiative and Self-Direction; Social and Cross-Cultural Skills; Productivity and Accountability; Leadership and Responsibility	Ways of thinking. Creativity, critical thinking, problem-solving, decision-making, and learning Ways of working. Communication and collaboration Tools for working. Information and communications technology (ICT) and information literacy Skills for living in the world. Citizenship, life and career, and personal and social responsibility