Andreas Liening

Complexity and Entrepreneurship

Complexity Research and Its Implications on Entrepreneurship Processes

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Preface to the Second Edition

Yes, just make a plan!
Just be a great light!
And then make a second plan
Neither of them will work. (Brecht, 1928)

The world in which we live is perceived as increasingly complex. We often feel as if we are in a fog. Complexity is more than just a "feeling", it is an empirical fact. And yet, we are constantly surprised by developments that we have not even remotely anticipated. This applies to events on the world stage, where financial crises, climate crises, state crises, and other escalations up to and including military conflicts catch us unprepared. Nevertheless, major events that begin as perceived crises can also be positive, as the "fall of the Berlin Wall" shows, which no one had anticipated.

However, complex, unpredictable events exist not only on a large scale, but also on a small scale, as in the context of one's own individual experiences:

Everyone, for example, writes a "CV", a Curriculum Vitae, which, when viewed as a reader, reveals a "common thread". In retrospect, much can be interpreted sensibly and you can assign a meaning to the developments of life. However, if you are at the beginning of those developments that later appear so sensible and logical, you may well have the impression that this supposed "common thread" does not exist at all.

Often, it is rather the small coincidences, but of course also a lot of hard work and perseverance, close friends who have supported and accompanied you, decisions that you made in a certain situation, but certainly not a long-term plan, which could be assumed from the example of the curriculum vitae, that characterises one's professional career.

Quite the contrary: Particularly significant, major events in one's own life often happen unplanned and seemingly randomly, and are rarely the result of a sophisticated plan. And this applies not only to your profession: For example, you may have met your life partner because you happened to "end up" in a certain club with a few friends; you contemplate at a fork in the road whether to turn left or right, spontaneously decide for one direction and against the other, and encounter an interesting person, and your life is steered in a completely new direction, etc.

Often, we can only make sense of developments in retrospect, and then we speak of a "red thread". Ex post, everything can be explained, but ex ante, it is often much more difficult to find an explanation.

If, on the one hand, we experience a negative event, for example, we often speak of "bad luck" or "fate" to indicate that it was beyond our control.

We often overlook the fact that much of what "happens" to us was brought about by ourselves, and even when we admit our own mistakes as such, we often overlook the fact that it is precisely these mistakes that make us stronger and more confident in mastering life.

On the other hand, if the experiences are positive, we tend to attribute the success much more to ourselves alone, rather than remembering who and what, but also which unforeseen circumstances, helped us to achieve this success. Only sometimes are the experiences of such great importance to us that we elevate them and speak of "providence", "fate" or "destiny".

In English, the idea that positive experiences sometimes have to be attributed to things that are beyond our control and may not even have been intended finds expression in the beautiful word: *Serendipity*—Fortunate coincidence.

Both belong together, the negative as well as the positive, only in the interplay of the two poles is balance achieved: Hard and soft—*Go* and *Ju*—*Goju,* as karateka of a certain style would describe their way of life, the Karate-Do, where the path (Do) itself, which one bravely takes step by step, is the actual goal.

But it's not just in our personal lives and on a small scale; many things can't be planned, especially in public and on a larger scale. Let's take the German national economy as an example and the enormous task of coordinating supply and demand, which affects millions of individuals in Germany every day. Those who thought they could plan this centrally for decades were not only mistaken, their whole (political and economy) system ultimately failed. How can a central authority possibly know *which* goods or services should be provided individually, *when* and *where*, in which *quantity* and in which *quality* should be provided to each individual?

However, despite the limitations that plans obviously have, we cannot do without plans.

For example, macroeconomic coordination and other related issues can only be solved with the help of the decentralised knowledge of hundreds of thousands of entrepreneurs, managers, and merchants "on the spot". This requires meticulous planning at the decentralised level, which ultimately represent tools for solving a factual problem. But plans also have important functions beyond that as they

- … are *communicative:* They make the intentions of the planner and his approach clear to other people.
- … enable *coordination:* Thus, the activities of several people can be coordinated with each other using a plan against the backdrop of a common goal.
- … *commit:* By trusting that we will adhere to our plans, we allow others to rely on the outcomes of our planned projects. Thus, plans create certainty of action.

- … ensure *transparency:* They make goals and procedures clear and discussable, they enable an improvement of action frameworks, possibly even a change of goals.
- … *relieve:* Once a plan is in place, you don't have to constantly recall all the details or rethink them over and over again (Strohschneider, von der Weth et al., 2001, p. 12 ff.).

Predictability from the perspective of startups can, for example, include knowledge of the supply and demand situation in existing markets. In this context of an existing market in which new start-ups are emerging, we speak of a *Red Ocean*. The ocean turns red because suddenly new fish appear in the sea that eat the others. But plans have their limits. This is always the case when situations, processes, etc. no longer move "in calm waters", when past data no longer helps us to develop solutions for the future, but when we get into complex situations, when turbulence, dynamic leaps, and chaos occur.

Of all entrepreneurial fields of action, entrepreneurship is particularly affected by these boundaries. Especially in startups, the limits of plans become apparent very quickly. Especially in uncertain times like these, where also the resurgence and advancement of artificial intelligence (AI), coupled with an explosive increase in its usage, enable entirely new business models, the limitations of traditional planning become more than apparent. No wonder that the founder of Amazon, Jeff *Bezos,* says that a business plan will not survive its first encounter with reality. It will always be different. Reality will never match the plan, according to *Bezos* (Mochere, 2022). After all, startups have little chance of foresight, especially since they are often the cause of unpredictable changes and disruption themselves.

Newly emerging markets cannot possess a past. Before the invention of significant innovations—from steam power to smartphone technology—no one could have obtained detailed information about demand or supply—they simply did not exist. The tearing down of old markets, the creation of new markets, in short—as *Schumpeter* once said—*creative destruction,* is indeed a hallmark of a startup.

The chaos that we encounter in the environment of a startup is therefore rather positive for the startup itself: As chaos and complexity are signs of great uncertainty and only enable the development of the startup, enable creative destruction, while order only generates habit and relies on what has been proven so far, which is swept away by the startup. Here we speak of the *Blue Ocean,* the blue ocean, which still wants to be explored. Creating new markets is like sailing over the blue ocean to an undiscovered land that is not marked on any map. In this process, a classic business administration, which is oriented towards existing companies, often fails because it needs past data to predict the future. However, startups that have just been founded do not have any past values.

Furthermore: Even if numerous (past) data are available, they do not help us if we venture into complex waters. The *Laplacian Demon,* who can predict everything because he knows all initial conditions and has all necessary information, and who also

permeates our thinking, must fail in complex phases, as is explained and justified in great detail in this book. One could say: In the realm of complexity, the future lies between chance and order like in a fog.

Yes, even more so, complexity itself *is*, figuratively speaking, like fog: Short-term forecasts are indeed possible, but then the so-called butterfly effect sets a clear limit to predictability.

Complex phenomena are often popularly described with this *'butterfly effect'*, which states that the flap of a butterfly's wings in Brazil could, after some time, trigger a tornado in Texas.

And with that, we do not see the turbulence coming, we cannot predict it. The classic methods are blind to complexity: And even if we could capture all the smallest snippets of information, the collection, analysis, and evaluation would still take more time than it would take to get the results in time, so that the events would roll over us one way or another. Certainly, one can analyse the fog with classic methods. But, to stay with this image, even after the most thorough analysis, the fog is still there.

But there is hope—There are indeed scientific answers to the question of how we can walk safely step by step through the fog:

For several decades, there have been established quantitative, respectively mathematical-empirical methods that can be successfully applied to questions of complexity between order and randomness. And it becomes clear: Chaos is not a coincidence on the one hand, and on the other hand, chaos and order belong together, like two sides of a coin, just as *Ying* and *Yang* are inconceivable without each other and condition each other.

While the first edition of this book focussed on theory, the second edition places particular emphasis on the integration of empirical approaches, whether to better understand behavioural anomalies in simple, complicated, and also complex systems through case studies, or to make the different levels of the "Synergetic Entrepreneurship Model (SEM)" developed in this book, which considers both the individual, organisational, and macroeconomic phenomena of entrepreneurship, accessible for empirical investigation through complexity science analyses of concrete time series.

For example, abrupt changes in system behaviour, which are described as phase transitions, can be diagnosed and investigated in complex processes by including the calculation of *Lyapunov* exponents, (fractal) dimensions, permutation entropies, *Recurrence* plots, and other methods, whereby these methods are previously described and analysed in detail theoretically. This makes it possible to investigate complex processes of structure genesis, as can be observed in the development of business models up to entire entrepreneurial ecosystems.

Often, entrepreneurs stand as if in front of a fog wall, where they need to test or realign their thinking. There is probably no "red thread" that they can cling to, no 'Business Plan' that could last five years. For a long time, people oriented themselves towards a mechanistic model, whose mathematics successfully shaped economics for many years, but reaches its limits in complex phases. Some economists therefore advocate a departure from mathematics in economics. However, this is not necessary,

because "Where there is danger, the saving power also grows," writes the poet (Hölderlin, 2015). And so the established quantitative, empirical-mathematical methods of complexity sciences, which are increasingly also making their way into economics, help us find a path through the complex tangle and safely shape our startup: The fog is lifting.

Dortmund Univ.-Prof. Dr. Andreas Liening
January 2024

Reference

Brecht, B. (1928). *Ballad of the insufficiency of human planning*. Threepenny Opera.
Hölderlin, F. (2015). *Poems*. CreateSpace Independent Publishing Platform.
Mochere, V. (2022). *Best quotes from Jeff Besoz*. Retrieved March 10, 2022.
Strohschneider, S., et al. (2001). Ja, mach nur einen Plan! - Pannen und Fehlschläge - Ursachen, Beispiele, Lösungen. Hogrefe AG.

Preface

For a long time, economics had a perhaps silent, but certainly large role model: Physics with its Newtonian mechanics. It allows us to calculate the trajectory of a rocket to the moon, or even to land instruments precisely on a distant tiny asteroid. The basis is the famous Newtonian law: Force = Mass times Acceleration. The force is the *cause* for a very specific *effect:* The acceleration, i.e., the force determines the latter. Even more: A double force produces a twice as large acceleration, i.e., a *linear* relationship applies. It is more than evident how tempting it is to transfer this reaction to economic events. For example, predictability means complete planning, which found its expression in Marxist economic theory and large-scale practice with its devastating consequences.

Now, this "worldview" of physics, just presented, has experienced two severe shocks in the last century:

1. Quantum physics, without which the entire information technology we use daily would not exist, but whose fundamental processes are not at all deterministic.
2. The second, probably more in the spotlight and brought to a wider audience, is the theory of chaos. However, the sensational term "chaos" used by scientists is not accurate: it refers to aesthetically highly appealing and *highly ordered* structures generated on computers. But this also obscures a fundamental aspect of chaos research: Although their equations are strictly deterministic, the smallest inaccuracy grows enormously in the course of the calculation process, so that predictability is fundamentally not possible. The failure of weather forecasting beyond several days is the glaring example almost inevitably cited in this context.

Around the same time as the emergence of chaos theory, I founded synergetics—the study of interaction. How do the individual parts of a complex system manage to interact in such a way that ordered structures or functions are created? Even with a system from physics, the then novel light source laser, it turned out that entirely new concepts were needed to represent the self-organisation of laser light that occurs here. Determinism is replaced by a strange combination of certainty and chance, linearity is replaced by non-linearity, causality is replaced by "circular causality", and finally: instead of direct

control, indirect influence. I do not want to anticipate the author's explanations, Prof. Andreas Liening. However, it is clear that new perspectives on economics emerge from here, with only an economist being able to draw and present the appropriate conclusions. These are vital for the entrepreneur in an increasingly complex world. I am pleased that Prof. Liening mentions the significant economists Robert Solow and Friedrich August von Hayek in his conclusion. I have met both scientists several times. As I learned from Solow, he dealt with synergetics. Hayek advocated the concept of market self-organisation with its collective intelligence, which could not be predicted. So, with both scientists, Prof. Liening is "in good company". I am sure that the comprehensive work of Prof. Andreas Liening will not only be groundbreaking for entrepreneurs, but will also have a significant influence on the further development of economics.

Stuttgart, Germany
February 2016

Univ.-Prof. Dr. rer. nat. Dr. h.c. mult. Hermann Haken

Greetings

None other than *Seneca* once said:

> "Reading is, I believe, absolutely necessary for me:
> Firstly, so that I do not have to be content with myself alone, secondly, to become acquainted with the knowledge of others, thirdly, so that I can form an opinion about what they have discovered and think about the questions that still need to be answered." (Seneca, approx. 4 BC–65 AD)

In this regard, I thank the research team at my chair for the tremendous support. Special thanks go to Hermann *Haken* for his foreword and the inspiration through his life's work.

I wish the readers much joy in reading, interesting insights, and hopefully many ideas and questions about the complexity between order and randomness and their significance for entrepreneurship research!

Your

Prof. Dr. Andreas Liening

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