Springer Proceedings in Mathematics & Statistics

Dimitrios Vlachos *Editor*

Mathematical Modeling in Physical Sciences

12th IC-MSQUARE, Belgrade, Serbia, August 28–31, 2023

Springer

Springer Proceedings in Mathematics & Statistics

Volume 446

This book series features volumes composed of selected contributions from workshops and conferences in all areas of current research in mathematics and statistics, including data science, operations research and optimization. In addition to an overall evaluation of the interest, scientific quality, and timeliness of each proposal at the hands of the publisher, individual contributions are all refereed to the high quality standards of leading journals in the field. Thus, this series provides the research community with well-edited, authoritative reports on developments in the most exciting areas of mathematical and statistical research today.

Dimitrios Vlachos
Editor

Mathematical Modeling in Physical Sciences

12th IC-MSQUARE, Belgrade, Serbia, August 28–31, 2023

Springer

Editor
Dimitrios Vlachos
Department of Economics
University of Peloponnese
Tripoli, Greece

ISSN 2194-1009 ISSN 2194-1017 (electronic)
Springer Proceedings in Mathematics & Statistics
ISBN 978-3-031-52964-1 ISBN 978-3-031-52965-8 (eBook)
https://doi.org/10.1007/978-3-031-52965-8

Mathematics Subject Classification: 74P15, 60G35, 62M10, 68W20, 92D25, 65C05, 03E72

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Paper in this product is recyclable.

International Scientific Committee

Morten Hjorth-Jensen, University of Oslo, Norway
Jacob Barhen, Oak Ridge Natl. Lab, Oak Ridge, USA
Georgios Loukes-Gerakopoulos, TPI, FSU Jena, Germany
Javier Hernández-Andrés, University of Granada, Spain
Maia Martcheva, University of Florida, USA
Theocharis Kosmas, University of Ioannina
Angelo Tartaglia, Politecnico di Torino, Italy
Yoshiharu Yamamoto, The University of Tokyo, Japan
Amaury Mouchet, Université de Tours, France
Mairi Sakellariadou, King's College of London, UK
Kostas Skenderis, University of Amsterdam, The Netherlands
Orfeu Bertolami, University of Porto, Portugal
Saurya Das, University of Lethbridge, Canada
Michel A. Van Hove, City University of Hong Kong, Hong Kong
Rho Shin Myong, Gyeongsang National University, Republic of Korea
Sergey Nazarenko, University of Warwick, UK
Emilio Elizalde, CSIC, Barcelona, Spain
Theodosios Christodoulakis, University of Athens, Greece
Bari Ferenc, University of Szeged, Hungary
Mark Cross, Swansea University, UK
Kostas Kokkotas, Eberhard Karls University of Tübingen, Germany
Shiyi Chen, Peking University, People's Republic of China
Heinrich Begehr, Germany
Gerard R. Lemaitre, France
Robert Finn, Stanford University, USA

Organizing Committee

Chair: Evangelia D. Vlachou, Ionian University, Greece
Amanda Mavrogianni, Sivitanideios School, Greece
Ioannis Torakis, National and Kapodistrian University of Athens
Sotirios-Neilos D. Vlachos, National Technical University of Athens, Greece

Preface

The International Conference on Mathematical Modeling in Physical Sciences (IC-MSQUARE) is an interdisciplinary scientific event which aims to promote the knowledge and the development of high-quality research in mathematical fields that have to do with the applications of other scientific fields and the modern technological trends that appear in them, these fields being those of Physics, Chemistry, Biology, Medicine, Economics, Sociology, Environmental Sciences, etc.

The 12th IC-MSQUARE took place in Belgrade, Serbia, between August 28 and 31, 2023. In addition, (because in previous years the conference, due to the pandemic, took place only with online presentations), the possibility of participation via the internet was also adopted this time. Thus, apart from the talks and workshops held on site, we also had a large collection of pre-recorded presentations that were available to the participants through the conference website.

The Conference was attended by more than 350 participants and hosted about 320 oral and virtual presentations while counted more than 1000 pre-registered authors. The 12th IC-MSQUARE consisted of different and diverging workshops and thus covered various research fields where Mathematical Modeling is used, such as Theoretical and Mathematical Physics, Neutrino Physics, Non-integrable Systems, Dynamical Systems, Computational Nanoscience, Biological Physics, Computational Biomechanics, Complex Networks, Stochastic Modeling, Fractional Statistics, DNA Dynamics, Macroeconomics, Social Modeling, etc.

The scientific program was rather heavy, however, according to all attendees, the program was excellent with high level of talks and the scientific environment was fruitful, thus all attendees had a creative time.

All papers submitted to the Conference have been reviewed using a single-blind peer-review process. The review process adopted a scoring system (0–5 points) who rated the Relevance, Scientific Quality, Contribution, Title and Paper Presentation, Tables and Figures, Language, and the References of each paper. The threshold for an article to be accepted was a score above 4. Additionally, each paper was tested for similarity with already published content using the Turnitin software. All sources of similarities were checked and only original content was accepted. In this way, out

of more than 120 papers that were evaluated for this volume, only 51 were accepted for publication.

We would like to thank the Reviewers, the Members of the International Scientific Committees as well as the Members of the Organizing Committee.

Tripoli, Greece

Dimitrios Vlachos
The Conference Chairman

Contents

Mathematical Methods in Physical and Engineering Applications

Single and Multi-material Topology Optimization of Continuum Structures: ABAQUS Plugin

Dhaval Patel, Thomas Rockenbauer, Sandra Schlögl, and Margit Lang

Abstract This research addresses the need for versatile topology optimization techniques capable of optimizing both single and multi-material designs. The extended Python code incorporates the modified Bi directional Evolutionary Structural Optimization (BESO) algorithm and a material interpolation scheme to enhance its capabilities. Engineers and designers can utilize this improved approach to optimize the performance of structures, especially in the context of multi-material configurations. To increase its practicality, the code is converted into ABAQUS plugin, seamlessly integrating it with the widely used finite element analysis software. Validation examples, conducted in the ABAQUS environment, demonstrate the compatibility and accuracy of the code. This research provides an efficient and accurate solution for topology optimization, addressing the demands of multi-material designs in various engineering applications. Examples of such applications include lightweight design in automotive and aerospace industries, customized implants in biomedical engineering, and optimal material distribution in architectural structures.

Keywords Topology optimization · ABAQUS plugin · Bi-directional evolutionary structural optimization (BESO) · Multi-materials optimization

1 Introduction

Structural optimization is a method used in engineering and design to determine the most efficient material layout and shape of a structure, considering specific design criteria and constraints. This technique originated in the 1980s and has progressed as technology has advanced. The three primary techniques for structural optimization are: (1) topology optimization, which optimizes the material distribution for a desired structural performance, (2) shape optimization, which enhances the design geometry

D. Patel (✉) · S. Schlögl · M. Lang
Polymer Competance Center Leoben GmbH, Leoben, Austria
e-mail: dhaval.rasheshkumar.patel@pccl.at

T. Rockenbauer
Luxinergy GmbH, Leoben, Austria

D. Vlachos (ed.), *Mathematical Modeling in Physical Sciences*, Springer Proceedings in Mathematics & Statistics 446, https://doi.org/10.1007/978-3-031-52965-8_1

to improve performance (e.g. to avoid stress concentration), and (3) size optimization, which determines the optimal dimensions (e.g. thickness) of a component. By utilizing topology optimization, engineers can design structures that are stronger, lighter, and more efficient, while meeting performance requirements such as durability and stability. This results in improved structural performance. The ability to remove redundant material from areas that do not carry significant loads, furthermore increases material efficiency by offering the possibility to use resources in a more sustainable way. Consequently, the use of topology optimization is expanding in various fields, including aerospace, automotive, civil and biomedical engineering. As computational power and optimization algorithms continue to improve, it is expected that topology optimization will become even more prevalent in engineering design.

Topology optimization encompasses various techniques that have unique strengths and limitations. Some of the most commonly used methods include Solid Isotropic Material with Penalization (SIMP) [1], Evolutionary Structural Optimization (ESO) [2], and Level Set Method (LSM) [3]. Each method offers a different approach to solving topology optimization problems and is selected based on the specific design objectives and constraints. SIMP is a commonly used topology optimization technique that assigns material density to each element in the design domain and penalizes intermediate densities to obtain a clear "black-white" design. However, it can lead to so-called checkerboard patterns [4] and numerical instabilities [5]. To address these issues, various restriction methods have been developed, such as perimeter control, gradient constraints, or blurring filters [6–8]. ESO is a topology optimization technique that mimics the process of natural selection to obtain an optimal topology by gradually removing "inefficient" material and redistributing loads. It is useful in conceptual design and can produce organic-looking shapes. Bi-directional evolutionary structural optimization (BESO) is an extension of ESO that enables the addition of "efficient" material while removing "inefficient" material. However, both methods suffer from numerical instability issues that can be addressed by incorporating perimeter control or filters [6–8]. The most distinct difference with respect to SIMP is that with ESO or BESO no intermediate densities are produced. For ESO and BESO basically only two "material states" exist—"0 = void" and "1 = full material". Therefore, no interpretation of intermediate densities is necessary for the design engineer. LSM is a topology optimization technique that utilizes a level set function to represent the design domain, enabling straightforward manipulation of the topology. LSM is beneficial for problems with multiple load cases or design objectives, and it can create smooth and continuous designs. However, it is computationally costly and may be mesh-dependent [9].

Over the past four decades, based on these methods, numerous educational articles have been published with the objective of introducing fundamentals of various topology optimization algorithms through computer program implementations. For instance, Sigmund developed a 99-line MATLAB code for the SIMP method [1], while Andreessen et al. developed an 88-line MATLAB code that is an extension of Sigmund's code with improved efficiency [10]. Yongsheng et al. presented an efficient 137-line MATLAB code [11] for 2D compliance minimization utilizing the BESO method presented by Huang and Xie [12], and Liu and Tovar presented

a modified SIMP model for 3D topology optimization for linear structures with regular eight-noded elements [13]. Moreover, Zhi Hao Zuo et al. [14] developed a compact and simple Python code for complex 3D topology optimization utilizing the BESO method. Additionally, several commercial and open-source tools for topology optimization are available, as stated in the comparative study published in article [9].

As mentioned before, most studies and applications of topology optimization have been limited to single material structures. As a result, there is a noticeable gap in research regarding topology optimization for structures with multiple materials. Several researchers have attempted to address this issue through different approaches. For instance, Zhou and Wang [15] developed a phase-field method with a generalized Cahn-Hilliard model for the optimization of multi-material structural topology. An algorithm was introduced by Huang and Xie [16] that utilizes the BESO method for topology optimization of continuum structures. The algorithm is capable of handling both single and multiple materials and incorporates the material interpolation approach from the SIMP method. Hvejsel and Lund [17] proposed two multi-material interpolation schemes based on SIMP and (RAMP) Rational Approximation of Material Properties material interpolation schemes to enable topology and multi-material optimization formulated within a unified parametrization. A novel problem formulation was proposed by Gao and Zhang [18] to address the topology optimization of lightweight structures comprising multiple phases of materials. The formulation introduces a mass constraint to guide the optimization process. Gaynor and Yang [19] combined the force flow path with topology optimization to provide topology optimization methods for two- and three-dimensional reinforced concrete structures. Wang et al. [20] proposed a new Multi-Material Level Set (MM-LS) topology description model for topology and shape optimization of structures involving multiple materials. A multi-material topology optimization method utilizing a reaction-diffusion equation based on the level set method was proposed by Cui et al. [21]. Recently, a multi-material topology optimization method based on BESO has been introduced. This method specifically addresses structures with multiple materials that possess notably different mechanical properties in tension and compression [22, 23].

Despite many research efforts, there is still a lack of open-source codes available for multi-material topology optimization. To overcome this issue, the author is motivated by the works of Zuo and Xie in [14] and Huang and Xie in [16] and intends to extend the Python code available from [14]. The objective is to make it more computationally efficient and to enable multi-material topology optimization using the formulation of the modified BESO method introduced in the research article [16]. This endeavour aims to contribute to the advancement of multi-material topology optimization research and provide an efficient tool for the scientific community. The code can be found in this GitHub repository: https://github.com/dhavalrpatel2511/Topology_Optimization.git.

2 Methodology

This research article focuses on the implementation of the modified BESO method for single and multi-material topology optimization, which is based on the previously published work by X. Huang et al. [16]. While the formulation of the topology optimization problem is not extensively discussed in this article, readers can refer to the original work [14, 16] for a more detailed explanation. Nonetheless, the essential information regarding the topology optimization problem shall be provided here.

2.1 Problem Statements

The objective of the topology optimization problem is to minimize the compliance of a structure while satisfying a constraint on the volume of the material to be used in the final design. Mathematically, this approach to single-material optimization can be expressed as:

$$\begin{aligned} \min_X : C_m(\mathbf{X}) &= \frac{1}{2}\mathbf{F^T U} = \frac{1}{2}\mathbf{U^T K U} \\ \text{subject to: } \mathbf{X} &= [x_e];\ x_e = 1 \text{ or } x_{min},\ \forall e = 1, ..., N \\ \mathbf{F} &= \mathbf{KU} \\ V(\mathbf{X}) &= \sum_X x_e V_e = V^* \end{aligned} \tag{1}$$

The mean compliance $C_m(\mathbf{X})$, which is the objective function, represents the energy required to deform the structure under a given set of loads, and it is proportional to the strain energy stored in the material. $\mathbf{F}$ represents the global force vector, $\mathbf{U}$ is the displacement vector, $\mathbf{K}$ is the stiffness matrix and V($\mathbf{X}$) refers to the volume of the design domain occupied by the material. The binary design variable x_e denotes the density of eth element. To avoid the singularity of the stiffness matrix, a small value of x_{min}, for example, 0.001 is used to denote the void elements.

With respect to multi-material optimization the objective is to find the optimal distribution of multiple materials with differing modulus of elasticity $E_1, E_2, ..., E_n$ (where $E_1 > E_2 > ... > E_n$). The corresponding optimization problem can be formulated as follows:

$$\begin{aligned} \min_X : C_m(\mathbf{X}) &= \frac{1}{2}\mathbf{F^T U} = \frac{1}{2}\mathbf{U^T K U} \\ \text{subject to: } \mathbf{X} &= [x_{ij}];\ x_{ij} = 1 \text{ or } x_{min},\ (j = 1,\ 2,\ ...,\ n-1) \\ \mathbf{F} &= \mathbf{KU} \\ V_j^* - \sum_{i=1}^{n} V_i x_{ij} &- \sum_{i=1}^{j-1} V_i^* = 0 \end{aligned} \tag{2}$$

where x_{ij} denotes the density of the ith element for the jth material, and V_j* refers to the prescribed volume of the jth material.

2.2 Sensitivity Numbers

In the work of Huang and Xie [16], the material interpolation scheme from the SIMP method and sensitivity numbers for various topology optimization variants (such as single material void topology optimization, two non-zero material, or multi-material topology optimization) are briefly described. However, for the purpose of implementation, the equations for computing the sensitivity numbers for all these different variants of topology optimizations are provided here.

The sensitivity number in the BESO algorithm is a measure of how the overall structural performance is influenced by a change in "design" of a specific element. Based on the sensitivity number the algorithm decides which elements to modify or remove during the optimization process to achieve an optimal material distribution for the given objective. The sensitivity number for topology optimization with a single material and void is defined as follows:

$$\alpha_i = -\frac{1}{p}\frac{\partial C}{\partial x_i} = \begin{cases} \frac{1}{2}\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{u}_i & \text{when } x_i = 1 \\ \frac{x_{min}^{p-1}}{2}\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{u}_i & \text{when } x_i = x_{min} \end{cases} \tag{3}$$

The sensitivity numbers for topology optimization with two non-zero materials and with multi-materials are similar in form and can be defined as follows:

$$\alpha_i = \begin{cases} \frac{1}{2}\left[1 - \frac{E_2}{E_1}\right]\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i^1\mathbf{u}_i & \text{formaterial1} \\ \frac{1}{2}\frac{x_{min}^{p-1}(E_1 - E_2)}{x_{min}^p E_1 + (1 - x_{min}^p)E_2}\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i^2\mathbf{u}_i & \text{formaterial2} \end{cases} \tag{4}$$

$$\alpha_{ij} = \begin{cases} \frac{1}{2}\left[1 - \frac{E_{j+1}}{E_j}\right]\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i^j\mathbf{u}_i & \text{for material } 1, \ldots, \text{j} \\ \frac{1}{2}\frac{x_{min}^{p-1}(E_j - E_{j+1})}{x_{min}^p E_j + (1 - x_{min}^p)E_{j+1}}\mathbf{u}_i^{\mathbf{T}}\mathbf{K}_i^{j+1}\mathbf{u}_i & \text{for material j+1}, \ldots, \text{n} \end{cases} \tag{5}$$

The equations described above indicate that the sensitivity of voids in single material topology optimization, as well as other materials in various topology optimization methods, except for the first material, is affected by the selection of the penalty exponent. In particular, as the penalty exponent approaches infinity, the sensitivity values of these materials or voids tend to approach zero.

2.3 Filter Scheme and Convergence Criterion

The sensitivity numbers defined in the previous section are often adapted to obtain a mesh-independent solution. In order to achieve this, a blurring filter can be utilized, such as the one proposed by Huang and Xie [6, 8]. In this paper, a simplified elemental sensitivity filtering scheme [14] is employed to maintain the solution accuracy while avoiding the need for a newly developed computationally slower nodal sensitivity filtering scheme presented in the study by Huang et al. [16]. Nodal sensitivity analyzes localized information, while elemental sensitivity provides a broader understanding of design impacts for large-scale optimization. The simplified elemental sensitivity filtering scheme is used in this paper as follows:

$$\alpha_e = \frac{\sum_j w(r_{ej})\alpha_j}{\sum_j w(r_{ej})} = \sum_j \left(\frac{w(r_{ej})}{\sum_j w(r_{ej})}\alpha_j\right) = \sum_j \eta_j \alpha_j \tag{6}$$

$$w(r_{ej}) = max(0, r_{min} - r_{ej}) \tag{7}$$

Equation (6) uses the distance between the centers of elements e and j (r_{ej}), a weight function (w) to average the raw sensitivities, and a filter radius (r_{min}). It is important to note that the weight factor $w(r_{ej})$ is predetermined and does not depend on the sensitivity values, see Eq. (7).

The element sensitivity filter scheme requires the computation of the distance between each element that lies within the filter radius, which is used to calculate the weighting factor based on the relative distances. However, calculating the distance between each individual element and all other elements is computationally inefficient and time-consuming. Therefore, to improve efficiency, the *getByBoundingBox(...)* function, available in ABAQUS, is used in this study. Thereby, only the distances between elements that lie within the filter radius, instead of the distances of the individual element to all other elements, as described in the original publication [14], are considered.

To ensure convergence of the solution, it is advised to incorporate the historical information of the sensitivities when using discrete methods like BESO [12]. This can be achieved by averaging the sensitivity of the current iteration with that of the previous iteration as shown in Eq. (8), where k is the current iteration number.

$$\alpha_e = \frac{\alpha_e^k + \alpha_e^{k-1}}{2} \tag{8}$$

Convergence criteria are needed in the BESO algorithm to determine when the optimization process has reached a satisfactory solution. They provide a stopping criterion, ensuring the algorithm doesn't run infinitely. Convergence criteria monitor specific parameters or metrics, such as changes in the objective function or design

variables, to assess when the optimization has converged to an acceptable level of accuracy. This helps save computational resources and ensures the reliability and quality of the optimized topology.

$$Error = \frac{\left|\sum_{i=1}^{N}(C_{k-i+1} - C_{k-N-i+1})\right|}{\sum_{i=1}^{N}(C_{k-N-i+1})} \leq \tau \tag{9}$$

The criterion shown in Eq. (9) uses a tolerance value τ and an integer value N (fixed at 0.01% and 5 respectively in this paper) to ensure that subsequent iterations do not significantly improve the design. Specifically, the convergence criterion requires a stable compliance over a minimum of ten consecutive iterations, which reduces the impact of above equation on the final solution. The simplified form of Eq. (9) can be represent as follows:

$$Error = \frac{C[k-4:k+1] - C[k-9:k-4]}{C[k-9:k-4]} \leq \tau \tag{10}$$

2.4 Optimization Procedure

A step-by-step explanation of topology optimization procedure is given below.

Step 1: Mesh the design domain with finite elements and apply appropriate boundary conditions and loads.
Step 2: Determine the coordinates of the center of each element and compute the weight factor for each element based on the distance to neighboring elements within the filter radius.
Step 3: Provide the inputs such as Volume fraction, Evolution rate, Young's modulus, Poisson's ratio and most important the Design-Case, which refers to the variants of topology optimization (1. solid-void, 2. two non-zero materials and 3. Multi-Material topology optimization).
Step 4: Perform finite element analysis (FEA) to determine the compliance of the entire model and the strain energy of each element.
Step 5: Compute the sensitivity number for each element based on the respective design domain variation.
Step 6: Bi-directional Evolutionary Structural Optimization (BESO) procedure.
Step 7: Update material properties and design domain for next optimization iteration based on current sensitivity numbers and material distribution.
Step 8: Perform steps 4–7 iteratively until the desired volume is reached and the convergence criteria are met.

The flowchart (Fig. 1) shows a graphical representation of the step-by-step procedure of this topology optimization process. The BESO algorithm exhibits similarities

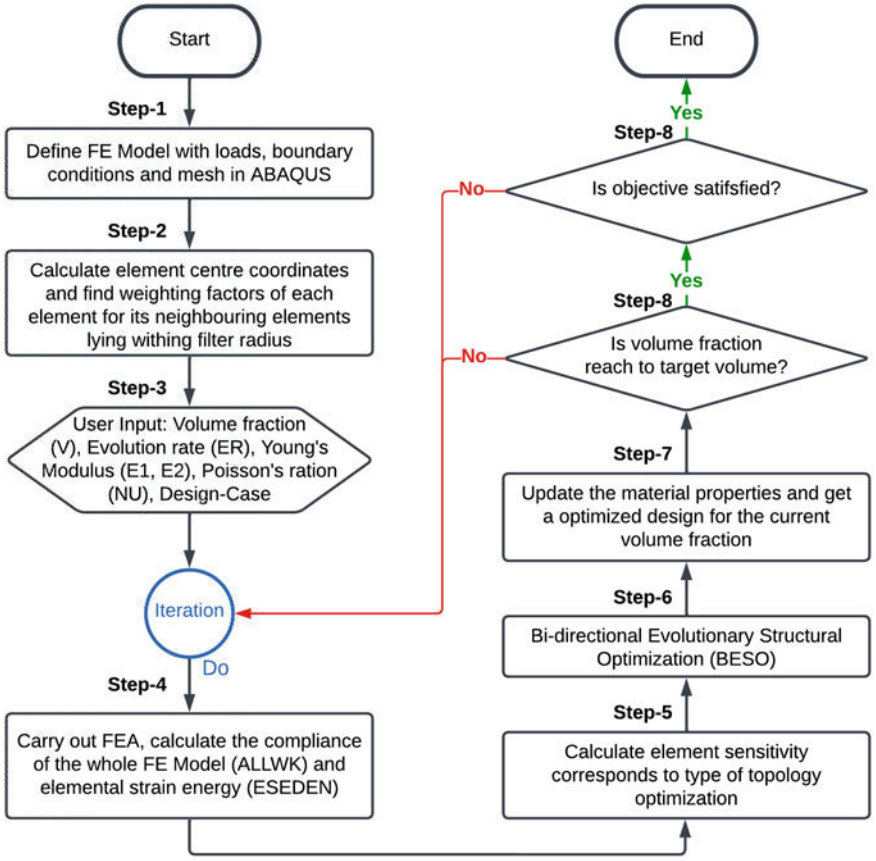


Fig. 1 Flowchart illustrating the topology optimization procedure

between the solid-void design and the two non-zero materials design variants. However, the BESO algorithm for multi-materials design variants has some differences. A detailed explanation of this BESO variant is provided in the pseudo-code (1). The multi-material BESO procedure starts with a full design containing material 1. It utilizes the evolutionary ratio (ER) to trigger the volume reduction of material 1. Simultaneously, the volume of material 3 gradually increases until the predefined volume fraction is reached. Afterwards, the volume of material 3 remains constant and the volume of material 2 increases until the target volume is achieved. The step-wise procedure is outlined in the pseudo-code, where lines 1–26 handle the optimization for material 1, lines 27–52 for material 2.

Algorithm 1 Modified BESO Algorithm for Multi-Materials Topology Optimization

Find the minimum (min1) and maximum (max1) values of elemental sensitivity numbers based on stiff and void material properties ($Ae1_i$).
Calculate the target volume of the void material in the next optimal design ($T_v1 = V_f1$ * Total number of elements).
repeat
  Update the threshold ($\epsilon 1_k$ = (max1+min1)/2.0)
  for each element i in the design domain **do**
    if $Ae1_i > \epsilon 1_k$ **then**
      Set $x1_i = 1.0$ (fully dense)
    else
      Set $x1_i = 0.00001$ (fully porous)
    end if
  end for
  if sum($x1_i$) - $T_v1 > 0$ **then**
    Set min1 = $\epsilon 1_k$
  else
    Set max1 = $\epsilon 1_k$
  end if
until (max1-min1)/max1 > 0.00001
Initialization of empty lists for Void group of elements.
Initialize the temporary dictionary of elemental sensitivity numbers based on stiff and soft material properties ($Ae2_i$).
for each element i in the design domain **do**
  if $x1_i = 0.00001$ **then**
    Void.append(i)
  else
    $TAe2(i) = Ae2(i)$
  end if
end for

Find the minimum (min2) and maximum (max2) values of elemental sensitivity from temporary dictionary ($Ae2_i$).
Calculate the target volume of the stiff material in the next optimal design ($T_v2 = V_f2$ * Total number of elements)
repeat
  Update the threshold ($\epsilon 2_k$ = (max2+min2)/2.0)
  for each element i in the design domain **do**
    if $TAe2_i > \epsilon 2_k$ **then**
      Set $x2_i = 1.0$ (Stiff Material)
    else
      Set $x2_i = 0.00001$ (Soft Material)
    end if
  end for
  if sum($x2_i$) - $T_v2 > 0$ **then**
    Set min2 = $\epsilon 2_k$
  else
    Set max2 = $\epsilon 2_k$
  end if
until (max2-min2)/max2 > 0.00001
Initialization of empty lists for Stiff and Soft groups of elements.
for each element i in the design domain **do**
  if $x2_i = 1.0$ **then**
    Stiff.append(i)
  else
    Soft.append(i)
  end if
end for
Section assignment for both Solid and Void groups of elements.

3 Results

Within this section, the focus lies on investigating the various design optimization variants as listed in Table 1. Specifically, the topology optimization problem

Table 1 Various design optimization variants

Variants	Stiff material volume (%)	Soft material volume (%)	Void volume (%)
3.1	30	–	70
3.2	50	50	–
3.3	15	25	60

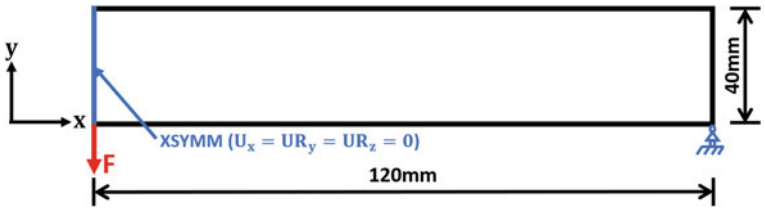


Fig. 2 Design domain of the right half of the full beam model

Table 2 Material properties

	Stiff material	Soft material	Void
Young's modulus	1000 MPa	100 MPa	1e-9 MPa
Poisson's ratio	0.3	0.3	0.3

is analyzed, concerning a beam structure with dimensions of 240×40 mm. However, exploiting the beam's symmetry, we shall solely examine the right half, which possesses a domain size of 120×40 mm, see Fig. 2. The bottom right corner of the right half is subjected to support, while the left edge adheres to X-symmetry boundary conditions. Additionally, a 10N load (**F**) is applied at the bottom left corner, as illustrated in Fig. 2. The application of X-symmetry boundary conditions, in this context, serves to accurately replicate the overall behavior of the entire beam structure under the specified load. These boundary conditions effectively constrain three degrees of freedom at the nodes along the left edge of the beam: the displacement along the X-axis (U_x), the rotation along the Y-axis (UR_y), and the rotation along the Z-axis (UR_z). The domain is discretized using four-node plane stress elements (CPS4) with a size of 1 mm.

As mentioned earlier, the topology optimization is performed using specific parameters. These parameters include a minimum element density (xmin) of 0.0001, a filter radius (rmin) of 3.0, a penalty exponent (p) set to infinity, and an Evolutionary Ratio (ER) of 2.0%. Furthermore, Table 2 includes the material properties of the stiff material (Material 1), soft material (Material 2), and void. The properties include the respective values for Young's modulus and Poisson's ratio assuming liner elastic material behavior.

3.1 Optimized Solid-Void Design

This section focuses on optimizing the beam structure, shown in Fig. 2, using a single material. The objective is to design the structure with maximum stiffness by minimizing its mean compliance while ensuring that only 30% of the material is present in the final design domain. The topology parameters mentioned earlier are used for this optimization process. The material properties for the stiff material and void can be referred to in Table 2.

Figure 3a shows the optimal design achieved using the newly developed BESO algorithm as described in the reference paper [16]. The mean compliance of this optimal design is measured to be 7.26 Nmm. On the other hand, Fig. 3b shows the optimal design obtained using the author's extended algorithm, which exhibits a mean compliance of 7.25 Nmm. Although both optimal designs differ slightly in appearance, the design from this paper fulfills the objective function and demonstrates slightly lower mean compliance, indicating increased stiffness. Moreover, this design bears some resemblance to the optimal design obtained through the continuation method described in Fig. 3c from the reference paper [16], which achieves a mean compliance value of 7.18 Nmm.

3.2 Optimized Design with Non-zero Materials

In this section, the topology optimization problem, illustrated in Fig. 2, is investigated, considering two non-zero materials in the optimal design, excluding the void. Again, the objective is to maximize the stiffness of the structure by minimizing its mean compliance while ensuring an optimal distribution of the two materials. Each material

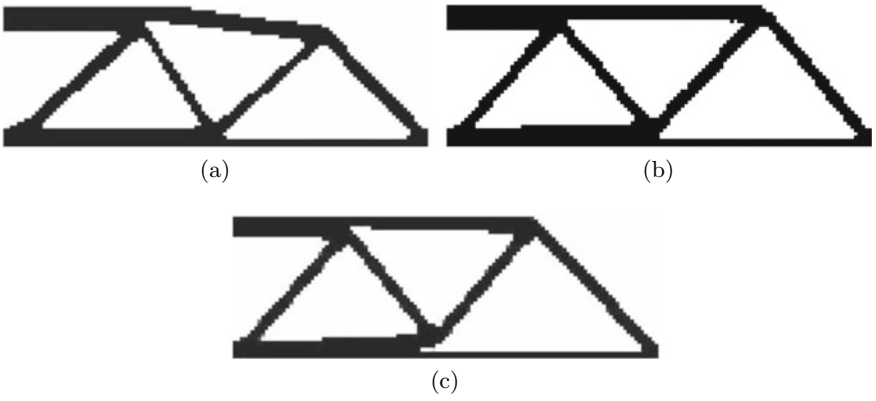


Fig. 3 Comparative optimal designs obtained from: **a** reference paper [16], **b** current research paper, and **c** continuation method [16]

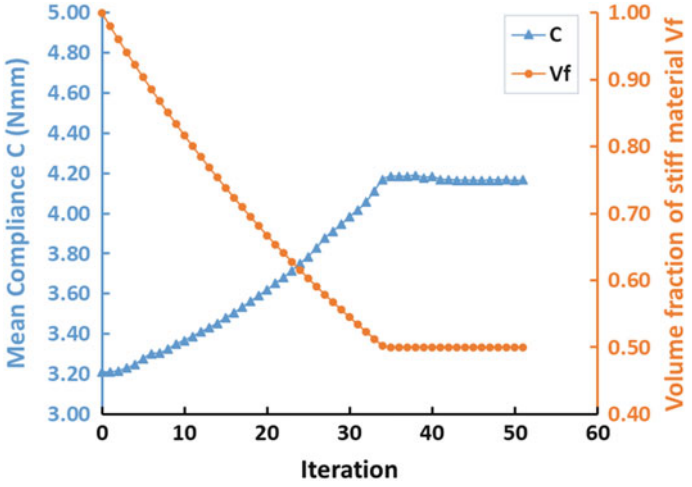
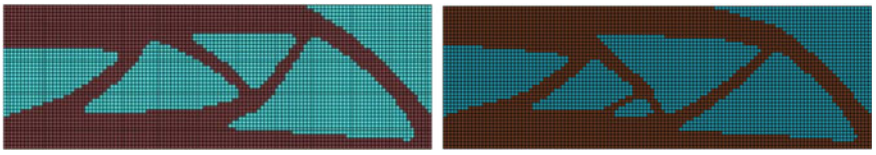


Fig. 4 Evolutionary histories of the mean compliance and volume fraction



(a) (b)

Fig. 5 Comparative two material optimal designs obtained from: **a** reference paper [16], **b** current research paper

is allocated an equal contribution of 50% in the final design domain. Figure 4 shows that the mean compliance of the whole model increases initially as the volume fraction of the stiff material decreases, and then it converges to an almost constant value after the objective volume is achieved. The topology parameters mentioned earlier are utilized for this optimization process, and the properties of the non-zero materials (stiff and soft materials) can be found in Table 2.

Figure 5a) presents the optimal design obtained using the two non-zero materials, as outlined in the reference paper [16]. The mean compliance of this design is determined to be 4.21 Nmm. Figure 5b) illustrates the optimal design attained using the author's code, which exhibits a mean compliance of 4.10 Nmm. While the material distribution in both optimal designs varies slightly, the design derived from this paper successfully meets the objective function and showcases a lower mean compliance, indicative of enhanced stiffness.

3.3 Optimized Design with Void and Two Materials

The author's code has demonstrated its effectiveness in addressing the optimization problem for designs involving two materials as well as solid-void configurations. It successfully integrates two non-zero materials, optimizes their distribution, and achieves the desired objectives, resulting in enhanced structural performance. While numerous open-source codes and software exist for similar purposes, the primary focus lies in achieving the same level of success for multi-material designs. This section aims to investigate and explore the optimization of multi-material designs including void, which presents unique challenges and requires specialized techniques.

In this section, the topology optimization problem, depicted in Fig. 2, considering two non-zero materials and the void is analyzed. The objective is to maximize the stiffness of the structure by minimizing its mean compliance, while ensuring an optimal distribution of the materials. The stiff and soft materials are allocated contributions of 15% and 25%, respectively, resulting in a final design domain consisting of 40% material and 60% void. The previously mentioned topology parameters are employed for this optimization process, and the properties of all the materials can be found in Table 2.

Figure 6a illustrates the optimal design obtained from the reference paper [16], with a mean compliance of 13 Nmm. Conversely, Fig. 6b displays the optimal design achieved using the author's code, which exhibits a mean compliance of 12.6 Nmm. Despite slight variations in material distribution, due to slightly different implementation (e.g. different sensitivity filter), the design presented in this paper meets the objective function and exhibits a lower mean compliance, indicating enhanced stiffness.

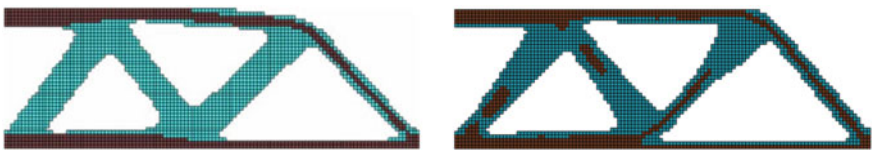


Fig. 6 Comparative multi-materials optimal designs obtained from: **a** reference paper [16], **b** current research paper

4 Conclusions

To conclude, this research paper introduces a comprehensive code that has been developed and implemented based on the formulation proposed in the reference paper authored by Huang and Xie [16]. The code effectively addresses three different variants of design optimization, namely solid-void, two materials, and multi-materials optimization. The primary objective of the code is to generate optimal designs that maximize stiffness by minimizing mean compliance while satisfying volume constraints. The results obtained from the code consistently show designs that exhibit slightly higher stiffness compared to those reported in the reference paper. This demonstrates the effectiveness of the code in achieving the desired objective of improved structural performance.

However, it is worth mentioning that the code does face certain challenges, particularly in terms of computational time. It requires more time to converge compared to the demonstrated results in the reference paper. This indicates an area for potential future improvements, with opportunities to enhance the code's efficiency and reduce the computational time required for optimization.

Furthermore, the code has been successfully converted into an ABAQUS plugin, enhancing its usability and convenience for users. This enables researchers and engineers to easily apply the code to their specific design problems within the ABAQUS framework. In the future, the code can be enhanced to optimize structures with complex geometries, multiple materials, and diverse design objectives and constraints. These improvements will broaden its application in structural engineering, addressing a wider range of optimization challenges.

Overall, this research paper contributes to the field of topology optimization by providing a comprehensive code that can be applied to different design variants, yielding stiffer designs and showcasing its potential for future advancements in the field.

Acknowledgements The research work was performed within the PdZ-project "3DFit4Wear - Neue Wege zur wirtschaftlichen Herstellung von adaptierbaren und wiederverwertbaren Orthesen über 3D Druck" at the Polymer Competence Center Leoben GmbH (PCCL, Austria) within the framework of the "Produktion der Zukunft"-program of the Federal Ministry for Climate Action, Environment, Energy, Mobility, Innovation and Technology and the Federal Ministry for Digital and Economic Affairs with contributions by Montan University Leoben (Chair of Chemistry of Polymeric Materials & Chair of Science and Testing of Polymer), In-Vision Technologies AG, and Luxinergy GmbH. The PCCL is funded by the Austrian Government and the State Governments of Styria, Lower Austria and Upper Austria.

References

1. Sigmund, O.: A 99 line topology optimization code written in matlab. Struct. Multidiscip. Optim. **21**, 120–127 (2001)
2. Nha Chu, D., Xie, Y.M., Hira, A., Steven, G.P.: Evolutionary structural optimization for problems with stiffness constraints. Finite Elements Anal. Des. **21**(4), 239–251 (1996)

3. Wang, M.Y., Wang, X., Guo, D.: A level set method for structural topology optimization. Comput. Methods Appl. Mech. Eng. **192**(1–2), 227–246 (2003)
4. Philip Bendsoe, M., Sigmund, O.: Topology Optimization: Theory, Methods, and Applications. Springer Science & Business Media (2003)
5. Sigmund, O., Petersson, J.: Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima. Struct. Optim. **16**, 68–75 (1998)
6. Bourdin, B.: Filters in topology optimization. Int. J. Numer. Meth. Eng. **50**(9), 2143–2158 (2001)
7. Sigmund, O.: Design of material structures using topology optimization. Ph.D. thesis, Technical University of Denmark Lyngby (1994)
8. Sigmund, O.: On the design of compliant mechanisms using topology optimization. J. Struct. Mech. **25**(4), 493–524 (1997)
9. Yago, D., Cante, J., Lloberas-Valls, O., Oliver, J.: Topology optimization methods for 3d structural problems: a comparative study. Arch. Comput. Methods Eng. **29**(3), 1525–1567 (2022)
10. Andreassen, E., Clausen, A., Schevenels, M., Lazarov, B.S., Sigmund, O.: Efficient topology optimization in matlab using 88 lines of code. Struct. Multidiscip. Optim. **43**, 1–16 (2011)
11. Han, Y., Bin, X., Liu, Y.: An efficient 137-line matlab code for geometrically nonlinear topology optimization using bi-directional evolutionary structural optimization method. Struct. Multidiscip. Optim. **63**, 2571–2588 (2021)
12. Huang, X., Xie, Y.M.: Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method. Finite Elem. Anal. Des. **43**(14), 1039–1049 (2007)
13. Liu, K., Tovar, A.: An efficient 3d topology optimization code written in matlab. Struct. Multidiscip. Optim. **50**, 1175–1196 (2014)
14. Zuo, Z.H., Xie, Y.M.: A simple and compact python code for complex 3d topology optimization. Adv. Eng. Softw. **85**, 1–11 (2015)
15. Michael Yu Wang and Shiwei Zhou: Synthesis of shape and topology of multi-material structures with a phase-field method. J. Comput. Aided Mater. Des. **11**, 117–138 (2004)
16. Xiaodong Huang and Yi Min Xie: Bi-directional evolutionary topology optimization of continuum structures with one or multiple materials. Comput. Mech. **43**, 393–401 (2009)
17. Christian Frier Hvejsel and Erik Lund: Material interpolation schemes for unified topology and multi-material optimization. Struct. Multidiscip. Optim. **43**, 811–825 (2011)
18. Gao, T., Zhang, W.: A mass constraint formulation for structural topology optimization with multiphase materials. Int. J. Numer. Meth. Eng. **88**(8), 774–796 (2011)
19. Yang, Y., Moen, C.D., Guest, J.K.: Three-dimensional force flow paths and reinforcement design in concrete via stress-dependent truss-continuum topology optimization. J. Eng. Mech. **141**(1), 04014106 (2015)
20. Wang, Y., Luo, Z., Kang, Z., Zhang, N.: A multi-material level set-based topology and shape optimization method. Comput. Methods Appl. Mech. Eng. **283**, 1570–1586 (2015)
21. Cui, M., Chen, H., Zhou, J.: A level-set based multi-material topology optimization method using a reaction diffusion equation. Comput. Aided Des. **73**, 41–52 (2016)
22. Li, Y., Min Xie, Y.: Evolutionary topology optimization for structures made of multiple materials with different properties in tension and compression. Composite Struct. **259**, 113497 (2021)
23. Li, Y., Yuan, P.F., Xie, Y.M.: Topology optimization of structures composed of more than two materials with different tensile and compressive properties. Composite Struct. **306**, 116609 (2023)