JÜRGEN ULM

Mathematical Methods 4 Electrotechnic Freaks

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Foreword

Mathematics is the universal tool for the scientist,

„... *for the mathematic is the basis of all exact scientific knowledge...*“

(David Hilbert, German mathematician, 1862-1943).

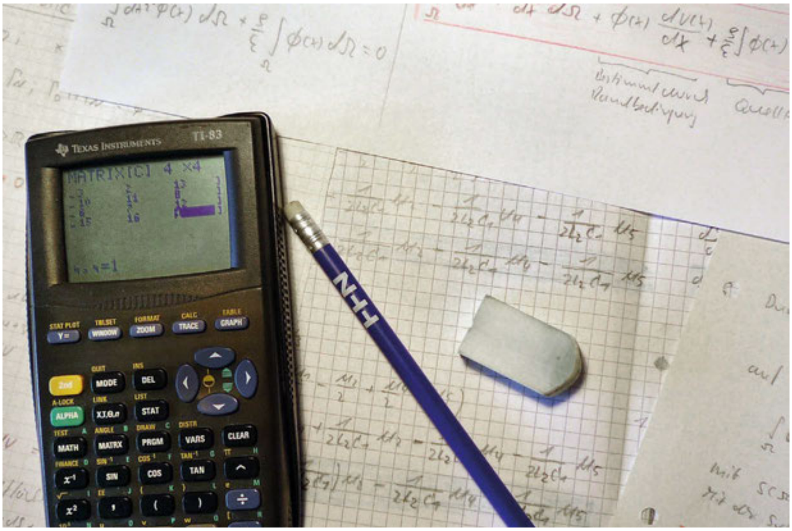
Special attention is therefore paid to learning how to use the tool. As is so often the case, the realisation of the necessity paired with the motivation of the user is in the foreground. If the declared aim is to describe physical relationships by means of mathematics, this does not necessarily require thematic rigour. The application of mathematical rigour is likely to be counterproductive to this concern. Furthermore, Gödel's incompleteness theorem of mathematics applies, which even shows mathematics itself its limitations. Experience has shown that the users' desire for mathematical rigour can be observed when they are convinced and enthusiastic about mathematics and its possibilities. For this reason, mathematical rigour should not be given the highest priority at the beginning. Mathematics lives from the joy of its users and applications!

„*It is impossible to adequately convey the beauties of the laws of nature if someone does not understand mathematics. I regret that, but it is probably so.*“

(Richard Feynman, physicist and Nobel Prize winner, 1918 1988),
denn

„*The book of nature is written in the language of mathematics.*“

(Galileo Galilei, 1564 1642).



Calculator, paper, pencil and eraser in combination with coffee form a good basis. Mathematics is the universal tool of electrical engineering. Selected mathematical methods are also used to deal with selected topics in electrical engineering. The work is carried out by presenting the basics, describing the task and solving the problem in detail. The target group of readers also results from this procedure. From the author's point of view, these are:

- Students of engineering sciences who would like to work on scientific topics using mathematical methods.
- Software engineers who want to implement differential equations in matrix form in microprocessors.
- Simulation engineers who would like to calculate something „on foot“.
- Measurement engineers who need a measurement value from a location where no sensor can be adapted and only calculations can be made for this location.
- Maths brave, pale in the face, survived and now want to try maths again.

Since our science has a mirror-image structure, it is worthwhile, for example, to familiarise oneself in depth with a scientific discipline. Here, electrical engineering is preferably recommended. By changing the coefficients of a differential equation, the enthusiastic reader of this book conquers another scientific discipline (hence the use of the term "mirror image"). For example, anyone who can solve electrical networks (meshes) can consequently also solve thermal, magnetic, mechanical and hydraulic networks. The mathematical basics include calculation rules, definitions, matrices, ordinary and partial differential equations and coordinate systems. They provide access to understanding the chosen mathematical methods and applications in electrical engineering. An elementary application in electrical engineering is the LCR oscillating circuit, which is described with differential equations and whose properties are presented. The integral transformation, the method of moments and Green's method have in common the formation of the inner product for the solution of differential equations. The last two methods are introduced in detail with the help of examples. With the method of moments, the transition to the finite element method (FEM) and finite difference method (FDM) is made using application examples. The method of moments is also used to introduce the eigenvalue problem. The development of infinite series by alternately applying the law of flow and the law of induction leads to Bessel functions as well as to the phenomenon of field displacement with the effect of current displacement in the conductor. Selected standards should provide the reader with hints for the preparation of scientific documentation. A note on the extended use of the book is permitted: New exercises can be generated by simply modifying the original problem that has already been solved. The modification of the original task should be done in such a way that its solution is already known in advance. This gives the possibility to compare the results and to further deepen the familiarisation. Because the following always applies

„*Uncertain are the calculations of the dispersible*“

(Wisdom Literature).

With kind regards the
author
autumn 2023

For more information on the institutes, see also Appendix B.

Contents

Symbols and abbreviations

Symbol	Meaning	Unit
A	coefficient, matrix	
A	area	m^2
B	coefficient, matrix	
$B, \vec{B}$	magnet. flux density, vector of magnet. flux density	Vs/m^2
B_h	interpolation, approach function	
C	coefficient, matrix	
C	capacity	As/V
C	heat capacity	J/K
D	coefficient, Charge	
D	charge density	As/m^2
D	discriminant	
E	coefficient, matrix	
$E, \vec{E}$	electric field strength, electric field strength	V/m
$\mathcal{E}$	length-related electric field strength	V/m^2
F	coefficient, function	
F	force	$N, kgm/s^2$
G	Green's function	
G	coefficient	
$H, \vec{H}$	magnet. field strength, vector of the magnet	A/m
H_Φ	field interpolation function, approach function	
I	current	A
$J, \vec{J}$	electr. current density, vector of electr. current density	A/m^2

Symbol	Meaning	Unit
K	constant	
L	inductivity	Vs/A
M	matrix	
N	number of nodes, line elements, running variable, number of turns	
P	power	W
P	polynomial function, evaluation point	
P'	source point, integration point	
Q	charge	As
R	residuum	
R	radius	m
R	resistance	Ω
S	matrix	
S_P	vertex	
U	voltage	V
V	volume	m^3
W	Wronski determinant	
X	reactance, reactance	Ω
$Z, \mid \underline{Z} \mid$	impedance, magnitude of the impedance	Ω
$\underline{Z}$	impedance (complex impedance)	Ω
a	coefficient	
a_0	acceleration	m/s^2
b	damping coefficient	kg/s
c	constant	
c	spring constant	N/m
c	speed of light	m/s
c	specific heat capacity	$J/(kgK)$
d	diameter	m
e	e-function	
$\vec{e}$	unit vector	
f	auxiliary variable, function, matrix, column vector	
g	auxiliary variable, function, matrix	
h	element length, distance, height	m

Symbol	Meaning	Unit
i	control variable	
i	current	A
j	control variable	
j	imaginary unit	$\sqrt{-1}$
$k, \underline{k}$	constant, complex constant	
l	length	m
l	matrix	
m	control variable	
m	mass	kg
n	normal, number of partial intervals	
p	impulse	$kg\ m/s$
p	variable, function	
r	radius	m
s	constant	
s	distance, length	m
t	time	s
u	function, interpolation, approach function	
u	voltage	V
$\hat{u}_0$	voltage amplitude	V
v	function, interpolation, approach function	
v	speed	m/s
w	weight, weighting, test, shape function	
x	coordinate, path	m
y	coordinate, path	m
y	function	
z	coordinate, path	m
Γ	edge of the FEM area	
Δ	delta, differential	
Θ	magnetomotive force	A
Φ	magnetic flux	$V s$
Ψ	chained magnetic flux	$V s$
Ω	area, sub-area, element	m^2

Symbol	**Meaning**	**Unit**
α	coefficient	
β	coefficient	
γ	Coefficient, boundary value	
δ	decay constant	
ε	permittivity	$As/(Vm)$
ε_0	permittivity of the vacuum $[8,8542\ 10^{-12}As/(Vm)]$	$As/(Vm)$
υ	temperature	$^\circ C$
κ	specific electrical conductivity	$m/(\Omega mm^2)$
λ	thermal conductivity	$W/(mK)$
λ	eigenvalue, Lagrange multiplier	
μ	permeability	$Vs/(Am)$
μ_0	permeability of the vacuum $[4\pi 10^{-7}Vs/(Am)]$	$Vs/(Am)$
ρ	density	kg/m^3
ρ	volume charge density	As/m^3
τ	time constant	s
υ_h	approach, test function	
φ	potential	V
φ	interpolation, approach function, angle	
φ	angle	rad
ϕ	development, base, triangular function	
ω	angular velocity, angular frequency	$1/s$
ΔA, $\Delta A'$	differential surface elements	m^2
Δx, Δy	differential line elements	m
dA	infinitesimal surface element	m^2
dx, dy	infinitesimal line elements	m
$\mathcal{L}$	linear operator	
$\mathcal{M}$	linear operator	
$\mathcal{O}$	zero operator	
$\mathcal{I}$	identity operator	
∇	Nabla operator	
Δ	Delta operator	

Chapter 1

Required mathematical basics

„*Last time I asked: What does mathematics mean to you?, and some people answered: The manipulation of numbers, the manipulation of structures. And if I had asked what music means to you, would you have answered: The manipulation of notes?*"

(Serge Lang, French-American mathematician, 1927-2005) from „*The beauty of doing Mathematics*". Serge Lang became known for his work on algebraic geometry and number theory and as the author of many textbooks.

The basics required for the numerical solution of differential equations have been compiled in this chapter. These essentially include matrices, definitions and classifications of differential equations as well as initial and boundary value problems and vector operators. Particularly recommended literature for this are [4], [60] and [67].

1.1 Logarithm

The logarithm of x (numerus, logarithmand) to the base a is the real number b (exponent), for which the following applies

$$\begin{aligned} \log_a x &= b \\ a^b &= x. \end{aligned}$$

The logarithm to the base 10 is called the decadic or Briggsian logarithm. It follows

$$\log_{10} x = \lg x$$

and it applies

$$\log\left(x \cdot 10^{\alpha}\right) \quad = \quad \alpha + \log x.$$

Examples of this are

- Example 1:

$$\begin{aligned} \log\left(5 \cdot 10^{1}\right) &= 1 + \log 5 \\ &= 1,698 \end{aligned}$$

- Example 2:

$$\begin{aligned} \log\left(5 \cdot 10^{2}\right) &= 2 + \log 5 \\ &= 2,69. \end{aligned}$$

Furthermore

$$\log a \quad = \quad \alpha + \log m$$

with the numerus or logarithm a, mantissa m and α the index of the logarithm, equal to the exponent of the place value of the first significant digit of the numerus. See also [1], S. 56. In summary, some more useful logarithmic laws are

- Multiplication of the independent parameters

$$\log_a(u \cdot v) \quad = \quad \log_a u \; + \; \log_a v$$

- Division of the independent parameters

$$\log_a\left(\frac{u}{v}\right) \quad = \quad \log_a u \; - \; \log_a v$$

- Exponentiation of the independent variable

$$\log_a u^v \quad = \quad v \log_a u$$

- Squaring of the independent variable

$$\log_a \sqrt[v]{w} \; = \; \log_a w^{\frac{1}{v}} \quad = \quad \frac{1}{v} \log_a w.$$

1.2 Matrices

The matrix notation summarises the calculations with functions and thus increases the overview. For this purpose, a vector operator summarises derivatives. These are marked with a simple symbol (Nabla or Laplace operator). The matrix notation (matrix equations) enables the numerical solution of linear systems of equations by means of the solution methods known in the literature. Therefore, matrix and matrices receive special attention. Selected matrix operations are presented here. These include the necessary matrix calculation rules, the inversion, multiplication of a matrix, matrix textures as well as determinant calculation rules, and much more. Recommended literature is [60], p. 268 ff. and [29], p. 12 ff. (Random matrices – new universal laws).

1.2.1 Arithmetic operations with matrices

Table 1.1 summarises the most important algebraic axioms.

Table 1.1: Summary of the most important calculation rules

Associative law	$\mathbf{A}\ (\mathbf{BC}) = (\mathbf{AB})\ \mathbf{C}$
Distributive law	$\mathbf{A}\ (\mathbf{B}+\mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
	$(\mathbf{A}+\mathbf{B})\ \mathbf{C} = \mathbf{AC} + \mathbf{BC}$
Transpose	$(\mathbf{AB})^T = \mathbf{B}^T\ \mathbf{A}^T$

Note that matrix multiplication is not commutative, which is

$$\mathbf{A} \cdot \mathbf{B} \neq \mathbf{B} \cdot \mathbf{A}.$$

1.2.2 Addition and subtraction of two matrices

Two matrices **A** and **B** of the same type are added or subtracted by adding or subtracting their corresponding elements

$$\mathbf{A} \pm \mathbf{B} = (a_{ik} \pm b_{ik}) = \mathbf{C},$$

with $i = 1, 2, 3, ..., m$ and $k = 1, 2, 3, ..., n$ and $\mathbf{C}$ the sum or difference matrix. Addition and subtraction are only defined for matrices of the same type (m, n).