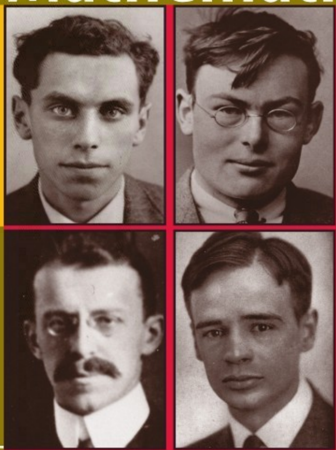
Alexander Soifer

The Mathematical Coloring Book



Mathematics of Coloring and the Colorful Life of Its Creators

Springer

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Mathematics of Coloring and the Colorful Life of its Creators

Forewords by Branko Grünbaum, Peter D. Johnson Jr., and Cecil Rousseau

Springer

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Cover illustration: The photographs on the front cover depict, from the upper left clockwise, Paul Erdős, Frank P. Ramsey, Bartel L. van der Waerden, and Issai Schur.

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This coloring book is for my late father Yuri Soifer,
a great painter, who introduced colors into my life.

To Paint a Bird

First paint a cage
With wide open door,
Then paint something
Beautiful and simple,
Something very pleasant
And much needed
For the bird;
Then lean the canvas on a tree
In a garden or an orchard or a forest –
And hide behind the tree,
Do not talk
Do not move. . .
Sometimes the bird comes quickly
But sometimes she needs years to decide
Do not give up,
Wait,
Wait, if need be, for years,
The length of waiting –
Be it short or long –
Does not carry any significance
For the success of your painting
When the bird comes –
If only she ever comes –
Keep deep silence,
Wait,
So that the bird flies in the cage,
And when she is in the cage,
Quietly lock the door with the brush,
And without touching a single feather
Carefully wipe out the cage.
Then paint a tree,
And choose the best branch for the bird
Paint green leaves

Freshness of the wind and dust of the sun,
Paint the noise of animals in the grass
In the heat of summer
And wait for the bird to sing
If the bird does not sing –
This is a bad omen
It means that your picture is of no use,
But if she sings –
This is a good sign,
A symbol that you can be
Proud of and sign,
So you very gently
Pull out one of the feathers of the bird
And you write your name
In a corner of the picture.

by Jacques Prévert[1]

[1] [Pre]. Translation by Alexander Soifer and Maurice Stark.

Foreword

This is a unique type of book; at least, I have never encountered a book of this kind. The best description of it I can give is that it is a mystery novel, developing on three levels, and imbued with both educational and philosophical/moral issues. If this summary description does not help understanding the particular character and allure of the book, possibly a more detailed explanation will be found useful.

One of the primary goals of the author is to interest readers—in particular, young mathematicians or possibly pre-mathematicians—in the fascinating world of elegant and easily understandable problems, for which no particular mathematical knowledge is necessary, but which are very far from being easily solved. In fact, the prototype of such problems is the following: If each point of the plane is to be given a color, how many colors do we need if every two points at unit distance are to receive distinct colors? More than half a century ago it was established that the least number of colors needed for such a coloring is either 4, or 5, or 6 or 7. Well, which is it? Despite efforts by a legion of very bright people—many of whom developed whole branches of mathematics and solved problems that seemed much harder—not a single advance towards the answer has been made. This mystery, and scores of other similarly simple questions, form one level of mysteries explored. In doing this, the author presents a whole lot of attractive results in an engaging way, and with increasing level of depth.

The quest for precision in the statement of the problems and the results and their proofs leads the author to challenge much of the prevailing historical "knowledge." Going to the original publications, and drawing in many cases on witnesses and on archival and otherwise unpublished sources, Soifer uncovers many mysteries. In most cases, dogged perseverance enables him to discover the truth. All this is presented as following in a natural development from the mathematics to the history of the problem or result, and from there to the interest in the people who produced the mathematics. For many of the persons involved this results in information not available from any other source; in lots of the cases, the available publications present an inaccurate (or at least incomplete) data. The author is very careful in documenting his claims by specific references, by citing correspondence between the principals involved, and by accounts by witnesses.

One of these developments leads Soifer to examine in great detail the life and actions of one of the great mathematicians of the twentieth century, Bartel Leendert van der Waerden. Although Dutch, van der Waerden spent the years from 1931 to 1945 in the Nazi Germany. This, and some of van der Waerden's activities during that time, became very controversial after Word War II, and led Soifer to examine the moral and ethical questions relevant to the life of a scientist in a criminal dictatorship.

The diligence with which Soifer pursues his quests for information is way beyond exemplary. He reports exchanges with I am sure hundreds of people, via mail, phone, email, visits – all dated and documented. The educational aspects that begin with matters any middle-school student can understand, develop gradually into areas of most recent research, involving not only combinatorics but also algebra, topology, questions of foundations of mathematics, and more.

I found it hard to stop reading before I finished (in two days) the whole text. Soifer engages the reader's attention not only mathematically, but emotionally and esthetically. May you enjoy the book as much as I did!

University of Washington Branko Grünbaum

Foreword

Alexander Soifer's latest book is a fully fledged adult specimen of a new species, a work of literature in which fascinating elementary problems and developments concerning colorings in arithmetic or geometric settings are fluently presented and interwoven with a detailed and scholarly history of these problems and developments.

This history, mostly from the twentieth century, is part memoir, for Professor Soifer was personally acquainted with some of the principals of the story (the great Paul Erdős, for instance), became acquainted with others over the 18 year interval during which the book was written (Dima Raiskii, for instance, whose story is particularly poignant), and created himself some of the mathematics of which he writes.

Anecdotes, personal communications, and biography make for a good read, and the readability in "Mathematical Coloring Book" is not confined to the accounts of events that transpired during the author's lifetime. The most important and fascinating parts of the book, in my humble opinion, are Parts IV, VI, and VII, in which is illuminated the progress along the intellectual strand that originated with the Four-Color Conjecture and runs through Ramsey's Theorem via Schur, Baudet, and Van der Waerden right to the present day, via Erdős and numbers of others, including Soifer. Not only is this account fascinating, it is indispensable: it can be found nowhere else.

The reportage is skillful and the scholarship is impressive – this is what Seymour Hersh might have written, had he been a very good mathematician curious to the point of obsession with the history of these coloring problems.

The unusual combination of abilities and interests of the author make the species of which this book is the sole member automatically endangered. But in the worlds of literature, mathematics and literature about mathematics, unicorns can have offspring, even if the offspring are not exactly unicorns. I think of earlier books of the same family as "Mathematical Coloring Book" – G. H. Hardy's "A Mathematician's Apology", James R. Newman's "The World of Mathematics", Courant and Robbins' "What Is Mathematics?", Paul Halmos' "I Want to Be a Mathematician: an Automathography", or the books on Erdős that appeared soon after his death – all of them related at least distantly to "Mathematical Coloring Book" by virtue of the attempt to blend (whether successfully or not is open to debate) mathematics with history or personal memoir, and it seems to me that, whatever the merits of those works, they have all affected how mathematics is viewed and written about. And this will be a large part of the legacy of "Mathematical Coloring Book" – besides providing inspiration and plenty of mathematics to work on to young mathematicians, a priceless source to historians, and entertainment to those who are curious about the activities of mathematicians, "Mathematical Coloring Book" will (we can hope) have a great and salutary influence on all writing on mathematics in the future.

Auburn University Peter D. Johnson

Foreword

What is the minimum number of colors required to color the points of the Euclidean plane in such a way that no two points that are one unit apart receive the same color? *Mathematical Coloring Book* describes the odyssey of Alexander Soifer and fellow mathematicians as they have attempted to answer this question and others involving the idea of partitioning (coloring) sets.

Among other things, the book provides an up-to-date summary of our knowledge of the most significant of these problems. But it does much more than that. It gives a compelling and often highly personal account of discoveries that have shaped that knowledge.

Soifer's writing brings the mathematical players into full view, and he paints their lives and achievements vividly and in detail, often against the backdrop of world events at the time. His treatment of the intellectual history of coloring problems is captivating.

Memphis State University | Cecil Rousseau

Acknowledgments

My first thank you goes to my late father Yuri Soifer, a great painter, who introduced colors into my life, and to whom this book is gratefully dedicated. As the son of a painter and an actress, I may have inherited artistic genes. Yet, it was my parents, Yuri Soifer and Frieda Hoffman, who inspired my development as a connoisseur and student of the arts. I have enjoyed mathematics only because it could be viewed as an art as well. I thank Maya Soifer for restarting my creative engine when at times it worked on low rpm (even though near the end of my work, she almost broke the engine by abandoning the car). I am deeply indebted to my kids Mark, Isabelle, and Leon, and to my cousin and great composer Leonid Hoffman for the support their love has always provided. I thank my old friends Konstantin Kikoin, Yuri Norstein and Leonid Hoffman for years of stimulating conversations on all themes of high culture. I am deeply grateful to Branko Grünbaum, Peter D. Johnson Jr., and Cecil Rousseau, the first readers of the entire manuscript, for their kind forewords and valuable suggestions. My 16-year old daughter Isabelle Soulay Soifer, an aspiring writer, did a fine copy-editing job – thank you, Isabelle!

This is a singular book for me, a result of 18 years of mathematical and historical research, and thinking over the moral and philosophical issues surrounding a mathematician in the society. The long years of writing have produced one immense benefit that a quickly baked book would never fathom to possess. I have had the distinct pleasure to communicate on the mathematics and the history for this book with senior sages Paul Erdős, George Szekeres, Esther (Klein) Szekeres, Martha (Wachsberger) Svéd, Henry Baudet, Nicolaas G. de Bruijn, Bartel L. van der Waerden, Harold W. Kuhn, Dirk Struick, Hilde Brauer (Mrs. Alfred Brauer), Hilde Abelin-Schur (Issai Schur's daughter), Walter Ledermann, Anne Davenport (Mrs. Harold Davenport), Victor Klee, and Branko Grünbaum. Many of these great people are no longer with us; others are near or in their 80s. Their knowledge and their memories have provided blood to the body of my book. I am infinitely indebted to them all, as well as to the younger contributors Ronald L. Graham, Edward Nelson, John Isbell, Adriano Garsia, James W. Fernandez, and Renate Fernandez, who are merely in their 70s.

Harold W. Kuhn wrote a triple essay on the economics of Frank P. Ramsey, John von Neumann and John F. Nash Jr. especially for this book, which can be found in Chapter 30. Steven Townsend wrote and illustrated a new version of his proof especially for this book (Chapter 24). Kenneth J. Falconer wrote a new clear exposition of his proof especially for this book (Chapter 9). Your contributions are so great, and I thank you three so very much!

I am grateful to several colleagues for their self-portraits written for this book: Nicolaas de Bruijn, Hillel Furstenberg, Vadim Vizing, Kenneth Falconer, Paul O'Donnell, Vitaly Bergelson, Alexander Leibman, and Michael Tarsi.

I am thrilled to be able to present results and writings of my colleagues I have known personally: Paul Erdős, Paul O'Donnell, Rob Hochberg, Kiran Chilakamarri, Ronald L. Graham, Joel H. Spencer, Paul Kainen, Peter D. Johnson, Jr., Nicholas Wormald, David Coulson, Paul Seymour, Neil Robertson, Robin J. Wilson, Edward Pegg, Jr., Jaroslav Nešetřil, Branko Grünbaum, Saharon Shelah, Dmitry Raiskii, Dmytro Karabash, Willie Moser, George Szekeres, Esther Klein, Alfred W. Hales, Heiko Harborth, Solomon W. Golomb, Jan Mycielski, Miklós Simonovits, and colleagues I have gotten to know through correspondence: Bruce L. Rothschild, Vadim G. Vizing, Mehdi Behzad, Douglas R. Woodall, and Jan Kyncl.

I am grateful to George Szekeres and MIT Press for their permission to reproduce here the most lyrical George's *Reminiscences* (part of Chapter 29). I thank Bartel L. van der Waerden and Academic Press, London, for their kind permission to reproduce here Van der Waerden's insightful *How the Proof of Baudet's Conjecture Was Found* (Chapter 33).

I thank all those who have provided me with the rare, early photographs of themselves: Paul Erdős (as well as a photograph with Leo Moser); George Szekeres and Esther Klein; Vadim Vizing; Edward Nelson; Paul O'Donnell. Hilde Abelin-Schur has kindly provided photographs of her father Issai Schur. Dorith van der Waerden and Theo van der Waerden have generously shared rare photographs of their uncle Bartel L. van der Waerden and the rest of their distinguished family. Henry Baudet II has generously supplied photographs of his father P. J. H. Baudet and also of his family with the legendary world chess champion Emanuel Lasker. Ronald L. Graham has kindly provided a photograph of him presenting Timothy Gowers with the check for $1000. Alice Bogdán has provided a photograph of her great brother-in-law Tibor Gallai.

I thank numerous archives for documents and photographs and permissions to use them. Dr. Mordecai Paldiel and Yad Vashem, The Holocaust Martyrs' and Heroes' Remembrance Authority, Jerusalem, provided documents related to the granting Senta Govers Baudet the title of a "Righteous Among the Nations." John Webb and Mathematics Department of the University of Cape Town provided a photograph of Francis Guthrie. By kind permission of the Provost and Scholars of King's College, Cambridge, I am able to reproduce here two photographs of Frank Plumpton Ramsey, from the papers of John Maynard Keynes. The Board of Trinity College Dublin has kindly allowed me to reproduce the correspondence between Augustus De Morgan and William R. Hamilton. Humboldt University of Berlin shared documents from the personnel file of Issai Schur.

The following archivists and archives provided invaluable documents and some photographs related to Bartel L. van der Waerden: John Wigmans, *Rijksarchief in Noord-Holland*; Dr. Peter J. Knegtmans, The University Historian, *Universiteit van Amsterdam*; Prof. Dr. Gerald Wiemers and Martina Geigenmüller, *Universitätsarchiv Leipzig*; Prof. Dr. Holger P. Petersson and his personal archive; Gertjan Dikken, *Het Parool*; Madelon de Keizer; Dr. Wolfram Neubauer, Angela Gastl and Corina Tresch De Luca, *ETH-Bibliothek, ETH*, Zürich; Dr. Heinzpeter Stucki, *Universitätsarchiv, Universität Zürich*; Drs. A. Marian Th. Schilder, *Universiteitsmuseum de Agnietenkapel*, Amsterdam; Maarten H. Thomp, *Centrale Archiefbewaarplaats, Universiteit Utrecht*; Nancy Cricco, University Archivist, and her graduate students assistants, New York University; James Stimpert, Archivist, Milton S. Eisenhower Library, Special Collections, The Johns Hopkins University; Prof. Mark Walker; Thomas Powers; Nicolaas G. de Bruijn; Henry Baudet II, Dr. Helmut Rechenberg, Director of the Werner Heisenberg Archiv, Munich; G. G. J. (Gijs) Boink, *Het Nationaal Archief*, Den Haag; Library of Congress, Manuscript Division, Washington D.C.; *Centrum voor Wiskunde en Informatica*, Amsterdam; Prof. Nicholas M. Katz, Chair, Department of Mathematics, Princeton University; and Mitchell C. Brown and Sarah M. LaFata, Fine Library, Princeton University.

My research on the life of Van der Waerden could not be based on the archival material alone. I am most grateful to individuals who provided much help in this undertaking: Dorith van der Waerden; Theo van der Waerden; Hans van der Waerden; Helga van der Waerden Habicht; Annemarie van der Waerden; Prof. Dr. Henry Baudet (1919–1998); Prof. Dr. Herman J. A. Duparc (1918–2002); Prof. Dr. Nicolaas G. de Bruijn; Prof. Dr. Benno Eckmann; Prof. Dr. Dirk van Dalen; Prof. Dirk J. Struik (1894–2000); Dr. Paul Erdős (1913–1996); Reinhard Siegmund-Schultze; Thomas Powers; Prof. Mark Walker, Prof. James and Dr. Renate Fernandez; Dr. Maya Soifer; and Princeton University Professors Harold W. Kuhn, Simon Kochen, John H. Conway, Edward Nelson, Robert C. Gunning, Hale Trotter, Arthur S. Wightman, Val L. Fitch, Robert Fagles, Charles Gillispie and Steven Sperber.

I thank my wonderful translators of Dutch, Dr. Stefan van der Elst and Prof. Marijke Augusteijn; and of German: Prof. Simon A. Brendle; Prof. Robert von Dassanowsky; Prof. Robert Sackett; Prof. Dr. Heiko Harborth; and Prof. Dr. Hans-Dietrich Gronau.

I thank those who converted my pencil doodles into computer-aided illustrations: Steven Bamberger, Phil Emerick and Col. Dr. Robert Ewell.

The research quarterly *Geombinatorics* provided a major forum for the essays related to the chromatic number of the plane. Consequently, it is cited numerous times in this book. I wish to thank the editors of *Geombinatorics* Paul Erdős, Branko Grünbaum, Ron Graham, Heiko Harborth, Peter D. Johnson, Jr., Jaroslav Nešetřil, and János Pach.

My University of Colorado bosses provided support on our campus and opportunity to be away for long periods of time at Princeton and Rutgers Universities—I thank Tom Christensen, Tom Wynn, and Pam Shockley-Zalabak.

I am grateful to my Princeton-Math colleagues and friends for maintaining a unique creative atmosphere in the historic Fine Hall, and Fred Roberts for the tranquility of his DIMACS at Rutgers University. Library services at Princeton have provided an invaluable swift service: while working there for 3 years, I must have read thousands of papers and many books.

I thank my Springer Editor Mark Spencer, who initiated our contact, showed trust in me and this project based merely on the table of contents and a single section, and in 2004 proposed to publish this book in Springer. At a critical time in my life, Springer's Executive Editor Ann Kostant made me believe that what I am and what I do really matters—thank you from the bottom of my heart, Ann!

There is no better place to celebrate the completion of the book than the land of Pythagoras, Euclid, and Archimedes. I thank Prof. Takis Vlamos for organizing my visit and lectures on the Island of Corfu, Thessaloniki, and Athens.

Contents

Greetings to the Reader

I bring here all: what have I lived thru,
And that what keeps my soul alive,
My rectitude and aspirations,
And what have seen my own eyes.
– Boris Pasternak, *The Waves*, 1931[2]

When the form is realized, it is here to live its own life.
– Pablo Picasso

Pasternak's epigraph describes precisely my work on this book—I gave it all of myself, without reservation. August Renoir believed that just as many people read one book all their lives (the Bible, the Koran, etc.), so can he paint all his life one painting. Likewise I could write one book all my life—in fact, I almost have, for I have been working on this book for 18 years.

It is unfair, however, to keep the book all to myself—many colleagues have been waiting for the birth of this book. In fact, it has been cited and even reviewed many years ago. The first mention of it appears already in 1991 on page 336 of the book by Victor Klee and Stan Wagon [KW], where the authors recommend the book for "survey of later developments of the chromatic number of the plane problem." On page 150 of their 1995 book [JT], Tommy R. Jensen and Bjarne Toft announced that "a comprehensive survey [of the chromatic number of the plane problem]. . .will be given by Soifer [to appear]." Once in the 1990s my son Mark told me that he saw my *Mathematical Coloring Book* available for $30 for special order at the Borders bookstore. I offered to buy a copy!

I started writing this book when copies of my *How Does One Cut a Triangle?* [Soi1] arrived from the printer, in early 1990. I told my father Yuri Soifer then

[2] [Pas], Translated for this book by Ilya Hoffman. The original Russian text is:
Здесь будет все: пережитое,
И то, чем я еще живу,
Мои стремленья и устои,
И виденное наяву
.

that this book would be dedicated to him, and so it is. This coloring book is for my late father, a great painter and man. Yuri lived with his sketchpad and drawing utensils in his pocket, constantly and intensely looking at people and making sharp momentary sketches. He was a great artist and my lifelong example of searching for and discovering life around him, and creating art that challenged "real" life herself. Yuri never taught me his trade, but during our numerous joint tours of art in museums and exhibitions, he pointed out beauties that only true artists could notice: a dream of harvest in Van Gogh's "Sower," Rodin's distortions in a search of greater expressiveness. These timeless lessons allowed me to become a student of beauty, and discover subtleties in paintings, sculptures, and movies throughout my life.

This book includes not just mathematics, but also the process of investigation, trains of mathematical thought, and where possible, psychology of mathematical invention. The book does not just include history and prehistory of Ramsey Theory and related fields, but also conveys the process of historical investigation—the kitchen of historical research if you will. It has captivated me, and made me feel like a Sherlock Holmes—I hope my reader will enjoy this sense of suspense and discovery as much as I have.

The epigraph for my book is an English translation of Jacques Prévert's genius and concise portrayal of creative process—I know of no better. I translated it with the help of my friend Maurice Starck from *Nouvelle Caledonie*, the island in the Pacific Ocean to which no planes fly from America, but to paraphrase Rudyard Kipling, *I'd like to roll to Nouvelle Caledonie some day before I'm old*!

This book is dedicated to problems involving colored objects, and results about the existence of certain exciting and unexpected properties that occur regardless of how these objects (points in the plane, space, integers, real numbers, subsets, etc.) are colored. In mathematics, these results comprise *Ramsey Theory*, a flourishing area of mathematics, with a motto that can be formulated as follows: any coloring of a large enough system contains a monochromatic subsystem of given in advance structure, or simply put, absolute chaos is absolutely impossible. Ramsey Theory thus touches on many fields of mathematics, such as combinatorics, geometry, number theory, and addresses new problems, often on the frontier of two or more traditional mathematical fields. The book will also include some problems that can be solved by inventing coloring, and results that prove the existence of certain colorings, most famous of the latter being, of course, The Four-Color Theorem.

Most books in the field present mathematics as a flower, dried out between pages of an old dusty volume, so dry that the colors are faded and only theorem–proof narrative survives. Along with my previous books, *Mathematical Coloring Book* will strive to become an account of a live mathematics. I hope the book will present mathematics as a human endeavor: the reader should expect to find in it not only results, but also portraits of their creators; not only mathematical facts, but also open problems; not only new mathematical research, but also new historical investigations; not only mathematical aspirations, but also moral dilemmas of the times between and during the two horrific World Wars of the twentieth century. In my view, mathematics is done by human beings, and knowing their lives and cultures enriches our understanding of mathematics as a product of human activity, rather than an abstraction which exists separately from us and comes to us exclusively as a catalog of theorems and formulas. Indeed, new facts and artifacts will be presented that are related to the history of the Chromatic Number of the Plane problem, the early history of Ramsey Theory, the lives of Issai Schur, Pierre Joseph Henry Baudet, and Bartel Leendert van der Waerden.

I hope you will join me on a journey you will never forget, a journey full of passion, where mathematics and history are researched in the process of solving mysteries more exciting than fiction, precisely because those are mysteries of real affairs of human history. Can mathematics be received by all senses, like a vibrant flower, indeed, like life itself? One way to find out is to experience this book.

While much of the book is dedicated to results of Ramsey Theory, I did not wish to call my book "Introduction to Ramsey Theory," for such a title would immediately lose young talented readers' interest. Somehow, the playfulness of *Mathematical Coloring Book* appealed to me from the start, even though I was asked on occasion whether 5-year olds would be able to color in my book between its lines. To be a bit more serious, and on advice of Vickie Kern of the Princeton University Press, I created a subtitle *Mathematics of Coloring and the Colorful Life of Its Creators*. This book is not a "dullster" of traditional theorem–proof–theorem–proof kind. It explores the birth of ideas and searches for its creators. I discovered very quickly that in conveying "colorful lives of creators," I could not always rely on encyclopedias and biographical articles, but had to conduct historical investigations on my own. It was a hard work to research some of the lives, especially that of B. L. van der Waerden, which alone took 12 years of archival research and thinking over the assembled evidence. Fortunately this produced a satisfying result: we have in this book some definitive biographies, of Bartel L. van der Waerden, Pierre Joseph Henry Baudet, Issai Schur, autobiography of Hillel Furstenberg, and others.

I always attempt to understand who made a discovery and how it was made. Accordingly, this book tries to explore biographies of the discoverers and the psychology of their creative processes. Every stone has been turned: my information comes from numerous archives in Germany, the Netherlands, Switzerland, Ireland, England, South Africa, the United States; invaluable and irreplaceable now interviews conducted with eyewitnesses; discussions held with creators. Cited bibliography alone includes over 800 items—I have read thousands of publications in the process of writing this book. I was inspired by people I have known personally, such as Paul Erdős, James W. Fernandez, Harold W. Kuhn, and many others, as well as people I have not personally met, such as Boris Pasternak, Pablo Picasso, Herbert Read—to name a few of the many influences—or D. A. Smith, who in the discussion after Alfred Brauer's talk [Bra2, p. 36], wrote:

> Mathematical history is a sadly neglected subject. Most of this history belongs to the twentieth century, and a good deal of it in the memories of mathematicians still living. The younger generation of mathematicians has been trained to consider the product, mathematics, as the most important thing, and to think of the people who produced it only as names attached to theorems. This frequently makes for a rather dry subject matter.

Milan Kundera, in his *The Curtain: An Essay in Seven Parts* [Kun], said about a novel what is true about mathematics as well:

> A novelist talking about the art of the novel is not a professor giving a discourse from his podium. Imagine him rather as a painter welcoming you into his studio, where you are surrounded by his canvases staring at you from where they lean against the walls. He will talk about himself, but even more about other people, about novels of theirs that he loves and that have a secret presence in his own work. According to his criteria of values, he will again trace out for you the whole past of the novel's history, and in so doing will give you some sense of his own poetics of the novel.

I was also inspired by the early readers of the book, and their feedback. Stanisław P. Radziszowski, after reviewing Chapter 27, e-mailed me on May 2, 2007:

> I am very anxious to read the whole book! You are doing great service to the community by taking care of the past, so the things are better understood in the future.

In his unpublished letter, Ernest Hemingway in a sense defended my writing of this book for a very long time:[3]

> When I make country, or a city, or a river in a novel it is slow work because you have to always *make* it, then it is alive. But nobody makes anything quickly nor easily if it is any good.

Branko Grünbaum, upon reading the entire manuscript, wrote in the February 28, 2008 e-mail:

> Somehow it seems that 18 years would be too short a time to dig up all this information!

This book will not strike the reader by completeness or most general results. Instead, it would give young active high school and college mathematicians an accessible introduction to the beautiful ideas of mathematics of coloring. Mathematics professionals, who may believe they know everything, would be pleasantly surprised by the unpublished or unnoticed mathematical gems. I hope young and not so young mathematicians alike will welcome an opportunity to try their hand—or mind—on numerous open problems, all easily understood and not at all easy to solve.

If the interest of my colleagues and friends at Princeton-Math is any indication, every intelligent reader would welcome an engagement in solving historical mysteries, especially those from the times of the Third Reich, World War II, and de-Nazification of Europe. Historians of mathematics would find a lot of new information and old errors corrected for the first time. And everyone will experience seeing, for the first time, faces they have not seen before in print: rare photographs of the creators of mathematics presented herein, from Francis Guthrie to Issai Schur as a young man, from young Edward Nelson to Paul O'Donnell, from Pierre Joseph Henry Baudet to Bartel L. van der Waerden and his family, and documents, such as

[3] From the unpublished 1937 letter. Quoted from *New York Times*, February 10, 2008, p. AR 8.

the one where Adolph Hitler commits a "micromanagement" of firing the Jew, Issai Schur, from his job of professor at the University of Berlin.

This is a freely flowing book, free from a straight jacket of a typical textbook, yet useable as a text for a host of various courses, two of which I have given to college seniors and graduate students at the University of Colorado: *What is Mathematics?*, and *Mathematical Coloring Course*, both presenting a "laboratory of a mathematician," a place where students learn mathematics and its history by researching them, and in the process realizing what mathematics is and what mathematicians do.

In writing this book, I tried to live up to the high standard, set by one of my heroes, the great Danish film director Carl Theodore Dreyer [Dre]:

> There is a certain resemblance between a work of art and a person. Just as one can talk about a person's soul, one can also talk about the work or art's soul, its personality. The soul is shown through the style, which is the artist's way of giving expression of his perception of the material. The style is important in attaching inspiration to artistic form. Through the style, the artist molds the many details that make it whole. Through style, he gets others to see the material through his eyes. . . Through the style he infuses the work with a soul – and that is what makes it art.

Mathematics is an art. It is a poor man's art: Nothing is needed to conceive it, and only paper and pencil to convey.

This long work has given me so very much, in Aleksandr Pushkin's words, "the heavenly, and inspiration, and life, and tears, and love."[4] I have been raising this book for 18 years, and over the past couple of years, I felt as if the book herself was dictating her composition and content to me, while I merely served as an obedient scribe. At 18, my book is now an adult, and deserves to separate from me to live her own life. As Picasso put it, "When the form is realized, it is here to live its own life." Farewell, my child, let the world love you as I have and always will!

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[4] In the original Russian it sounds much better:
"И божество, и вдохновенье,
И жизнь, и слезы, и любовь."

I
Merry-Go-Round

1
A Story of Colored Polygons and Arithmetic Progressions

'Have you guessed the riddle yet?' the Hatter said, turning to Alice again.
'No, I give it up,' Alice replied. 'What's the answer?'
'I haven't the slightest idea,' said the Hatter.
'Nor I,' said the March Hare.

– Lewis Carroll, A Mad Tea-Party
Alice's Adventures in Wonderland

1.1 The Story of Creation

I recall April of 1970. The thirty judges of the Fourth Soviet Union National Mathematical Olympiad, of whom I was one, stayed at a fabulous white castle, half way between the cities of Simferopol and Alushta, nestled in the sunny hills of Crimea, surrounded by the Black Sea. This castle should be familiar to movie buffs: in 1934 the Russian classic film *Vesyolye Rebyata* (*Jolly Fellows*) was photographed here by Sergei Eisenstein's long-term assistant, director Grigori Aleksandrov. The problems had been selected and sent to printers. The Olympiad was to take place a day later, when something shocking occurred.

A mistake was found in the only solution the judges had of the problem created by Nikolai (Kolya) B. Vasiliev, the Vice-Chair of this Olympiad and a fine problem creator, head of the problems section of the journal *Kvant* from its inception in 1970 to the day of his untimely passing. What should we do? This question virtually monopolized our lives.

We could just cross this problem out on each of the six hundred printed problem sheets. In addition, we could select a replacement problem, but we would have to write it in chalk by hand in every examination room, since there would be no time to print it. Both options were rather embarrassing, desperate resolutions of the incident for the Jury of the National Olympiad, chaired by the great mathematician Andrej N. Kolmogorov, who was to arrive the following day. The best resolution, surely, would have been to solve the problem, especially because its statement was quite beautiful, and we had no counter example to it either.

A. Soifer, *The Mathematical Coloring Book*,
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