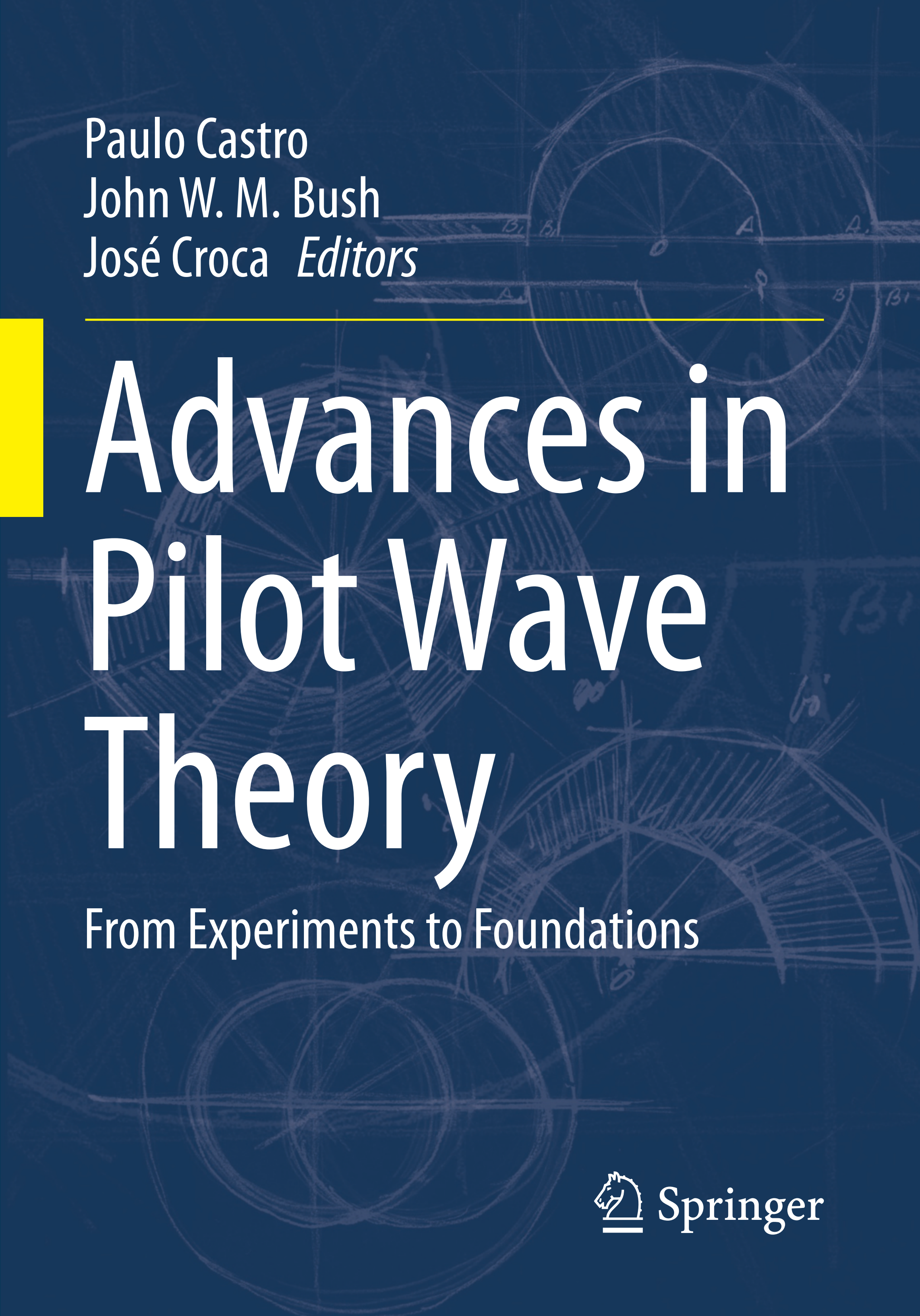

Boston Studies in the Philosophy and History of Science 344

Paulo Castro
John W. M. Bush
José Croca *Editors*

Advances in Pilot Wave Theory

From Experiments to Foundations

Springer

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Volume 344

The series *Boston Studies in the Philosophy and History of Science* was conceived in the broadest framework of interdisciplinary and international concerns. Natural scientists, mathematicians, social scientists and philosophers have contributed to the series, as have historians and sociologists of science, linguists, psychologists, physicians, and literary critics.

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Editors

Advances in Pilot Wave Theory

From Experiments to Foundations

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Contents

Chapter 1
Pilot Wave Theory in the Twenty-First Century

Paulo Castro, John W. M. Bush, and José R. Croca

This book is the result of the International Conference on Advances in Pilot Wave Theory, hosted by the Centre for Philosophy of Sciences of the University of Lisbon (CFCUL), Portugal, held from 26 to 30 July 2021 (International Conference on Advances in Pilot Wave Theory—Concurrently Hosting Hydrodynamic Quantum Analogs HQA-2021 n.d.; Advances in Pilot Wave Theory & HQA2021—YouTube n.d.). The conference concurrently hosted the Hydrodynamic Quantum Analogs meeting (HQA-2021) owing to the common interests of these seemingly disparate fields. The meeting took place online due to the COVID Pandemic lockdown and attracted participants from distant points of the globe, including the United States, Canada, Brazil, Mexico, Portugal, the United Kingdom, Italy, France, the Netherlands, Germany, Russia, Australia, and Israel.

It was a friendly and highly constructive meeting between physicists and philosophers of science that generally remain dissatisfied with the unintelligibility of the explanations provided by the Copenhagen interpretation of quantum mechanics. The meeting invoked the merits of the Pilot wave theoretical approach to quantum

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dynamics that Louis de Broglie put forward almost a century ago in his double-solution theory.

Pilot wave theory vindicates an ontological belief about the nature of quantum phenomena, wherein corpuscles and waves are two very real physical entities, coexisting and interacting at all times. The wave guides or pilots the quantum corpuscle, and in turn, the corpuscle reenforces the wave, acting as a moving oscillator. This physical picture of the quantum world is very different from that put forward by the Bohrian complementarity view, according to which the undulatory and corpuscular aspects always present themselves in a mutually exclusive way. Louis de Broglie ontology is also distinct from that put forward by David Bohm; specifically, Bohmian mechanics does not posit the physical existence of a particle-generated quantum wave: the pilot-wave is the wave function of standard quantum theory and only in de Broglie's ontology would it make sense to discuss the possibility of a quantum wave detection. Something that is addressed in this volume (Croca, ch. 17) and that has also been recently treated in another publication (Croca et al. 2023).

Although this is not a book strictly about Hydrodynamic Quantum Analogs (HQA), it seems epistemologically pertinent to say a few words about the field. Until 2005, when Yves Couder, S. Protière, Emmanuel Fort and A. Boudaoud (Paris, France) published their findings (Couder et al. 2005), the pilot wave phenomena did not seem to be a feasible possibility, despite the physical diversity of the world. However, it just happened that Couder *et al*, in an astonishing act of experimental insight, found that under proper conditions, an oil droplet could generate a quasi-monochromatic wave field on an oil bath surface. This wave field propagates with the droplet, making it follow a certain trajectory, by way of a piloting or guiding effect. In certain circumstances, the droplet follows a path quite different from the usual Newtonian trajectory expected in the absence of the piloting wave effect. The discovery of the pilot wave hydrodynamic system (Bush 2015) can be said to represent a considerable historical milestone in modern physics. Its importance lies in the fact that it testifies, without a shred of doubt, that de Broglie's insight about a pilot-wave effect provides valuable insight into quantum phenomena. While it is still a macroscopic scale system described using Newtonian physics, it engenders new physics that can serve as an analogical guide to counter the opacity of the Bohrian ontological black box approach to quantum processes. Over the last 15 years, researchers in the emergent field of Hydrodynamic Quantum Analogs (HQA) have been studying, through experimental and computational means, systems that can be seen as being analogous to quantum phenomena (Bush and Oza 2021). Examples include the tunneling effect (Eddi et al. 2009; Tadrist et al. 2020), the quantum corral effect (Harris et al. 2013; Cristea-Platon et al. 2018), Landau levels (Fort et al. 2010; Oza et al. 2014), Friedel oscillations (Sáenz et al. 2020), diffraction from slits (Couder and Fort 2006; Pucci et al. 2018; Ellegaard and Levinsen 2020), spin states (Labousse et al. 2016; Oza et al. 2018), Zeeman splitting (Eddi et al. 2012), the quantum harmonic oscillator (Kurianski et al. 2017; Perrard and Labousse 2018), and the Hong-Ou-Mandel effect (Valani et al. 2018).

The most important and striking feature in Hydrodynamic Pilot-wave phenomena is the fact that the information about the past trajectory of the droplet is embedded in the wave field. Thus, as the droplet bounces and creates new waves, these overlap with earlier waves, producing an overall field that in turn alters the droplet's future path. This feature of the pilot-wave hydrodynamic system implies the existence of a path wave-memory for the droplet. It also represents a new example of physical structures capable of storing information about the behavior of other physical structures, something typically associated with biology or computation. In HQA experiments the oil bath must be acted on from below by a vibrating force, producing an amplitude acceleration below the Faraday threshold acceleration. This is the acceleration value above which the pilot wave phenomena will no longer exist, giving way to standing Faraday waves on the oil surface (Couder et al. 2005). Related with the path wave-memory property, another revealing fact about pilot wave hydrodynamics is that the decay time of the past waves defines the field's memory storage capacity, which is experimentally controlled by the vibrational acceleration of the bath. If, on one hand, the decay time is very rapid (low acceleration), the pilot wave effect will be less stringent. On the other hand, if the decay time is extended (using vibrational accelerations either near or above the Faraday threshold), the pilot wave phenomena will exhibit other behaviors. These will include dynamic regimes analogous to the semi-classical quantum chaos situation (Stöckmann 1999), where a droplet can bounce unpredictably over the bath. All this suggests that the wave memory encoding process has a storage limit, strongly influencing the droplet's behavior. HQA research seems to point to a new way of thinking about the ontological status of the quantum wave, in so much that it can be addressed as a kind of "nomological" memory (nomological, from the Greek: *nomos*, law, and *logos*, reason, order), containing information about the possible behaviors of corpuscles. An idea that it is also developed in the present volume (Castro, ch. 12).

The book begins with contributions focusing on the Physics of Pilot wave theory, assuming a more philosophical and historical tone as it progresses to the end. The second chapter covers the state-of-the-art in the Hydrodynamic Quantum Analogs (HQA) research field, and how it can help solve conceptual difficulties concerning non-locality and quantum entanglement (Bush, Frumkin and Papatryfonos, ch. 2). The next two chapters, also coming from HQA, provide a computational analysis about long-range pilot-wave interactions (Nachbin, ch. 3) and a deterministic hydrodynamically-inspired ensemble interpretation for free relativistic particles (Dagan, ch. 4). From the Stochastic mechanics approach we get, in chapter five, an answer to a long-standing criticism that multi-time correlations in stochastic mechanics differ from those predicted in quantum theory (Derakhshani and Bacciagaluppi, ch. 5). Within the de Bohm-Broglie framework we find in chapter six a version of de Broglie's double solution theory, reproducing Landau's quantization in a uniform magnetic field (Jamet and Drezet, ch. 6) and, in chapter seven, a comparative study between the deterministic de Broglie-Bohm formalism and the stochastic formalism of Bohm, Hiley and Nelson, concerning quantum equilibrium (Hatifi, Willox and Durt, ch. 7). Chapter eight features a work about configuration-space density frameworks, relating pilot-wave theory trajectories and configuration space representations (Roser and Scoggins, ch. 8). Chapter nine presents us with a description about quantized behaviors at the macroscopic level, observable in what is called the Doubochinski pendulum (Tennenbaum, ch. 9).

The more philosophically focused contributions start in chapter ten with a proposal to complete the quantum ontology with the electromagnetic zero-point field (de la Peña, Cetto, ch. 10). The next chapter provides an analysis of the possibility of hidden variables theories (Vervoort, ch. 11). In chapter twelve we find the Wave-memory interpretation of quantum mechanics, according to which the quantum wave is a four-dimensional memory structure, carrying probabilistic information about the corpuscle behavior. An alternative to the Copenhagen interpretation is suggested, trying to provide a conceptual basis to develop a four-dimensional realistic theory of quantum mechanics (Castro, ch. 12). Chapter thirteen critically addresses the measurement problem in quantum mechanics, arguing that it is a false problem for scientific realists. An historical account due to the positivistic climate of the time, is given for why a three-dimensional, microscopic ontology of quantum mechanics was not adopted. A possible alternative for such an ontology is speculated as way to avoid most paradoxes and puzzles of quantum mechanics (Allori, ch. 13). The next chapter offers several historical parallels between modern quantum theory and thermodynamics in the 1800s, which are relevant to the hidden variable debate (Kay, ch. 14). Chapter fifteen suggests the use of a general methodological approach, engendered in the pilot-wave framework, to study other domains of Nature, namely, biology (Magalhães, ch. 15). Chapter sixteen presents us with John Bell's unpublished notes on de Broglie's Pilot Wave hypothesis (Garuccio and Laurora, ch. 16). The book ends with an account of the experimental feasibility of quantum waves detection, including a history of the subject and a set of state-of-the-art experiments designed to accomplish such a feat (Croca, ch. 17).

Being a nonsystematic gathering of new ideas from physicists and philosophers of science about Nature itself, this volume should be considered a work in Natural Philosophy about quantum phenomena. It should also be read as a contemporary sign that the pilot wave approach to quantum phenomena is very much alive and that it can be developed and advanced, to the point that it can also address entanglement. This much, either through the existence of a wave-memory encoded correlation, transmitted by the wave field from one corpuscle to the other (Bush et al, ch. 2; Nachbin, ch. 3; Vervoort, ch. 11), or otherwise accepting a type of non-local necessitarianism from the nomological information encoded in the wave, as suggested in a more philosophical approach to the topic (Castro, ch. 12).

The reader is free to start from any chapter in the book, as each contribution is self-contained, although multiple relations can be established between them. With the present volume we hope to provide inspirational motivation to think beyond the restrictions imposed by the Copenhagen School interpretation. We also hope that in the near future a more realistic approach to quantum waves may emerge, from both an experimental and theoretical point of view, favoring new ways to look at Nature and allowing for new technological advances prompted by an improved understanding of the quantum realm.

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References

Advances in Pilot Wave Theory & HQA2021—YouTube. n.d. Retrieved February 14, 2023, from https://www.youtube.com/playlist?list=PLXKRQSlglfTWfdIeG2OmnLmWBM3kx6jfH

Bush, J.W.M. 2015. Pilot-wave hydrodynamics. *Annual Review of Fluid Mechanics* 47: 269–292.

Bush, J.W.M., and A.U. Oza. 2021. *Hydrodynamic quantum analogs. Reports on Progress in physics*. IOP Publishing.

Couder, Y., and E. Fort. 2006. Single particle diffraction and interference at a macroscopic scale. *Physical Review Letters* 97: 154101.

Couder, Y., S. Protière, E. Fort, and A. Boudaoud. 2005. Walking and orbiting droplets. *Nature* 437: 208.

Cristea-Platon, T., P.J. Sáenz, and J.W.M. Bush. 2018. Walking droplets in a circular corral: Quantisation and chaos. *Chaos* 28: 096116.

Croca, J.R., P. Castro, M. Gatta, and R. Moreira. 2023. Proposed experiments to clarify the real nature of the quantum waves. *Foundations of Physics* 53: 14.

Eddi, A., E. Fort, F. Moisy, and Y. Couder. 2009. Unpredictable tunneling of a classical wave–particle association. *Physical Review Letters* 102: 240401.

Eddi, A., J. Moukhtar, S. Perrard, E. Fort, and Y. Couder. 2012. Level splitting at a macroscopic scale. *Physical Review Letters* 108: 264503.

Ellegaard, C., and M.T. Levinsen. 2020. Interaction of wave-driven particles with slit structures. *Physical Review E* 102: 023115.

Fort, E., A. Eddi, A. Boudaoud, J. Moukhtar, and Y. Couder. 2010. Path-memory induced quantization of classical orbits. *Proceedings of the National Academy of Sciences of the United States of America* 107: 17515–17520.

Harris, D.M., J. Moukhtar, E. Fort, Y. Couder, and J.W.M. Bush. 2013. Wavelike statistics from pilot-wave dynamics in a circular corral. *Physical Review E* 88: 011001.

International Conference on Advances in Pilot Wave Theory—Concurrently Hosting Hydrodynamic Quantum Analogs (HQA-2021). n.d.. Retrieved February 14, 2023, from https://pilotwave.campus.ciencias.ulisboa.pt/

Kurianski, K.M., A.U. Oza, and J.W.M. Bush. 2017. Simulations of pilot-wave dynamics in a simple harmonic potential. *Physical Review Fluids* 2: 113602.

Labousse, M., S. Perrard, Y. Couder, and E. Fort. 2016. Self attraction into spinning eigenstates of a mobile wave source by its emission back-reaction. *Physical Review E* 94: 042224.

Oza, A.U., D.M. Harris, R.R. Rosales, and J.W.M. Bush. 2014. Pilot-wave dynamics in a rotating frame: On the emergence of orbital quantization. *Journal of Fluid Mechanics* 744: 404–429.

Oza, A.U., R.R. Rosales, and J.W.M. Bush. 2018. Hydrodynamic spin states chaos. *Chaos: An Interdisciplinary Journal of Nonlinear Science* 28: 096106.

Perrard, S., and M. Labousse. 2018. Transition to chaos in wave memory dynamics in a harmonic well: Deterministic and noise-driven behavior. *Chaos* 28: 096109.

Pucci, G., D.M. Harris, L.M. Faria, and J.W.M. Bush. 2018. Walking droplets interacting with single and double slits. *Journal of Fluid Mechanics* 835: 1136–1156.

Sáenz, P.J., T. Cristea-Platon, and J.W.M. Bush. 2020. A hydrodynamic analog of Friedel oscillations. *Science Advances* 6: 20.

Stöckmann, H.-J. 1999. *Quantum chaos: An introduction*. Cambridge: Cambridge University Press.

Tadrist, L., T. Gilet, P. Schlagheck, and J.W.M. Bush. 2020. Predictability in a hydrodynamicpilot-wave system: Resolution of walker tunneling. *Physical Review E* 102: 013104.

Valani, R.N., A.C. Slim, and T. Simula. 2018. Hong–Ou–Mandellike two-droplet correlations. *Chaos* 28: 096104.

Chapter 2
The State of Play in Hydrodynamic Quantum Analogs

John W. M. Bush, Konstantinos Papatryfonos, and Valeri Frumkin

Abstract We describe the manner in which the physical picture emerging from hydrodynamic quantum analogs (HQAs) may serve to resolve some of the long-standing difficulties of quantum mechanics. We enumerate some of the most significant intellectual cul-de-sacs of quantum mechanics, and the manner in which HQA suggests a route past them. Particular attention is given to enumerating the many guises of quantum nonlocality as it appears in the standard quantum interpretations. We illustrate how one might misinfer such nonlocality from the walking-droplet system if one had an incomplete description of the system dynamics, if the variables required for its complete description were hidden rather than in plain sight. We highlight recent work that illustrates how phenomena typically attributed to nonlocality in quantum systems may be rationalized in terms of classical, pilot-wave dynamics. Finally, we define the frontiers of the field of hydrodynamic quantum analogs, including attempts to achieve classical entanglement by demonstrating Bell violations in pilot-wave hydrodynamics, and attempts to develop a model of quantum dynamics informed by the walking-droplet system.

Keywords Pilot-wave hydrodynamics · Walking droplets · Quantum analogs · Local realism

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2.1 Introduction

The walking-droplet system discovered in 2005 by Yves Couder and Emmanuel Fort (Couder et al., 2005; Couder and Fort, 2006) has since captured many features previously thought to be exclusive the the microscopic, quantum realm (Bush, 2015; Bush and Oza, 2020). The growing grocery list of hydrodynamic quantum analogs includes single-particle diffraction and interference (Couder and Fort, 2006), quantized orbits (Fort et al., 2010; Perrard et al., 2014), unpredictable tunneling (Eddi et al., 2009a), Friedel oscillations (Sáenz et al., 2020), spin lattices (Sáenz et al., 2021), statistical projection effects (Sáenz et al., 2018) in corrals (Harris et al., 2013), surreal trajectories (Frumkin et al., 2022) and superradiance (Papatryfonos et al., 2022a; Frumkin et al., 2023). This pilot-wave hydrodynamic system has extended the range of classical systems and so provided a platform for delineating between what can and cannot be understood about quantum systems from a classical perspective.

The hydrodynamic pilot-wave system consists of a millimetric liquid droplet suspended on the surface of a vibrating liquid bath (Couder et al., 2005) (Fig. 2.1a–c). In certain parameter regimes (specifically, for certain drop sizes, liquid properties, and vibrational accelerations), the bouncing droplet achieves resonance with the bath's most unstable Faraday wave mode and destabilizes into a walking state in which it is guided or 'piloted' by its own wave field. The resulting 'walker' then consists of a droplet self-propelling along the bath surface, dressed in a quasi-monochromatic pilot-wave field (Fig. 2.1c). The key features of the system are two-fold (Bush and Oza, 2020). First, the resonance between the droplet and its wave ensures that

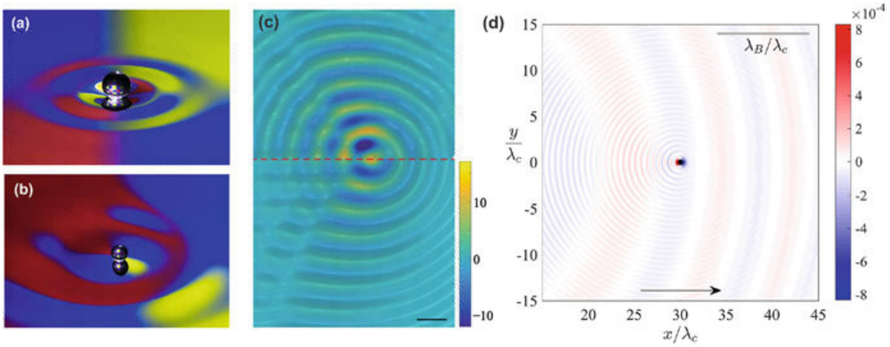

Fig. 2.1 (**a**) A millimetric droplet bounces in place on a vibrating bath. (**b**) When the droplet's bouncing frequency matches that of the vibrating bath's most unstable Faraday mode, the drop destabilizes into a 'walker', and is self-propelled across the bath by its pilot wave. (**c**) Comparison between experimental measurements (top) and simulation (bottom) of the wave field accompanying a walker moving from left to right (Damiano et al., 2016). Color bar indicates the wave height in microns. (**d**) The pilot-wave field computed for a quantum particle propelled by a wave field that evolves according to the Klein-Gordon equation and is excited locally by the particle vibrating at the Compton frequency, $\omega_c = mc^2/\hbar$ (Durey and Bush, 2020)

the pilot-wave field is quasi-monochromatc, a feature that is responsible for the emergence of quantized states in many settings. Second, the persistence of this pilot-wave field renders the droplet dynamics non-Markovian: computing its trajectory requires that one consider its history. The droplet is thus endowed with 'path-memory' (Eddi et al., 2011), and navigates a quasi-monochromatic potential of its own making, as is ultimately responsible for all of the system's emergent quantum features (Bush and Oza, 2020).

The most valuable aspect of the walking-droplet system is that it furnishes a macroscopic example of wave-particle interaction, and so a physical picture for how quantum dynamics might conceivably look. Importantly, this physical picture, of a vibrating particle moving in resonance with its own wave field, is not new and has features of several extant realist theories of quantum dynamics, including de Broglies double-solution pilot-wave theory (de Broglie, 1926, 1970, 1987), stochastic electrodynamics (de la Peña et al., 2015) and the Zitterbewegung theory of quantum mechanics (Hestenes, 1990). Each of these theories invokes the particles internal vibration at the Compton frequency as the source of its guiding wave. The commonality of this physical picture has allowed the HQA community to connect to others attempting to make sense of quantum mechanics. Moreover, it has motivated the development of a generalized classical pilot-wave theory that allows one to explore parameter regimes inaccessible in the laboratory with the walking-droplet system (Bush, 2015; Oza et al., 2018; Durey and Bush, 2021), as well as forge links with and extend extant quantum pilot-wave theories (Dagan and Bush, 2020; Durey and Bush, 2020).

The formalisms of pilot-wave hydrodynamics and quantum mechanics are markedly different. In the former, the HQA community has developed a hierarchy of progressively more sophisticated theories to describe the droplet and wave dynamics, both of which are required for an adequate desription of the droplet's trajectory (Turton et al., 2018). The resulting particle statistics are viewed as an emergent feature, the rationalization of which is not always straightforward. Conversely, quantum mechanics provides an explicit theory for the evolution of the system's statistics, as prescribed by the wave function. According to the standard Copenhagen Interpretation, there is no notion of an underlying dynamics, so no need for a trajectory equation. Bohmian mechanics furnishes a trajectory equation by positing that the particle moves in response to the quantum wavefunction (Holland, 1995). Throughout this review, we describe recent attempts to reconcile the very different theoretical formalisms developed to describe pilot-wave hydrodynamics and quantum mechanics. Specifically, we describe recent attempts to develop a statstical theory for pilot-wave hydrodynamics and a dynamical theory for quantum mechanics informed by the walker system.

We proceed by enumerating a number of concepts that may seem beguiling from the point of view of quantum mechanics, but become less problematic when considered from the new perspective offered by HQA. We advance in increasing order of difficulty, commencing with notions that transform from inscrutable to trivial, such as wave-particle duality and wave-function collapse. We move on to show how quantized states and coherent wave-like statistics naturally emerge from classical pilot-wave dynamics. We highlight the manner in which quantum nonlocality in its various guises might be misinfered from the walking-droplet system. Finally, we discuss current attempts to demonstrate violation of Bell's inequality with the pilot-wave hydrodynamic system, which remains the central challenge to any local realist theory of quantum mechanics. We conclude with a discussion of the benefits of the perspective and enhanced physical intuition offered by HQAs on quantum mechanics and quantum foundations.

2.2 Wave-Particle Duality and Complementarity

> Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles. Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time—the two concepts are too different.
>
> – Heisenberg, 'The Physical Principles of the Quantum Theory' (Heisenberg, 1930).

Wave-particle duality is a notion common in both optics and quantum mechanics. It first arose in the debate over the nature of light, where it became apparent that light sometimes behaves like a particle, other times like a wave. Huygens and Fresnel were two of the most prominent proponents of the wave nature of light. In 1678, Huygens proposed that every point on a wavefront of a light beam may be seen as a new source, emitting spherical waves in the forward direction (Miller, 1991). These secondary wave sources interfere with each other to produce the advancing wavefront. Huygens thus explained linear and spherical wave propagation and derived the laws of reflection and refraction, but failed to rationalize other optical phenomena, such as the diffraction from an edge or an aperture (Miller, 1991). More than a century later, Fresnel combined his own theory of interference with Huygens's principle, which enabled him to rationalise these diffraction effects (Santos et al., 2018). Newton championed the corpuscular view, that light consisted of a series of discrete, localized corpuscles, now known as photons, 'skipping on the ether like stones on a pond' (Newton, 1704). Subsequently, Thomas Young's ripple tank experiments demonstrated that the diffraction of light was consistent with its having a wave nature. The wave view became dominant when Maxwell (1873) demonstrated that all forms of light (infrared, visible and ultraviolet) could be described as electromagnetic waves oscillating at different frequencies. When the wave-particle debate seemed all but settled, it was again revived in 1905 by Einstein's explanation of the photoelectric effect in terms of 'light quanta', a critical step in the development of quantum mechanics (McKagan et al., 2009).

The concept of complementarity was introduced by Niels Bohr as an essential feature of quantum theory (Folse, 1985). It asserts that a complete knowledge of quantum phenomena requires a simultaneous description of particle and wave properties, hence a quantum version of wave-particle duality. Specifically, prior to being measured, a single quantum particle is described in terms of its wave function that evolves according to Schrodinger's equation. Measurement forces the wave to collapse to a particle, for example when particles passing through slits arrive at the detection screen. Bohr asserted that it is impossible to observe both wave and particle aspects simultaneously, but both notions need be retained. Finally, quantum systems have certain pairs of 'complementary' properties that cannot be observed simultaneously. These are also known as non-commuting observables and include, for example, position and momentum, and different components of a particle's spin.

De Broglie's theory of matter waves was built upon his premise that the universe is composed of two elements, light and matter. Since light has both corpuscular and wave aspects, so too must matter. His was an attempt to reconcile quantum mechanics with Einstein's theory of relativity. He proposed that a particle of mass m has an internal frequency that may be deduced by equating its rest mass energy mc^2 to its wave energy $\hbar\omega$. The resulting de Broglie-Einstein equation defines the frequency of particle vibration to be the Compton frequency, $\omega_c = mc^2/\hbar$. He thus envisioned microscopic particles generating, then moving in response to, their own wave fields. He proposed that a free particle moves in response to gradients in the phase of its monochromatic guiding or 'pilot' wave, with a wavelength prescribed by the de Broglie relation, $p = \hbar k$. He imagined, but never proved, that this pilot-wave dynamics might give rise to statistical behavior consistent with that described by the standard formulation of quantum mechanics. His theory thus involved two waves, the real pilot-wave responsible for guiding the particle, and the emergent statistical wave, and his unfinished theoretical program was known as the double-solution pilot-wave theory (Hatifi et al., 2018; Colin et al., 2017). De Broglie's answer to the question of 'particle or waves?' was thus simply 'particle and wave' (Bell, 1987). On the basis of this physical picture, he predicted electron diffraction and was awarded the Nobel prize in 1929. Nevertheless, this physical picture has been largely ignored by the physics community (Bell, 1987) until its recent revival by the HQA community (Bush, 2015; Bush and Oza, 2020).

The walking droplet is inarguably a classical realization of wave-particle duality (Figs. 2.1b and 2.2). The 'walker' has both particle and wave aspects. Without the droplet, there would be no source of waves, and without the accompanying wave field, the droplet wouldn't self propel. The walker is, moreover, an embodiment of the physical picture proposed by de Broglie in his double-solution pilot-wave theory (Bush, 2015). In the walker system, the Faraday frequency plays the role of the Compton frequency, the Faraday wavelength that of the de Broglie wavelength. The walker moves in response to gradients in the wave amplitude rather than phase, but the resonance condition respected by the free walker effectively renders this distinction a moot point. The pilot-wave of the walker is quasi-monochromatic, of a distinctive, horseshoe-like form (Fig. 2.1c), while de Broglie envisaged a monochromatic plane wave of the form illustrated in Fig. 2.1d (Durey and Bush, 2020).

2.3 Quantized States

> I wish first to show in the simplest case of the hydrogen atom that the usual rates for quantization can be replaced by another requirement, in which mention of "whole numbers" no longer occurs. Instead the integers occur in the same natural way as the integers specifying the number of nodes in a vibrating string.
>
> – Schrödinger, 'Quantisierung als Eigenwertproblem' (Schrödinger, 1926).

In the pilot-wave hydrodynamic system, quantized dynamical states are a central feature, apparent in both the structure of droplet aggregates (Fig. 2.2) and orbital dynamics (Fig. 2.3). Bouncing droplets may form either static bound states, pairs, trios, rings or lattices (Eddi et al., 2009b), or dynamic bound states comprised of ratcheting (Eddi et al., 2008; Galeano-Rios et al., 2018) orbiting (Couder et al., 2005; Protière et al., 2008; Oza et al., 2017) or promenading pairs (Borghesi et al., 2014; Arbelaiz et al., 2018). In all such states, the interdrop distance is quantized, prescribed by the Faraday wave length. In both rings and lattices, the drop aggregates are most stable when the droplets bounce in local minima of their collective wave field (Couchman and Bush, 2020). A critical requirement for the emergence of quantization is the synchronization of the droplets, which ensures a coherent quasi-monochromatic wave form that serves as the trapping self-potential of the aggregate.

Quantization is also a key feature of orbital pilot-wave dynamics, as arises when walkers move subject to constraints. Three such systems have been explored

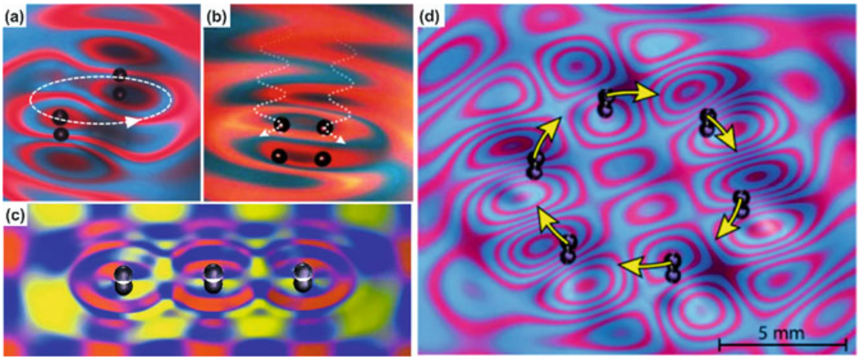

Fig. 2.2 Quantized bound states, both static and dynamic, arise owing to the quasi-monochromatic wave field generated by the droplets bouncing in resonance with the bath's most unstable Faraday mode. (**a**) Two walkers locked into a circular orbit. (**b**) A promenading pair: drops move together in the same direction, with the lateral distance between them varying periodically with time (Arbelaiz et al., 2018). (**c**) A colinear trio of stationary bouncers. (**d**) Rings of bouncing droplets may form with quantized radii. We show here the circular motion arising at the onset of instability of a stable ring induced as the driving acceleration exceeds a critical value (Couchman and Bush, 2020)

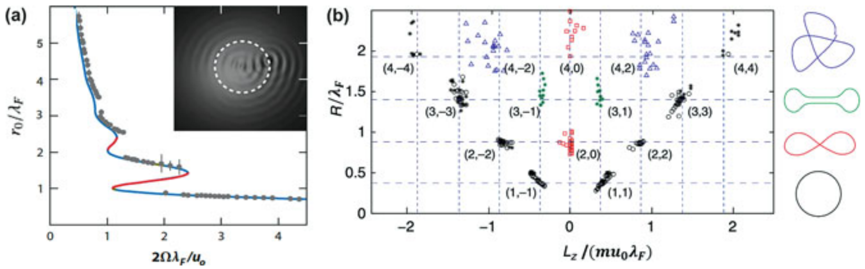

Fig. 2.3 Quantized orbital states arise for (**a**) walkers in a rotating frame or (**b**) walkers confined by a central spring force. (**a**) The solution curve for the orbital radius, r_0, as a function of rotation rate, Ω, has both stable (blue) and unstable (red) branches. The absence of stable solutions at certain radii leads to orbital quantization (Fort et al., 2010; Harris and Bush, 2014; Oza et al., 2014a). Inset: the walker moves along a circularly corrugated wave field, whose form imposes the quantization (Fort et al., 2010). (**b**) When a walker is confined by a radial spring force, a variety of periodic orbits are accessible, all of which are quantized in both mean radius $\bar{R} = R/\lambda_F$ and mean orbital angular momentum $\bar{L}_z = L_z/(mu_0\lambda_F)$ (Perrard et al., 2014; Labousse et al., 2014a)

both experimentally and theoretically. When walkers move in a rotating frame, the Coriolis force plays a role analogous to the Lorentz force acting on a charge moving in a uniform magnetic field (Fort et al., 2010). The expected continuum of circular inertial orbits arising at low memory are replaced by quantized circular orbits as the memory increases (Fort et al., 2010; Harris and Bush, 2014). The requirement for orbital quantization is that the orbital period of the drop is less than the memory time; thus, the drop feels its own capillary wake, which serves to quantize its orbital radius. The drop thus navigates its own potential, a circularly symmetric wave field with the Faraday wavelength centered on the orbital center (Fig. 2.3a, inset). In these quantized circular orbits, the Faraday wavelength plays a role analogous to the de Broglie wavelength in Landau levels (Fort et al., 2010).

When walkers move in a central force, in addition to quantized circular orbits, a family of more elaborate orbits arise, including lemniscates and trefoils (Perrard et al., 2014; Labousse et al., 2014a). As in the 2D quantum harmonic oscillator, these orbits are quantized in both energy and mean angular momentum. A similar progression of quantized orbits arise when a walker is confined to a small corral at relative low memory, when the bounding geometry plays the role of the confining potential (Cristea-Platon et al., 2018). Once again, the key requirement for the emergence of quantized states is resonance between drop and bath, as assures a quasi-monochromatic self-potential. Quantized orbital states emerge when the memory time exceeds the orbital period, so that the drop continuoulsy navigates a highly structured potential of its own making.

2.4 Single-Particle Diffraction

> A phenomenon which is impossible, absolutely impossible to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery.
>
> – Richard Feynman (Feynman, 1964).

The most prominent example of particle-wave duality is found in single-particle diffraction. The first experimental investigation of single-photon diffraction was undertaken by a graduate student, G.I. Taylor, who subsequently went on to become a prominent and influential fluid and solid mechanician (Batchelor, 2008). The question posed him by his supervisor, J.J. Thomson, was whether a diffraction pattern would emerge when light passed through a slit, even when the photons passed through one at a time. The results of his experiments were conclusive: a diffraction pattern ultimately emerged. So, despite the photons passing through the slits one at a time, their superposition corresponded to a continuous wave pattern (Taylor, 1909), indicating single-particle diffraction.

Feynman's (Feynman, 1964) insistence on the inscrutability of the electron double-slit experiments is somewhat puzzling in light of de Broglie's work on electron diffraction (de Broglie, 1926; Bell, 1987). The relevant experiments were first performed with electrons diffracting around a pair of obstacles, the complement of the double slit (Tonomura et al., 1989). The build-up of the resulting diffraction pattern, into alternating bright and dark bands, has been numbered among the most beautiful experiments in the history of physics, the beauty presumably being rooted in the common conception that the phenomenon is impossible to understand from a classical perspective. The mystery is not the particular form of the diffraction pattern: the mystery is that the pattern emerges at all.

The original experiments of walkers passing through slits (Fig. 2.4) and accompanying simulations of Couder and Fort (2006) clearly showed a diffraction pattern. As the drop approaches the slit, the distortion of its pilot wave by the barriers causes the drop to be deflected, with certain deflection angles being preferred. A more exhaustive series experiments performed in different laboratories have made clear that single-particle diffraction is indeed a robust feature of pilot-wave hydrodynamics (Andersen et al., 2015; Pucci et al., 2018; Ellegaard and Levinsen, 2020). Moreover, the presence of a second slit alters the emergent diffraction pattern, another keystone of its quantum counterpart. The fact that the emerging diffraction pattern does not generally conform to the Fraunhofer pattern, is neither troubling nor surprising when one considers that the pilot-wave form is markedly different in the walker systems than what one would expect to arise in de Broglie's mechanics (Fig. 2.1c, d). The diffraction of walking droplets reminds us that the physical picture of de Broglie's is sufficient to understand the basic mysteries of electron diffraction.

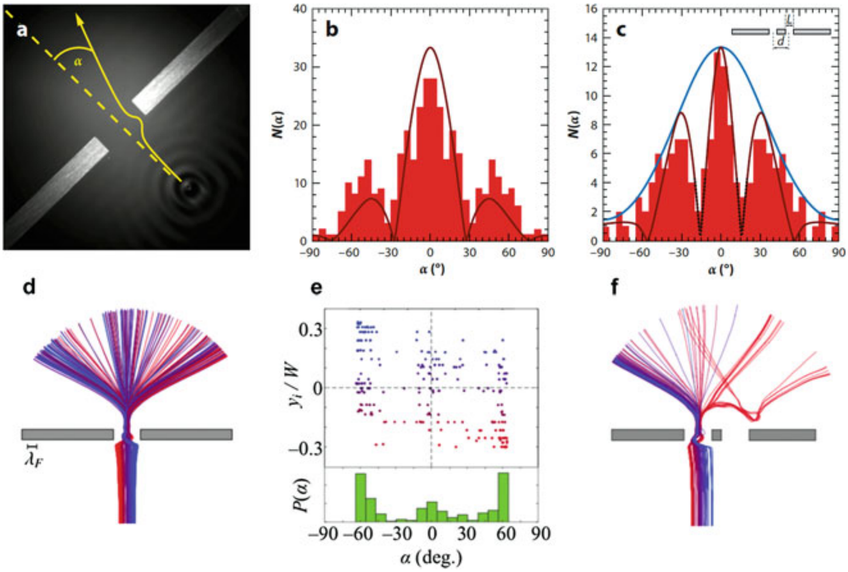

Fig. 2.4 Single-particle diffraction and interference of a walking drop. (**a**) A walker passing through a single slit is deviated from its initial path owing to the influence of the submerged topography on its pilot wave. Histogram for the final deflection angle α in (**b**) the single-slit experiments and (**c**) the double-slit arrangement. The droplet trajectories emerging from (**d**) the single-slit and (**f**) the double slit experiments of Pucci et al. (2018). (**e**) The distribution of deflection angles in the single-slit arrangement. In the double-slit arrangement, the presence of the second slit alters the diffraction pattern of walkers through the first slit because the pilot wave is influenced by both slits (Pucci et al., 2018). Panels (**a**)–(**c**) adapted from Couder and Fort (2006); panels (**d**)–(**f**) from Pucci et al. (2018)

2.5 Wave-Like Statistics

> The wave character of light is not vibrating stuff like a wave of water but rather a wavelike function encoding information about where you'll find the photon of light once it is detected.
>
> – Marcus du Sautoy, 'The Great Unknown: Seven Journeys to the Frontiers of Science' (Sautoy, 2017).

One of the most compelling features of pilot-wave hydrodynamics is that it naturally leads to the emergence of wave-like particle statistics reminiscent of those arising in many quantum systems. We have already seen the wave patterns arising in walker diffraction through slits (Fig. 2.4). Superpositions of dynamical states have been demonstrated in a number of settings involving chaotic pilot-wave hydrodynamics. In the orbital pilot-wave systems first considered by Couder and Fort, specifically walker motion in a rotating frame (Fort et al., 2010) and a central spring force (Perrard et al., 2014), analogs of a charge moving in a uniform magnetic field and the 2D quantum harmonic oscillator, respectively, quantized orbital states

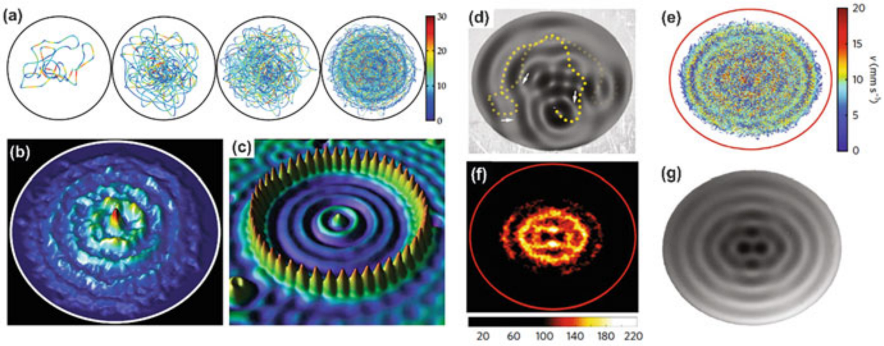

Fig. 2.5 The hydrodynamic corral consists of a single walker moving within a bounded domain, either circular (**a**)–(**b**) (Harris et al., 2013) or elliptical (**d**)–(**g**) (Sáenz et al., 2018). (**a**) The build-up of the particle trajectory, which is color-coded according to droplet speed. The resulting correlation between droplet position and speed gives rise to a statistical signature (**b**) strongly reminiscent of that arising in its quantum counterpart (**c**) (Crommie et al., 1993b). In the elliptical corral, similar speed maps (**e**) and histograms (**f**) emerge. The instantaneous wave field (**d**) is complex and time-dependent, while the mean pilot-wave (**g**) closely resembles the droplet histogram (**f**)

emerge as the memory is increased (Fig. 2.3). Eventually, at sufficiently high memory, these orbital states destabilize, giving rise to chaotic states marked by the drop switching intermittently between a number of finite accessible quantized orbital states (Harris and Bush, 2014; Oza et al., 2014b; Labousse et al., 2014a). The emergent wave-like statistics thus reflect a superposition of dynamical states, and the precise form of the emergent statistics reflects the relative instability of the accessible unstable orbits (Harris and Bush, 2014; Oza et al., 2014a; Labousse et al., 2014a).

Robust wave-like statistics also arose in Harris et al. (2013)'s experimental investigation of a walker in a circular corral (Fig. 2.5a–c). In the high-memory limit arising just below the Faraday threshold, the drop executes an erratic, chaotic trajectory. Ultimately, the correlation between drop position and speed give rises to a statistical signature of comparable form to the most unstable Faraday mode of the cavity. The emergent statistics in this ergodic system is virtually identical to that arising when electrons are confined to the quantum corral (Crommie et al., 1993b; Fiete and Heller, 2003), with the Faraday wavelength again playing a role analogous to the de Broglie wavelength. The walker corral is marked by three distinct timescales, those of droplet bouncing (∼0.01 s), droplet translation (∼2 s) and statistical convergence (∼1 h). Given the vast difference in scales between this experiment and its quantum counterpart (e.g. the corral diameter is 3 cm rather than 75 Angstrom), the ability to resolve all three timescales in the laboratory is quite remarkable.

While theoretical models have yet to capture satisfactorily the wave-like statistical behavior in corrals (Durey et al., 2020), its robustness was demonstrated in a subsequent study of an elliptical corral (Sáenz et al., 2018), where additional

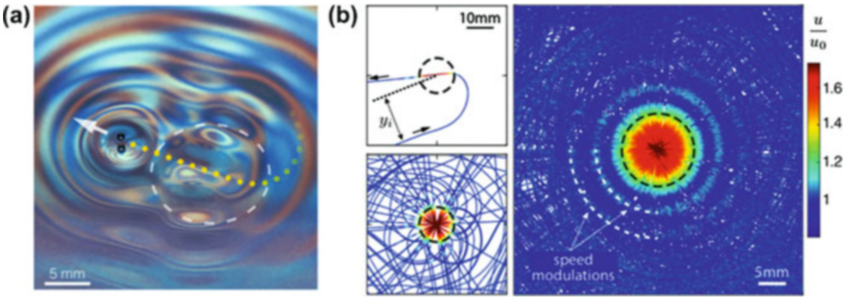

Fig. 2.6 The hydrodynamic analog of Friedel oscillations (Sáenz et al., 2020). (**a**) When a walker approaches a deep well, it is drawn inward along a spiral path, then exits the well radially. (**b**) In-line speed oscillations along its outgoing path are evident in the droplet's speed map. The associated correlation between droplet speed and radial position leads to a coherent statistical signature similar in form to Friedel oscillations (Crommie et al., 1993)

quantum-like features were revealed, including superposition of statistical states and statistical projection effects (Fig. 2.5d–g). Moreover, it was noted that the mean pilot wave was very similar in form to the emergent particle histogram, a result later rationalized by Durey et al. (2018, 2020), who demonstrated that the mean pilot-wave field for either periodic or ergodic walker mation may be deduced from a convolution of the particle histogram and the stationary bouncer wave field. This result provides the means to deduce the particle statistics from the mean pilot-wave form, so plays a role analogous to that of Born's Rule in quantum mechanics if one identifies the mean pilot wave with the wave function (Kutz et al., 2023). Bush and Oza (2020) discuss the relation between the mean-pilot-wave potential in the walker system and the quantum potential in Bohmian mechanics.

Robust wave-like statistics were also revealed in the hydrodynamic analog of Friedel oscillations (Sáenz et al., 2020, Fig. 2.6), waves in the density of states surrounding impurities in the electron sea on a metal surface (Crommie et al., 1993). The analog system consisted of a walker interacting with a deep well. The drop was drawn inwards along a spiral path until crossing the center of the well then exiting radially. As the drop exited the well, in-line speed oscillations were excited, and the resulting correlation between radial position and speed along the outgoing trajectory gave rise to a statistical signature identical to that arising in Friedel oscillations, with the Faraday wavelength again playing the role of the de Broglie wavelength. With this concrete physical mechanism for the emergent statistics in the analog quantum corral and Friedel oscillations, one can imagine that a similar mechanism might also be at play in their quantum counterparts. At the very least, one can conclude that the emergent statistics in both systems are not inconsistent with the notion of particle trajectories.

According to the Copenhagen Interpretation, the act of measurement causes the wave function to collapse to a particular eigenstate of the system, the associated probability cloud describing the particle position to collapse instantaneously to a point. Given the spatially extended nature of the associated wave form, this collapse would seem to violate the basic tenets of relativity (Einstein et al., 1935). Adherents to the Copenhagen Interpretation typically sidestep this issue by stating that the wave function collapse does not represent a physical process, but merely an update of information. However, if the wave function represents a complete description of a quantum system, there should be no new information according to which it need be updated (Hance and Hossenfelder, 2022). This criticism of the completeness of quantum theory was originally launched by Einstein et al. (1935), prompting an exchange now referred to as the Debate over the Nature of Reality. Despite the obscurity of Bohr's response (Bohr, 1935), history has judged him to be the winner (Bricmont, 2017, 2016).

2.6 The Measurement Problem

> The electron, as it leaves the atom, crystallises out of Schrödinger's mist like a genie emerging from his bottle.
>
> – Sir Arthur Stanley Eddington, Gifford Lectures (Eddington, 1927).

The state of a quantum system is described by the wave function, a vector in a Hilbert space that evolves deterministically in time according to the linear Schrodinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \tag{2.1}$$

where $\hat{H}$ is the system's Hamiltonian. Due to the linearity of the Schrodinger equation, any superposition (linear combination) of solutions will also be a solution and evolve in time according to Eq. (1). During measurement, the wave function "collapses" onto an eigenstate of the Hamiltonian that corresponds to a particular measurement outcome, the value measured in the laboratory (Griffiths, 2014). Unlike the time evolution described by Eq. (2.1), the collapse process is non-linear, non-deterministic, and typically non-local (Bassi et al., 2013). Importantly, the theory does not specify what precisely constitutes a measurement, which raises the following fundamental questions. What exactly happens during measurement? Is the "collapse" a physical process or merely an update of information? These puzzles in the foundations of quantum mechanics are collectively referred to as 'the measurement problem', and embodied in widely known paradoxes such as Schrodinger's cat (Schrödinger, 1935) and Wigner's friend (Wigner, 1995).

Attempts to solve the measurement problem have led to the various interpretations of quantum mechanics. In some, such as the de Broglie–Bohm theory (Bohm, 1952a, 1925b; Holland, 1995) (a.k.a. Bohmian mechanics, not to be confused with de Broglie's original double solution pilot-wave theory) and the Many Worlds Interpretation (Everett, 1957), there is no measurement problem since the wave function never undergoes a collapse process. In others, like the objective-collapse models, a stochastic term is added to the Schrodinger equation in order to induce a spontaneous collapse of the wavefunction at some characteristic time scale. The result is an emergent “classical” behavior for many particle systems, and an approximate quantum behavior for microscopic isolated systems (Bassi and Ghirardi, 2003). Thus, while collapse is continuously occurring, it is not the result of interaction with a measurement device.

An attempt to reconcile the Copenhagen interpretation with the measurement problem has given rise to the notion of decoherence. Roughly speaking, decoherence posits that, through interaction with the environment, a quantum system loses its coherent properties, resulting in a classical superposition of probabilities. In terms of the density matrix formalism, the off-diagonal terms that represent quantum interference disappear due to interaction with the environment, yielding a diagonal matrix with the Born probabilities as its entries (Zurek, 2003). Decoherence thus attempts to explain how quantum states evolve into classical probabilities, and so rationalize why we do not observe quantum superpositions in the laboratory. However, it does not explain why we do not observe classical superpositions instead. Schrodinger’s cat evolves from being simultaneously dead and alive, to being half dead and half alive. However, when we observe the cat, it is either 100% dead or 100% alive, corresponding to a density matrix consisting of a single non-zero entry on its diagonal. Thus, while decoherence describes how quantum probabilities become classical, it does not provide an entirely satisfactory resolution of the measurement problem (Adler, 2003).

If we adopt the physical picture suggested by de Broglie and the walking droplets, the measurement problem vanishes from consideration. In particular, wave function collapse becomes a nonproblem (Bush and Oza, 2020). Consider the robust wave-like statistics emerging in the hydrodynamic corrals (Harris et al., 2013; Sáenz et al., 2018) (Fig. 2.5) or the analog Friedel oscillations (Fig. 2.6). If one were to assert that this statistical waveform were a complete description of the system, then its collapse into a discrete droplet in response to the act of observation might be troubling. With the knowledge of the underlying pilot-wave dynamics, it is obvious that wave function collapse is a feature of any statistical theory. Bush and Oza (2020) refer to this as ‘statistical nonlocality’, the misinference of nonlocality owing to one’s insistence on the completeness of a statistical theory. Wave-function collapse may thus be seen as a dilemma only for those insistent on the completeness of a statistical theory, be it quantum or classical. Ditto for the measurement problem.

2.7 Quantum Superposition

> One cannot in the classical sense picture a system being partly in each of two states and see the equivalence of this to the system being completely in some other state. There is an entirely new idea involved, to which one must get accustomed and in terms of which one must proceed to build up an exact mathematical theory, without having any detailed classical picture.
>
> – P.A.M. Dirac, Principles of Quantum Mechanics (Dirac, 1958).

Due to the linearity of the Schrodinger equation (Eq. (2.1)), any linear combination of its solutions also constitutes a solution of the equation, and thus represent a valid quantum state. These linearly combined states are called superpositions, and they play a key role in the formalism and phenomenology of quantum mechanics. Formally, if ψ_1 and ψ_2 represent two different eigenstates of the Hamiltonian, then the superposed state, $\psi = a\psi_1 + b\psi_2$, is also a solution of the Schrodinger equation, with a, b being complex numbers such that $|a|^2 + |b|^2 = 1$. According to the Copenhagen interpretation, during measurement the wave function collapses onto one of the eigenstates ψ_1, ψ_2 with probabilities $|a|^2$, $|b|^2$ respectively. Notably, the coefficients a, b are generally complex numbers, and can assume negative values. As a result, the two eigenstates constituting the superposed state can destructively interfere with one another, yielding phenomena such as single particle interference as seen in the double-slit experiment (Sect. 2.4).

In their study of walker motion in an elliptical corral (Fig. 2.5d–g), Sáenz et al. (2018) demonstrated that in the high-memory, chaotic regime, the emergent statistics may be simply expressed in terms of the superposition of two cavity modes, one being the corrals most unstable Faraday mode at the systems driving frequency, the other being the most unstable mode at a nearby frequency. This then represents a superposition of statistical rather than dynamical states. By using bottom topography with high symmetry, they demonstrated that the relative weights of the two modes could be tuned. Specifically, by placing a submerged well at the focus of the ellipse, it favored one mode over the other. The analogous procedure in the quantum corral (Fiete and Heller, 2003), undertaken by manipulating magnetic impurities on the metal surface, leads to so-called 'statistical projection effects' (Moon et al., 2008; Manoharan et al., 2000). When an impurity is located at the focus of an ellipse, the preferred mode is that with extrema at the foci, leading to a projection effect referred to as the 'quantum mirage' (Manoharan et al., 2000). The walker system thus provides a rational means of interpreting both statistical projection and mirage effects.

We have seen that pilot-wave hydrodynamics can account for wave-particle duality and related phenomena such as single-particle interference. An important open question is thus whether one can derive a description of the emergent statistics equivalent to the Schrodinger formalism in quantum mechanics, from the dynamical description of the walker in HQA. The fact that the walker system exhibits both single-particle interference phenomena and the superposition of states suggests that such may be the case. In the HQA community, efforts are currently being made to develop a theory of walker statistics through consideration only of the pilot-wave field (Kutz et al., 2023). Specifically, for a walker confined to a one-dimensional well, the evolution of the pilot-wave field is characterized as the system memory (vibration forcing) is increased progressively. A discrete set of wave modes emerge sequentially, analogous to the new eigenstates of the wavefunciton arising as the particle energy is increased. Finally, the mean wavefield is inverted to yield the particle statistics, using Durey's convolution result (Durey et al., 2018, 2020). This recent study takes one step closer towards one current goal of the HQA community, developing a wave theory for the statistics of walking droplets (Kutz et al., 2023).

2.8 Nonlocality

> That particular interpretation (Bohmian mechanics) has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.
>
> – John S. Bell, On the Einstein Podolsky Rosen paradox (Bell, 1964).

The inference made from the experimental violation of Bell's Theorem (see Sect. 2.10) has generally been that any hidden variable theory of quantum mechanics must be non-local; thus, non-locality is seen by some as being a necessary feature of a theory of quantum dynamics (Maudlin, 2014). For example, having seen the experimental violations of Bell's Inequality (Aspect et al., 1982a), Bell was an advocate of Bohmian mechanics on the grounds that it was non-local (Bricmont, 2016). Nonlocality has different guises in the various interpretations of quantum mechanics. For example, we have already seen in §2.5 that proponents of the Copenhagen Interpretation must contend with the instantaneous collapse of the wave function, a form of nonlocality simply rationalized from the new perspective offered from HQAs (Bush and Oza, 2020).

Another manifestation of quantum nonlocality takes the form of an apparent action at a distance, as arises in Bohmian mechanics. Specifically, if the position of one particle in an entangled pair is changed, the position of its entangled counterpart is instantaneously affected. In HQA, a number of systems have been considered in which one would infer action-at-a-distance if the pilot-wave dynamics were not adequately resolved. For example, in the double-slit walker diffraction experiments, the change prompted by the influence of the second slit on a droplet passing through the first would be considered nonlocal (Fig. 2.4d). When walkers approach submerged pillars (Harris et al., 2018) and wells (Sáenz et al., 2020, Fig. 2.6), they experience long-range lift forces that depend in a particular fashion on the distance from the obstacle. Were it not known that such forces are wave-mediated, one might misinfer that they imply action at a distance.

In single-particle Bohmian mechanics, nonlocality enters through the quantum potential. In classical mechanics, Newtons law of gravitation and Coulombs law both express non-local force laws, specifically, action at a distance. While the motion of a massive or charged particle in response to either force field is local (in the sense that it responds only to the local field), the field itself is nonlocal, in the sense that its origins cannot be rationalized without appealing to deeper theoretical developments, specifically, quantum electrodynamics or general relativity. The same could be said of Bohmian mechanics, according to which a particle moves in response to the quantum potential, whose form is prescribed by the quantum wave function. While the particle's motion may be considered local, it is moving in response to a nonlocal field imposed by fiat.

A recent example illustrates how the HQA perspective allows one to achieve quantum effects and rationalize them without appealing to nonlocality. 'Surreal trajectories' is a term coined by Englert et al. (1992) (ESSW) to describe Bohmian

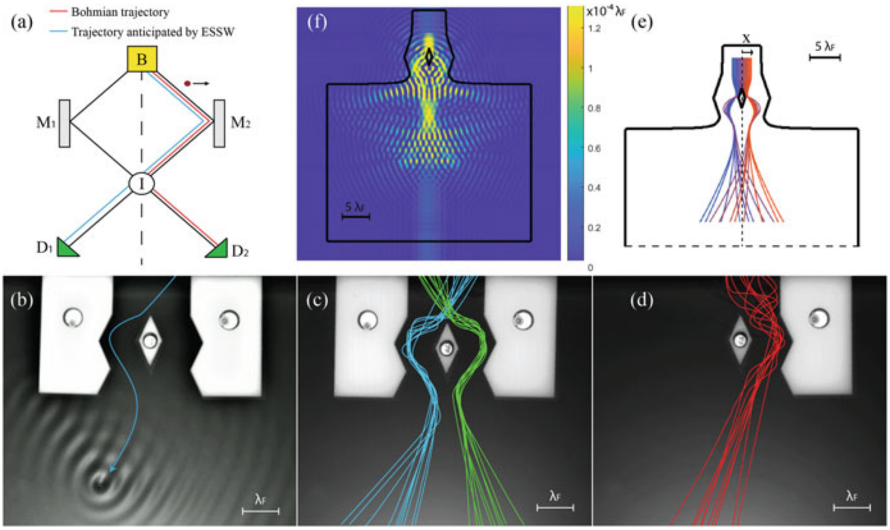

Fig. 2.7 Surreal trajectories in the hydrodynamic pilot-wave system (Frumkin et al., 2022). (**a**) A variant of the interferometer setup considered by ESSW (Englert et al., 1992). (**b**) A single particle trajectory, along with the instantaneous pilot wave field. (**c**) In a symmetric arrangement, the droplet enters the right or left channel with equal probability, after which it is deflected away from the system centerline, resulting in a 'surreal' trajectory. Twenty such trajectories are shown. (**d**) When one of the barriers is removed, the symmetry of the system is broken. The walking droplet is then reflected away from the remaining barrier, resulting in the trajectory that one might expect. (**e**) Numerical simulations of an ensemble of initially vertical trajectories with different values of the impact parameter x. (**f**) The mean pilot-wave field generated by averaging simulated droplet trajectories with a Gaussian distribution of initial impact parameters

trajectories predicted to arise in the interferometer arrangement illustrated in Fig. 2.7a. The term was intended to point out that the trajectories predicted by Bohmian mechanics are counterintuitive, and so cannot be real. Mahler et al. (2016) measured mean trajectories in the geometry proposed by ESSW via weak measurement, and found that they were consistent with those predicted by Bohmian mechanics. The authors concluded that 'the trajectories seem surreal only if one ignores their manifest nonlocality'. In Bohmian mechanics, surreal trajectories arise as a result of the particles being guided by the quantum potential, a non-local field. In the standard formulation, there is no notion of trajectories, but the experimental observations of mean trajectories consistent with surreal trajectories led (Mahler et al., 2016) to seek a rationale in terms of quantum nonlocality, specifically entanglement with the measurement device. In the walker system, 'real surreal trajectories' arise naturally from non-Markovian, classical dynamics in which the droplet navigates its pilot-wave field, a local potential of its own making. Nonlocality need not be invoked. Our study showed that the designation of Bohmian trajectories as surreal is based on misconceptions concerning the limitations of classical dynamics and a lack of familiarity with pilot-wave hydrodynamics (Frumkin et al., 2022). Moreover, it made clear that the physical picture furnished by the walker system allows one to see how to side-step the invocation of the non-local quantum potential required in Bohmian mechanics.

HQA suggests that one can avoid the invocation of a nonlocal field, as is done in Bohmian mechanics, provided one follows the suggestion of Holland (1995): "We can envisage a more active role for the particle, something which is not even admitted as conceivable in the conventional view. This may, for instance, enter as a source of the pilot-wave field through an inhomogeneous term in the wave equation". This conceptual leap would constitute a conformance to de Broglie's double-solution theory rather than the provisional theory now known as de Broglie-Bohm theory or Bohmian mechanics, and would seem to be a critical step in restoring locality to quantum pilot-wave theories. Considering the particle as the source of its own wave renders the resulting dynamics local: the particle generates its pilot wave and moves in response to it. This approach was recently followed by Dagan and Bush (2020) and Durey and Bush (2020), who considered particles with an internal oscillation at the Compton frequency generating, then moving in response to, waves satisfying a forced Klein-Gordon equation. While exploratory, the results yielded a striking result, a physical picture for the origins of the de Broglie relation. If a particle vibrating at the Compton frequency is dressed in a quasi-monochromatic wavefield that is a solution of the Klein-Gordon equation, then the group velocity of that wavefield must match the particle speed; thus, $p = \hbar k$. The emergent pilot-wave form for a free, uniformly translating particle, illustrated in Fig. 2.1d, roughly conforms to that envisaged by de Broglie (1987). The potential and limitations of this approach, of developing a model of quantum dynamics informed by the walker system, are currently being explored more widely.

2.9 Non-separable States

> Often a pair of quantum systems may be represented mathematically (by a vector) in a way each system alone cannot: the mathematical representation of the pair is said to be non-separable: Schrödinger called this feature of quantum theory entanglement.
>
> – Richard Healey, The Quantum Revolution in Philosophy (Healey, 2017).

Non-separable states arise in multi-partite systems when the state of the whole cannot be simply defined in terms of the state of its subsystems (Horodecki et al., 2009). For example, if a quantum system S is comprised of two subsystems, A and B, then the system will be considered non-separable if $\psi_S \neq \psi_A \otimes \psi_B$, where ψ_S, ψ_A and ψ_B are the wave functions of the respective systems, and $\otimes$ represents a tensor product. A canonical example is that of a singlet state, namely, two particles with opposite spins: $\psi = \frac{1}{\sqrt{2}} \left(|{\downarrow\uparrow}\rangle - |{\uparrow\downarrow}\rangle \right)$. Such states cannot be factored into a product of two independent states, specifically, there are no complex numbers a, b, c, d such that: $\psi = \left(a \left|\downarrow\right\rangle + b \left|\uparrow\right\rangle \right) \otimes \left(c \left|\downarrow\right\rangle + d \left|\uparrow\right\rangle \right)$. The singlet state is one of the maximally entangled Bell states (Bohm, 1951), and has