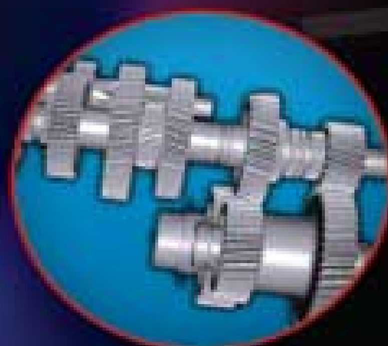
AUTOMOTIVE SERIES

VEHICLE GEARBOX NOISE AND VIBRATION

MEASUREMENT, SIGNAL ANALYSIS, SIGNAL PROCESSING AND NOISE REDUCTION MEASURES



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VEHICLE GEARBOX NOISE AND VIBRATION

MEASUREMENT, SIGNAL ANALYSIS, SIGNAL PROCESSING AND NOISE REDUCTION MEASURES

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To Magda, Lucie, Peter and Eva
To the memory of my parents and brother

Series Preface

The gearbox is one of the most critical components of the automobile. Indeed, it is one of the most critical components of many mechanical systems including those used in aerospace, maritime, construction and agricultural systems, to name a few. The intricate combination of rotating gears, bearing and shafts interacting in a wide variety of modes results in a complex set of dynamics defining the performance of the gearbox. This performance directly drives the ability to transmit power from the engine to the wheels. However, these interactions can also result in a significant amount of vibration and noise which can affect ride comfort, systems performance and even the safety of the overall vehicle due to issues such as durability and fatigue.

The *Automotive Series* publishes practical and topical books for researchers and practitioners in industry, and postgraduate/advanced undergraduates in automotive engineering. The series covers a wide range of topics, including design, manufacture and operation, and the intention is to provide a source of relevant information that will be of interest and benefit to people working in the field of automotive engineering. *Vehicle Gearbox Noise and Vibration* is an excellent addition to the series focusing on noise and vibration issues stemming from the gearbox. The text provides an excellent technical foundation for noise and vibration analysis based on significant past research and development efforts, as do many texts in this area. What makes this text unique is that the author expands upon the classical analysis techniques to integrate and make use of the latest, state-of-the-art technologies and concepts that are in use today. Finally, throughout the book, real world examples are given to demonstrate the application of the various techniques in combination with each other, providing the reader with some excellent insight into what can be expected when employing the various noise and vibration concepts.

As is mentioned in the beginning of this preface, *Vehicle Gearbox Noise and Vibration* is part of the *Automotive Series*; however, gearboxes are found on a wide variety of other systems outside of the automotive sector. Thus, the concepts presented in this text are applicable across a wide variety of fields. Issues related to noise and vibration of gearbox components such as shafts, gears and bearings further extend the utility of the concepts presented to a wide variety of rotating systems such as turbo pumps, aircraft engines and power generation, to name a few. Furthermore, the pragmatic signal processing techniques that are presented in the text are applicable to any physical engineering system, making the utility of this book quite far reaching.

Vehicle Gearbox Noise and Vibration nicely integrates a set of topics that are critical to rotating systems. It presents some very pragmatic applications of those techniques with real-world examples demonstrating the implementation of the presented concepts. It is state-of-the-art, written by a recognized expert in the field and is a valuable resource for experts in the field. It is a welcome addition to the *Automotive Series*.

Thomas Kurfess
January 2014

Preface

Many books deal with the calculation of gear geometry with respect to gear strength, selection of material, lubrication of teeth, alignment of gears and estimation of wear. However, there is less information available on how they are manufactured, and almost none is available on such details as the meshing cycle of teeth through the measurement of gearbox vibrations. The usual measurements of gearbox vibration and noise provide a frequency spectrum. The frequency spectrum does not give direct information about the meshing cycle. The first articles on the evaluation of toothmeshing in the time domain appeared about 25 years ago. In the frequency domain the responses of the loaded gears are separated from each other by different frequencies, but this does also have an impact on the time domain. The time course of vibrations becomes useful only when it is able to focus on a selected gear train and filters out the vibration responses of the other gear trains. This technique is known as synchronous filtration or synchronous averaging. Another name for it is signal enhancement. This method of signal processing requires the signal to be resampled synchronously with the rotational speed, which allows the angular vibrations during rotation to be determined.

A substantial part of this book discusses how the time domain analysis of the transmission unit is applicable to any rotating machine. This book also describes the practical measurement of angular vibration during rotation and how this is associated with the method of measuring transmission error of the gear train. However, many researchers, especially in the UK and the USA (for example

D. Smith, R.G. Munro, D. Hauser) have already carried out this measurement.

The gearboxes of the vehicles do not operate at a constant speed or a constant load. The variable speed requires changes in the gear meshing frequencies to be tracked and spectral peaks which are excited by meshing gears or due to the resonance of the mechanical structure to be distinguised. This book describes how the run-up and coast down of machines can be analysed using the time-frequency representation a multispectrum. An alternative way to analyse the transient states is to use tracking filters such as quadrature mixing and the Vold-Kalman tracking filtration.

This book describes how to interpret the composition of the real cepstrum. The difference between the cepstrum of harmonics, odd harmonics and the set of harmonics which contain the sidebands of carrier components is demonstrated. The fundamental frequency of the harmonics and odd harmonics is related to the zero frequency, while the fundamental frequency of the harmonic components as sidebands is related to the carrier component frequency.

The main topic of the book is a description of the research work which was done to reduce the gearbox noise of a heavy-duty vehicle. There are two possible solutions for keeping a transmission unit quiet. Introducing an enclosure for preventing noise radiation is the easiest one, but it has consequences, for example, low efficiency and maintenance difficulties. The more sophisticated and much more efficient solution is based on solving the noise problem at the source. It means introducing improvement aimed at the gear design and manufacturing, which results in the greatest reduction of noise level as is shown.

The final chapter of the book describes the process of deciding how to proceed when using the most effective noise control measures. The ratio of radiated noise power of the individual units such as engine, gearbox, axles and tyres to the overall noise level during the pass-by noise test of the vehicle is analysed. Based on the resulting statistics the effect of limiting the deviations during production is estimated. The need to increase the stiffness of the transmission housing has been demonstrated by measuring the vibrations at the different gear ratios. The final decision was to change the contact ratio of the gears from low (LCR) to high (HCR). To keep the radiated noise under control, the effect of load, the gear contact ratio and the tooth surface modification on noise and vibration are illustrated by measurement examples giving an idea of how to reduce transmission noise.

In addition to describing the noise problems of the vehicle gearbox the book is also a textbook of signal processing. The chapter which deals with the demodulation of the modulated signals is universally applicable to the diagnostics of machines. In particular, the described methods for the measurement of angular vibrations are not that well known and are waiting for further applications. The book contains the first detailed description of the Vold-Kalman order tracking filter.

Acknowledgements

I was first asked to write a book about gearbox vibration and noise by Debbie Cox at the 16th International Conference on Noise and Vibration, held in Krakow, Poland between 5–9 July 2009, where I presented a keynote lecture on this topic. To start with I was supervised by Debbie Cox and later by Tom Carter. I thank both for their help. I would like to mention all those who gave me the opportunity to work in the field of vehicle noise and vibration. Everything started when TATRA decided to solve the problem of noise from heavy-duty vehicles. My role in the project of the development of a quiet heavy-duty vehicle was to oversee noise and vibration measurements, signal processing, statistical investigation and the development of the software which supported measurements and evaluations. Fortunately, I could collaborate with very experienced designers, testing engineers and specialists working in technology and quality control of production.

I am grateful to Professor V.Moravec, the head of the team for developing the HCR gearing at TATRA at the beginning of the 1990s, for his help and valuable comments on gear design and accuracy. Not forgetting, fellow team members R. Kubena, V. Nykl and F. Sasin. Thanks to all.

Later, I started working at the VSB – Technical University of Ostrava, so I would like to thank colleagues from the Department of Mechanisms and Machine Parts and the Department of Control Systems and Instrumentation for cooperation in the development of methods for measuring transmission error. In particular, I must mention Professor Z. Dejl. For assistance with technical equipment there is Professor L. Smutný. Cooperation between the university and Tatra continues. Most valuable is the exchange of experiences with J. Jakubec, the chief designer of transmissions.

I am grateful to Professor M. J. Crocker, who, as editor-in-chief and co-author, invited me to publish a chapter in the Handbook of Noise and Vibration Control. This invitation helped me to start performing at international level.

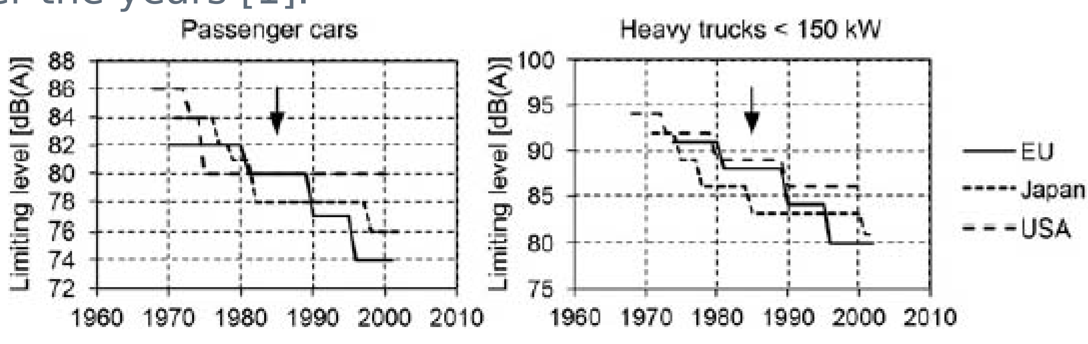
Research work on the transmission error measurement that has been carried out at the VSB – Technical University of Ostrava, was supported by the Czech Science Foundation.

1

Introduction

Various authorities aim to reduce the noise level in the environment by issuing requirements for the maximum noise level of critical noise resources. In transport, it is primarily motor vehicles which are subject to noise emission regulations. However, the strict limits cannot be introduced all at once, therefore the reduction is expected to be made gradually over at least 25 years. Newly manufactured vehicles which do not meet specified noise limits do not obtain permission to operate on public roads. Motor vehicle manufacturers have been given sufficient time to implement noise reduction innovations. The time line for noise limits for cars and trucks with an engine power of 150 kW and more is shown in Figure 1.1. Data was taken from the final report of the working party on noise emissions of road vehicles. The arrow pointing at 1985 indicates that in the EU there was a change in measuring procedure. For trucks, this corresponded to 2–4 dB of stricter requirements on top of the other changes; but for cars it corresponded to approximately 2 dB of less stringent requirements.

Figure 1.1 Development of vehicle noise emission limits over the years [1].



There is an international standard for the measurement of noise emitted into the environment. Details will be discussed in the last chapter of the book. For now, it is sufficient to note that under certain conditions the Sound Level Metre measures the maximum of the sound pressure level at the point which is at a distance of 7.5 m from the centreline of the track of the vehicle and 1.5 m above the road surface. The same sound pressure level is measured in the USA at the distance which is twice as far away, so limits for this country were raised to about 6 dB in the graph in Figure 1.1. This measurement relates to pass-by noise. The noise level in the vehicle cabin is a separate factor.

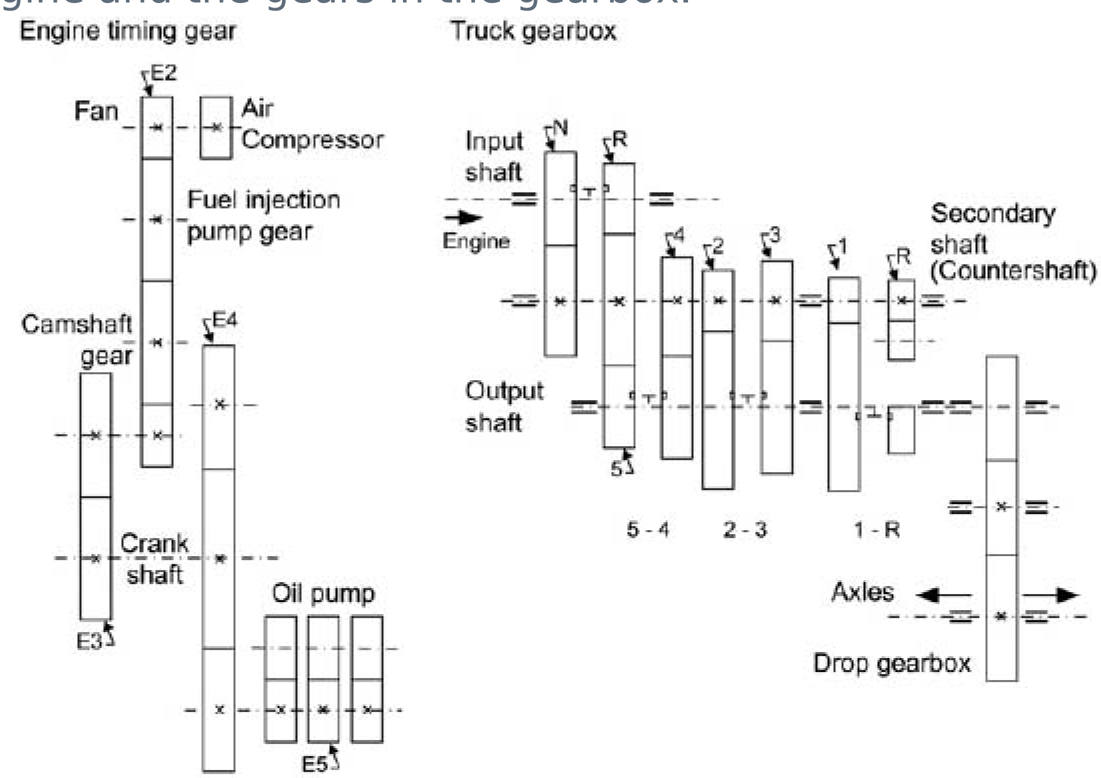
So began a race against time for manufacturers of heavy trucks. The sound pressure limit of 84 dB was not difficult to meet. But to produce a heavy-duty vehicle of 80 dB required changing the design. Transmissions can be put into an enclosure with a small reduction of 4 dB in the level of radiated noise or it is possible through a fundamental change in the parameters of gears [2, 3]. This book describes the difficult development which led to a substantial reduction in noise transmission by improving the design of gears. The theme of the book does not address the design, but describes the methods of measurement and signal processing which helped to determine the effect of design modifications or just to verify the correctness of the decision.

1.1 Description of the TATRA Truck Powertrain System

The theory of signal processing is illustrated by examples of the measurement of noise and vibration of the gearbox of the TATRA trucks. It is therefore appropriate to describe the transmission of these vehicles in detail. The truck powertrain system consists of the engine, gearbox, differentials and axles. All these units contain gears. Due to the high rotational speed and transferred torque, gears in a gearbox and axles play a key role in emitting noise. All gears in the TATRA gearbox are of the helical type and the gears in the axles are of the spiral bevel type. The problem of axle noise is serious, but this book does not propose to cover this area of research in detail. In Chapter 7 a method that enables the contribution of the noise level emitted by the axle to the overall noise level of the vehicle to be evaluated is discussed.

There are a number of gears which rotate in the truck as is shown in Figure 1.2. These include the timing gears of a diesel engine, but these are not a source of serious noise. The main source of noise which is produced by gears is the transmission unit. The older gearbox unit, including a drop or secondary gearbox, is in the left of Figure 1.2.

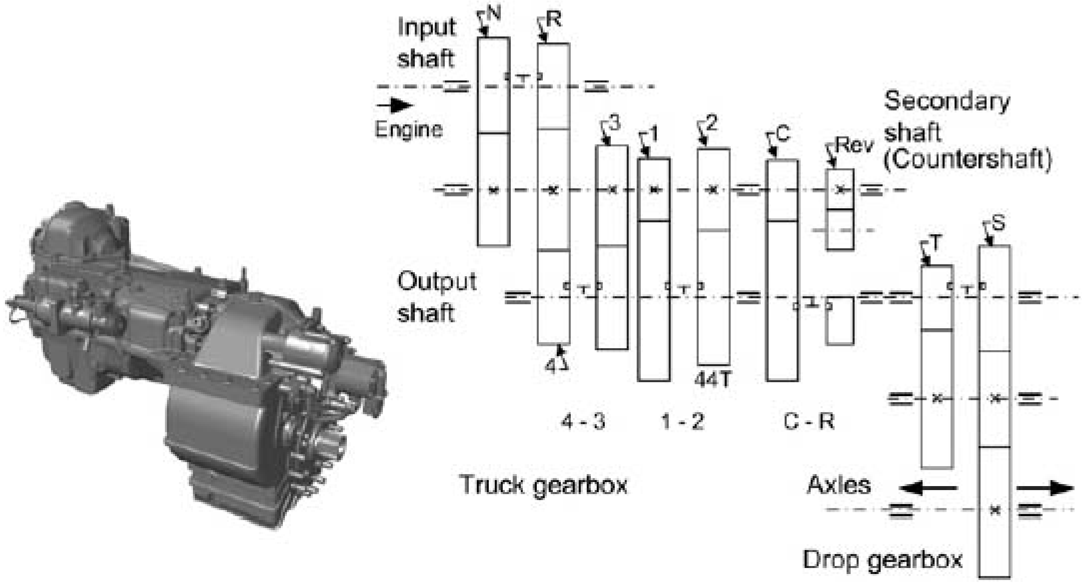
Figure 1.2 Kinematic scheme of the timing gears in the engine and the gears in the gearbox.



The secondary gearbox is sometimes called the drop gearbox due to the fact that this gearbox reduces the rotational speed. In the case of the TATRA trucks, the drop gearbox transfers power to the level of the central tube, which is the backbone of the chassis structure. The main gearbox comprises two stages and has five basic gears and reverse. As all the basic gears are split (R, N) the total number of the basic gears is extended to ten forward and two reverse gears. The gears are designated by a combination of the number character (1 up to 5 or 6) and letter (R or N), for example '3N'. According to the EEC regulations valid at the beginning of the 1990s, the basic gears selected for the pass-by tests are 3, 4 and 5. The drop gearbox is either the compound gear train with an idler gear or the two-stage gearbox, extending the number of gears to 12. TATRA does not use a planetary gearbox as the drop gearbox.

A kinematic scheme of the newest model of the TATRA gearbox is shown in Figure 1.3. The drop gearbox has two gear ratios in contrast to the old model of the gearbox. As is evident from the kinematic schemes both transmissions are manual and all the gears are synchronised.

Figure 1.3 Kinematic scheme of the newest model of the TATRA gearbox.

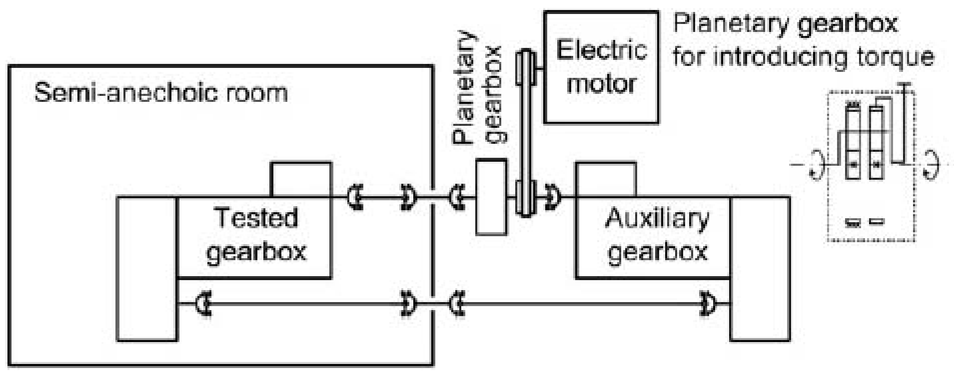


1.2 Test Stands

The operating conditions of gearboxes can be simulated using test rigs to drive the gearbox in a similar way to the pass-by noise test. The configuration of the closed loop is energy saving. With the use of an auxiliary planetary gearbox the torque is inserted in the closed circuit while an auxiliary electric motor spins the system at the operational speed. Power, which is the product of angular velocity and torque, then circulates inside the loop. If the auxiliary transmission adapts to different variants of the gearbox under test, then the power consumption of the test rig increases for example, up to 40% of the power that circulates in a closed loop.

An example of a closed circuit arrangement is shown in Figure 1.4. According to current standards for testing the radiated sound pressure level the volume of the chamber should be at least 200 times larger than the volume of the test gearbox. Microphones are placed on the sides of the gearbox in the direction of the truck movement at a distance of 1 m. Accelerometers that are attached on the surface of the gearbox housing near the shaft bearings can provide extensive information about the noise sources. A tacho probe, generating a string of pulses, is usually employed to measure the gearbox-primary-shaft rotational speed. A sensor for measuring the torque is also inserted into the closed loop.

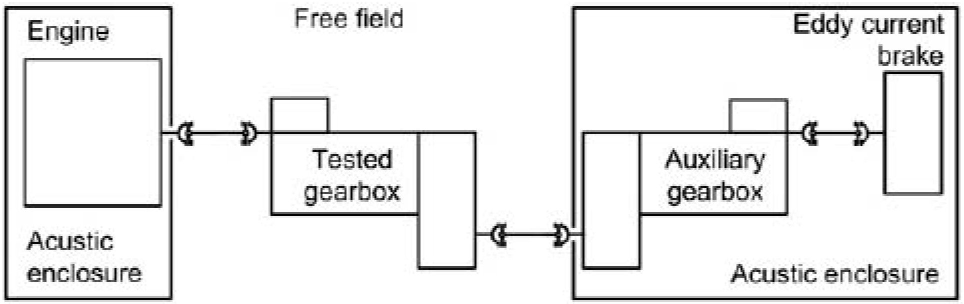
Figure 1.4 Closed loop test rig for testing noise in semi-anechoic room.



In contrast to the open loop test stand, the back-to-back test rig configuration saves drive energy. The torque to be transmitted by the gearbox is induced by a planetary gearbox. The gearbox under testing is enclosed in a semi-anechoic room with walls and ceiling absorbing sound waves and a reflective floor. The quality of the semi-anechoic room is of great importance for the reliability of the results. The reverberation time should be less than is required in the frequency range from at least 200 to 3 kHz. The input shaft speed is slowly increased from a minimal to maximal RPM while the gearbox is under a load corresponding to full vehicle ‘acceleration’. To simulate the gearbox operational condition during deceleration the noise test continues to slowly decrease from a maximal to minimal RPM.

The configuration for measuring an open loop is shown in Figure 1.5. Noise is measured in the open field with two microphones that are located in an anechoic chamber. Because the eddy current brake is used, it is necessary to use an auxiliary gearbox to increase the speed at which this type of the brake is able to effectively load the gearbox by a torque.

Figure 1.5 Open loop test rig for testing noise in free field.



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2

Tools for Gearbox Noise and Vibration Frequency Analysis

The signal $x(t)$ is a real or complex function of continuous time t. The other definition points to the fact that the signal contains information which transmits from the source to the receiver. But one of the signal types called a white noise does not formally contain any information. White noise is a totally random signal and the present samples do not depend on the past samples in any way. Signals describe the noise and vibration as time processes, and have common characteristics. This chapter deals with the theory of digitisation of analogue signals and different methods of signal processing in the time and frequency domain.

2.1 Theory of Digitisation of Analogue Signals

2.1.1 Types of Signals

Now we turn attention to the types of signals. The basic types of signals are deterministic and stochastic. There are deterministic periodic or non-periodic signals. The non-periodic signals can be broken down into almost periodic or transient signals. Simple tone seems to be deterministic, while multi-tonal sound seems to be stochastic (random). The signals in practice are a mixture of deterministic and random components. Further subdivision is shown in Table 2.1.

Table 2.1 Types of signals.

Deterministic				Random (stochastic)		
Periodic		Non-periodic		Stationary		Non-stationary
Sinusoidal	Complex periodic	Almost periodic	Transient	Ergodic	Non-ergodic	Special classification

Deterministic signals are defined as a function of time while random signals can be defined in terms of statistical properties. The deterministic signals can be predicted, while random signals, which have instantaneous values, are not predictable. The theory of random signals is based on the following system of naming. Names of random variables and processes are Greek letters:

variables: $\xi, \varepsilon, \ldots$ processes: $\xi(t), \varepsilon(t), \ldots$

while the measured waveforms, called realisations, are identified as Latin letters:

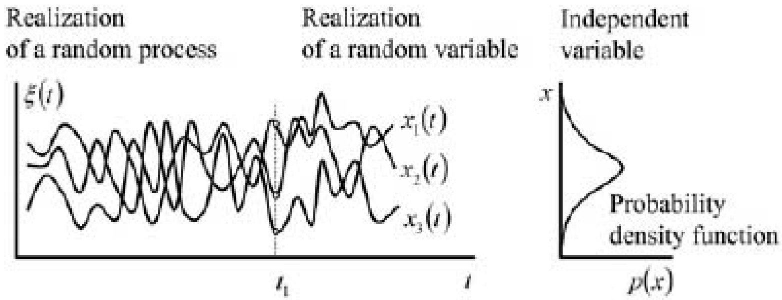
$x(t), y(t), \ldots$

A probability density function reflects the basic properties of random variables. Probability that a random variable ξ belongs to the interval of values greater than x and less than $x + \Delta x$ is proportional to the interval of the length Δx

(2.1) $P\{x < \xi \le x + \Delta x\} = p(x)\Delta x$

The coefficient of proportionality $p(x)$ is denoted as a probability density function (pdf). The function $p(x)$ is one-dimensional. There are also two and more dimensional pdf $p(x_1, x_2, \ldots)$, called joint probability. Between random variables and random signals (processes) is the relationship as it is documented in Figure 2.1. Values of the random process at time t_1 become a random variable.

Figure 2.1 Relationship between random processes and variables.



The probability density function is a basic property of random signals for the definition of the mean value μ and variance σ^2

$$\mu = \mathrm{E}\{\xi\} = \int_{-\infty}^{+\infty} x\, p(x)\, \mathrm{d}x$$

$$\sigma^2 = \mathrm{var}\{\xi\} = \mathrm{E}\left\{(\xi - \mu)^2\right\} = \int_{-\infty}^{+\infty} (x - \mu)^2\, p(x)\, \mathrm{d}x. \quad (2.2)$$

Square root of the variance is a standard deviation $\sigma = \sqrt{\mathrm{E}\left\{(\xi - \mu)^2\right\}}$.

An important property of random processes is stationarity, which is defined using the dependence of the probability density function on time. A stationary signal is a stochastic signal or process whose joint probability distribution does not change when shifted in time or space. As a result, parameters such as the mean and variance, if they exist, also do not change over time. The visual difference between the stationary and nonstationary signal is obvious from Figure 2.2.

The basic property of a stationary continuous signal $x(t)$ is that one-dimensional pdf and consequently the mean value of the random signal is independent of time

$$p(x_1, t_1) = p(x_1). \quad (2.3)$$

and the two-dimensional pdf depends only on the time difference $t_1 - t_2$ which is the time that elapses between these two time instants

$$p(x_1, x_2, t_1, t_2) = p(x_1, x_2, t_1 - t_2) \quad (2.4)$$

To understand the calculation of mean values and variances it is necessary to introduce the concept of ergodic processes or signals. An ergodic process is one which is complying with the ergodic theorem. This theorem allows the time average of a signal to be equal to the ensemble average. In practice this means that statistical sampling can be performed at one instant across a group of identical signals or sampled over time on a single signal with no change in the measured result.

$$\bar{x} = \int_{-\infty}^{+\infty} x p(x)\,\mathrm{d}x = \lim_{T\to+\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t)\,\mathrm{d}t. \quad (2.5)$$

This assumption is crucial to the process of measurements, because it allows practical results to be obtained.

2.1.2 Normal Distribution

The normal distribution of a random variable is defined by the formula

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \quad (2.6)$$

where μ is the mean value and σ is the standard deviation of the mentioned random variable which were defined above. For a random variable ξ the notation is as follows:

$$p(\xi) \sim N\left(\mu, \sigma^2\right).$$

The multivariate normal distribution of the random vector is defined by the formula

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{k}{2}} |\mathbf{P}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \mathbf{P}^{-1} (\mathbf{x}-\boldsymbol{\mu})\right) \quad (2.7)$$

where $\mathbf{x}$ is a vector of variables of the size $k \times 1$, $\boldsymbol{\mu}$ is a vector of mean values and $\mathbf{P}$ is a covariance matrix which is a positive definite symmetric matrix

$$\mathbf{x} = \left[x_1, x_2, x_3, \ldots, x_k\right]^T \quad \boldsymbol{\mu} = \left[\mu_1, \mu_2, \mu_3, \ldots, \mu_k\right]^T \quad \mathbf{P} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1,k} \\ \vdots & \ddots & \vdots \\ \sigma_{k,1} & \cdots & \sigma_k^2 \end{bmatrix}. \quad (2.8)$$

For a random variable $\boldsymbol{\xi}$ the notation is as follows: $p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \mathbf{P})$.

2.1.3 Mean Value and Standard Deviation (RMS) of a General Signal

In the case of signals, it is easier to calculate the time mean value from a sufficiently long record of a signal than from statistical data consisting of many realisations. Variance of an ergodic signal is given by the formula which is the similar to Eq. (2.5)

$$\sigma^2 = \int_{-\infty}^{+\infty} \left(x - \overline{x}\right)^2 p(x)\,\mathrm{d}x = \lim_{T\to+\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \left(x(t) - \overline{x}\right)^2 \mathrm{d}t. \quad (2.9)$$

Signal processing for signals such as sound pressure, vibration, voltage and electrical current is based on the calculation of the root mean square (RMS or rms). This abbreviation RMS describes the order of writing the detailed parts of the formula

$$RMS = \sqrt{\frac{1}{T}\int_{0}^{T} x(t)^2\,\mathrm{d}t}. \quad (2.10)$$

Because the mean value of signals such as sound pressure and vibration are equal to zero then the standard deviation and RMS are numerically identical.

If we process electrical signals, such as voltage $u(t)$ and current $i(t)$, then we can define instantaneous power $p(t) = u(t)i(t)$. If the electric current and voltage is related to the resistance R of the conductor, then the instantaneous power is given by the formula.

$$p(t) = Ri(t)^2 = \frac{u(t)^2}{R}. \quad (2.11)$$

In both cases the instantaneous power is proportional to the square of the current or voltage. The resistance R can be regarded as a scaling factor. Therefore it is useful to introduce the general power of the signal as the square of the signal. This power can be either instantaneous or an average of the instantaneous power for the selected time interval. The mean power PWR of the signal in a certain time interval of the length T is given by

(2.12) $$PWR = \frac{1}{T} \int_0^T x(t)^2 \, \mathrm{d}t = RMS^2.$$

As regards the units, the sound pressure, acceleration and the velocity have the units Pa, m/s^2 and m/s, respectively. The signal power of the sound pressure, acceleration and the velocity have the unit as follows Pa^2, (m/s^2)^2, (m/s)^2. A signal energy is the sum or integer with respect to time of the instantaneous power $\int x(t)^2 \, \mathrm{d}t$. The unit of the signal energy is multiplied by seconds, for example, Pa^{2}s, (m/s^2)^{2}s.

2.1.4 Covariance

The covariance between two real-valued random variables x and y is defined by the formula

(2.13) $$\mathrm{cov}(x, y) = \mathrm{E}\{(x - E\{x\})(y - E\{y\})\}.$$

If both the random variables x and y are identical, $x = y$, then the covariance becomes the variance, cov (x, x) = var (x). The covariance between random vectors **X** and **Y** of dimension ($m \times 1$) and ($n \times 1$), respectively, is a matrix defined by

(2.14) $$\mathrm{cov}(\mathbf{X}, \mathbf{Y}) = \mathrm{E}\left\{(\mathbf{X} - E\{\mathbf{X}\})(\mathbf{Y} - E\{\mathbf{Y}\})^T\right\} = E\left\{\mathbf{X}\mathbf{Y}^T\right\} - E\{\mathbf{X}\}\,E\{\mathbf{Y}\}^T.$$

There is a conflict in the notation of the variance and covariance random vector. Some statisticians use this notation

(2.15)
$$\mathrm{var}(\mathbf{X}) = \mathrm{cov}(\mathbf{X}) = \mathrm{E}\left\{(\mathbf{X} - E\{\mathbf{X}\})(\mathbf{X} - E\{\mathbf{X}\})^T\right\} = E\left\{\mathbf{X}\mathbf{X}^T\right\} - E\{\mathbf{X}\}\,E\{\mathbf{X}\}^T.$$

2.1.5 Mean Value and Standard Deviation (RMS) of a Sinusoidal Signal

A sinusoidal signal with the amplitude A and with the period of the length T is defined by the following formula