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The Language of Mathematics

Telling Mathematical Tales

Springer

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Dedication

This book is dedicated to my wife, Pip.

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Contents

PRELUDE: MAORI MATHEMATICS VOCABULARY

Abstract: The Maori language was adapted for mathematical discourse during the 1980s. Several issues arose from this intensive time of specific language development. The story of this development, with examples of difficulties is outlined.

Keywords: bilingual mathematics, Maori language, mathematical discourse

1987. New Zealand. A warm, stuffy room in an old school building. A group of mathematics teachers have been working for a week discussing mathematics education for the indigenous Maori people. They have been developing mathematical vocabulary in the Maori language, and this evening they are working on statistical terms. They are trying to explain the difference between continuous and discrete data to a Maori elder. Examples are given: heights and shoe sizes; temperatures and football scores; time and money. The concept is grasped easily enough, but the elder must put forward suggestions for Maori vocabulary for use in mathematics classes. He will not transliterate to produce Maori sounding versions of the English words: for example, he might have tried *konitinu* for continuous or *tihikiriti* for discrete. He does try existing words for some of the examples that are given: *ikeike (height)*, and *tae (score)*—but these terms are not representative enough for the mathematicians in the room, and are rejected. Then he begins to try metaphors. At each attempt a short discussion amongst those mathematics teachers who know the Maori language quickly reaches consensus that the metaphor suggested will not do. Then he suggests *rere* and *arawhata*. Those of us in the room with only a little Maori understand the common meanings of these words as ‘flying’ and ‘ladder’. It does not seem good enough for us. But the eyes of the good Maori speakers light up. They know that these words as a pair refer to the way a stream flows, either smoothly without a break, or in a series of little waterfalls over rocks. This mirrors the way that continuous data is information taken from a smooth stream of possible measurements, and discrete data is information that can only have particular values. Yes. New technical vocabulary is born.

Although I became aware of the importance of language in mathematics education while working in Swaziland in the late 1970s, my first serious involvement in this area was as part of this group of teachers developing vocabulary and grammar so that mathematics could be taught in the Maori language to the end of secondary education.

Maori is a Polynesian language brought to New Zealand by the first settlers over 1000 years ago. It was an oral language, and was not written down until European traders and missionaries came to New Zealand around 1800. As happened in other places in the world, significant European settlement signalled the start of a decline in the use of the indigenous language through familiar colonial processes. However, in the 1970s, a Maori cultural renaissance began. As part of this, bilingual primary schools were established, although mathematics and science were still mainly taught in English (Nathan, Trinick, Tobin, & Barton, 1993). Bilingual secondary schools developed during the 1980s, but Maori children remained alienated from mathematics and science. One response was the call for mathematics and science instruction in Maori (Fairhall, 1993; Ohia, 1993), and a small group was gathered together by the Department of Education to develop Maori mathematical language for this purpose (Barton, Fairhall & Trinick, 1995a). The group included teachers, mathematicians, mathematics educators, linguists, Maori elders, and Maori language experts. It worked under strict guidelines laid down by the Maori Language Commission, (these guidelines included a ban on the use of transliterations), and an imperative to ensure that any new language retained Maori grammatical structures.

This was a very exciting time for those involved. It felt as though we were in a crucible of language development, and we were all challenged both linguistically and mathematically. Linguistically the challenge was to produce vocabulary and grammar that had new uses (as far as Maori was concerned) but that was recognisably Maori in its structure, denotations, and connotations. There was a lot of use of metaphor, for example using *kauwhata* for a graphical framework or set of axes. *Kauwhata* refers to a rectilinear frame used for drying fish. Another vocabulary creation technique was to use standard Maori grammatical constructions, for example using standard suffixes for nominalising verbs, thus *pa* (to be related to, or concerning) is transformed to a noun, *panga*, with the meaning function. There was also an opportunity to resurrect old Maori words that had gone out of use with new (but related) technical meanings. The word *wariu* for 'value' had been used for many years, but was rejected as a transliteration. It was replaced by an old word, *uara*, that had fallen out of use, but meant the value or standing of someone.

Mathematically, those of us with expertise in the subject were challenged to accurately explain the meanings and functions of many mathematical terms and concepts. This proved more difficult than might be expected, particularly for the very basic concepts. For example, words like 'number' and 'graph' have meanings that shift in different contexts and at different stages of development of mathematical understanding. We were prompted to construct a genealogy of mathematical terminology that showed which words were base words in mathematical discourse and how other words could or should be derived from them. For example, 'multiple' is a child of 'number' and 'multiply'. This genealogical tree was not always obvious, nor is it unique.

The whole process was characterised by a cycle of collecting the terms being used in existing bilingual and immersion classrooms, taking the words and phrases back to Maori communities for their comment, writing up the results, and presenting this material to the Maori Language Commission for their decisions and ratification. The cycle was repeated three times over fifteen years, and the process and the resulting vocabulary and grammar have been published in a series of papers and dictionaries (Barton, Fairhall & Trinick, 1995a, 1995b, 1998; NZ Ministry of Education, 1991, 1994, 1995). It happened that the 'flowing' and 'waterfall' metaphors described above as words for 'discrete' and 'continuous' were eventually rejected in this process and replaced by words based on the Maori word *motumotu*—which means divided into isolated parts as islands are upon the sea.

So, was the Maori language successfully adapted to the teaching of mathematics? The answer is yes, ... and no. There is evidence that those taught mathematics in Maori are doing well (Aspin, 1995). Some students have been taught mathematics in Maori up to Year 13 (the final year of secondary school), but difficulties continue to exist in finding suitably qualified teachers (that is, those who are fluent in both Maori and mathematics), especially at senior levels.

However, those of us involved in the Maori mathematics language development had become increasingly uncomfortable with some aspects of our work. Somehow the mathematical discourse that had developed did not feel completely right, but we were unable to put our finger on why. We came to talk about this as the "Trojan Horse" phenomenon: mathematics education seemed to be a vehicle that led to the subtle corruption of the ethos of the Maori language (Barton, Fairhall & Trinick, 1998).

An example of grammatical corruption had happened during the vocabulary development process. It had been difficult to translate the concepts of positive and negative numbers. At the first meeting with the Maori Language Commission a discussion had resulted in a very rare agreement on the part of the Commission to alter the grammar of the language and use the direction-indicating adverbs *ake* (up) and *iho* (down) as adjectives for the noun *tau* (number). *Ake* and *iho* should only modify verbs, as in *heke iho* (fall down). But the adjectival uses *tau ake* (literally 'upwards number' for positive number) and *tau iho* (literally 'downwards number' for negative number) were to be permitted. Four years later, at the second meeting with the Commission, one member demanded that this decision be rescinded. She had heard some children in a school playground extend this grammatical misuse to their everyday discourse. A child had been heard to say "*korero ake*" (literally 'upwards talk') to refer to praise. *Ake* should not be used in this way as an adjective in correct Maori language. Under her angry imperative, an alternative formulation for positive and negative numbers was immediately found.

Our feeling that we had more fundamentally permanently changed the nature of the language was finally confirmed several years later. The example that epitomised the problem was that of the grammatical role of numbers. Classroom discourse that had developed during the 1980s used numbers grammatically very much as they are used in English. However, in Maori as it was spoken before European contact, numbers were verbal in their grammatical role (Trinick, 1999; Harlow, 2001; Waite, 1990).

What does "numbers were verbal in their grammatical role" mean? We are not familiar with numbers as verbs. A number does not seem to be an action. However it can be. In English there are verbal forms for the numbers 1 to 4: I can *single* someone out. I can *double* my bet. I can *triple* my earnings—well actually I can't, but someone else might be able to. A new school may even *quadruple* its enrolment over a few years. However, these forms are not the basis of our understanding of number. In everyday talk, numbers are usually used like adjectives. There are three bottles on the table. I have five fingers. Just as there might be green bottles on the table, and I have long fingers. (Technically, however, numbers are not adjectives. They are generally considered to have their own grammatical form).

In Maori, prior to European contact, numbers in everyday talk were like actions. The grammatical construction used would have been like saying that "the bottles are three-ing on the table", or that "my fingers five". Just as the bottles are standing on the table, or my fingers wiggle.

Our awareness of this old Maori grammar of number suddenly sharpened when we tried to negate sentences that used numbers. The construction that 'sounded right' was not the same as the construction that should logically follow from the classroom mathematics discourse.

Let us look at this in detail. To negate a verb in Maori the word *kaore* is used:

We are going to the house.	=	*E haere tatou ki te whare.*
We are not going to the house, we are returning.	=	*Kaore tatou e haere kit e whare, e hoki mai ke.*

Unlike English, where negating both verbs and adjectives requires the word 'not', in Maori to negate an adjective a different word is used, *ehara*:

This is a big house.	=	*He whare nui tenei.*
This is not a big house, it is a small house.	=	*Ehara tenei I te whare nui, he whare iti ke.*

In Maori, negating number uses the verbal form, *kaore*:

There are four hills.	=	*E wha nga puke.*
There are not four hills, there are three.	=	*Kaore e wha nga puke, e toru ke.*

Here was evidence that the classroom discourse that had been developed was against the original ethos of the Maori language. Numbers had been changed to become adjectival. While constructing the dictionaries and glossaries of mathematics vocabulary, the verbal nature of numbers was ignored, and a classroom discourse that treated numbers as they are in English was perpetuated. Thus the mathematics vocabulary process contributed to changes in Maori language use.

This experience led me to contemplate whether this had happened in other languages. I was interested in this example of the colonisation process, and I was concerned about the consequences for bilingual or multilingual mathematics education. But also, as a mathematician, I was curious about the mathematical concepts inherent in the original Maori usage of number. Would mathematics have developed differently if it had developed through languages in which numbers were verbal? More generally, I became curious about the way that mathematical ideas are presented differently in other languages.

So began a search for other examples, and an investigation into the mathematical consequences and the implications for mathematics education. I soon discovered that this material was not 'lost'. Many other people—linguists, anthropologists, mathematics educators, ethnomathematicians—had recorded and discussed unexpected ways of expressing mathematical thinking in many different languages. However these examples had not previously been considered from a mathematical point of view, and only briefly had educational consequences been considered (E.g. Pinxten, van Dooren, & Harvey, 1983, Chpt. 5). I quickly came to believe that there were important mathematical ideas to be found, and I began to change some of my views about mathematics itself. In addition, some of my thinking about mathematics education was being turned around. This book is the result.

INTRODUCTION

Abstract: An outline of the structure of the book is presented, making the argument that the language we use for everyday mathematical ideas presents us with valuable evidence and insights into the nature of mathematics.

Keywords: mathematical discourse, nature of mathematics

I begin the book by looking at the way people speaking different languages talk about mathematical ideas in their everyday conversation. I end up questioning some common beliefs about mathematics, its history, and its pedagogy.

The way we (English speakers) use numbers, the way we give directions, the way we express relationships, are all so commonplace that it is hard to imagine any other way of expressing these ideas. We take for granted the structures of the following sentences:

> There are four people in the room.
> The book costs forty-five dollars.
> Two and three are five.
> Turn left.
> Go straight on.
> The sun rises in the east.
> A dog is a mammal.
> He is not my father.
> I will either go shopping or read my book this afternoon.

But apparently simple English language statements turn out to be expressed quite differently in some other languages—so differently that it is often difficult to write in one language the equivalent of what is being said in another. Even when quantity is expressed in the simplest way—when we count—it is done in fundamentally different ways in different languages, as has been illustrated in the Preface. We are not talking about just different vocabulary. Nor is it a matter of differences in the underlying base of the number system, that is, whether it is a decimal system or one based on five or twenty. The variety occurs in the way languages express numbers, the grammar of mathematical discourse.

The first part of this book explores these differences. In order to further explore how other languages construct mathematical talk, I investigated languages as different as possible from my own first language of English. Distant languages are most likely to have unfamiliar structures. Unfamiliar structures are good clues in a search for different mathematical conceptions. Therefore most of the examples described are from indigenous languages rather than Indo-European languages: the Polynesian languages Maori, Hawaiian, and Tahitian; the Euskera language of the Basque people; Kankana-ey from the Cordillera region of The Philippines; Dhivehi from the Maldives; Kpelle from Liberia, and First Nation languages from North America.

The first part also includes some mathematical flights of fancy arising from the way various languages discuss numbers and shapes. The imaginings illustrate the possibility of different mathematical worlds. However the main point of this section is to lay down the evidence of language difference with respect to mathematical talk. I demonstrate the congruence between mathematics as we know it and the English language. Other languages are not so congruent.

Part II discusses what all this means for mathematics. Does it mean that mathematics as an academic discipline with very powerful practical applications is somehow different in different parts of the world? A bridge designed using mathematical theory surely stands (or falls) in the same way independently of the country it is built in, or of the language of the person who solved the equations of its design? Surely 1 + 1 = 2 in Alaska, Nigeria, Tahiti, and Singapore? I argue for alternative answers to conventional questions about mathematics—where it comes from, how it develops, what it does, what it means. I challenge the idea that mathematics is the same for everyone, that it is an expression of universal human thought—and explain the questions about the bridge and 1 + 1 posed incredulously above.

Another issue concerns the relationship between language and mathematical thought. Does the language we speak limit what we can say, do, and think mathematically? If this is so, we can infer serious consequences for mathematics if one language comes to dominate mathematical discourse, as English is doing within the international research arena. The question is wrongly posed. We probably do not need to focus on the limitations created by languages—languages are sufficiently creative as living structures to describe whatever we want to describe—but we should continue to explore the mathematical creativity embedded in other languages. New mathematical ideas (or old ideas given new roles) lie hidden in minority languages.

The third part of the book briefly discusses the consequences for the way we learn and teach mathematics. Can these linguistic insights into mathematics tell us anything about how we gain mathematical understanding? I make two fundamental suggestions. We should do more abstract activity, both in the early stages of learning mathematics, and when students are having difficulty. However, in saying this, the nature of useful abstract activity needs to be reconsidered. The second major suggestion is that undirected mathematical play is a good thing at all levels of education from early childhood to graduate level.

Does a better awareness of the links between mathematics and language lead us to practical strategies in mathematics classrooms? Educators have known for some time about the importance of talking, and the need for formal language development within the mathematics curriculum. And yet mathematics teachers do not universally use language activities. We re-examine the argument for these roles for language, and give some examples. In addition a plea is made for the importance of teaching about the nature of mathematics.

What about classrooms where more than one language is spoken, and what do the conclusions of Part I mean for students who learn mathematics in an unfamiliar language? Much writing on multilingual classrooms characterises such environments as full of problems. Without denying the complexity of the situation, the ideas in this book suggest that these classes have, rather, an abundance of resources. The question is how teachers can best utilise the linguistic potential therein.

Finally, having started with evidence collected from many languages of indigenous groups around the world, I end with a consideration of the particular issues faced by these groups with respect to mathematics education. A proper understanding of the link between language and mathematics may be the key to finally throwing off the shadow of imperialism and colonisation that continues to haunt education for indigenous groups in a modern world of international languages and global curricula.

For some time now, I have felt that many debates in mathematics education have been dominated by ideologies and theories, rather than comprehensively argued positions. These have sometimes reached ridiculous levels, such as the Math Wars in America where a professor went on a hunger strike, and people leapt into political action and lobbied with little regard for critical argument or evidence. I think that on a matter as important and deep-seated as this, there should be evidence of a more permanent kind that can clarify some of the debate. This book can be read as an attempt to interpret the evidence from language with respect to mathematics and mathematics education. The evidence presented here seems to me to support a weakly relativist philosophical position in that mathematics might have been created otherwise, and a social constructivist mathematics education position in that we develop mathematics in conjunction with our language. However readers would be mistaken to think that arguing these positions is what the book is about. The evidence is presented and interpreted.

Before we start, a short statement about what I mean by mathematics, and a few caveats. Mathematics is a tricky word, loaded, for the many non-mathematicians amongst us, with thoughts of schoolteachers and textbooks and homework exercises. For mathematicians the meaning is richer, although there is considerable disagreement over its exact reference (Davis & Hersch, 1981). The problem for this book is that I wish to talk about mathematical things in general, and in contexts in which formal mathematics has no part. For example, as far as I am aware, in pre-European Maori culture, there was no area of knowledge or discourse equivalent to mathematics as understood today. How then can I talk about aspects of that culture being mathematical? The problem is circumvented in this book by mentally replacing the words 'mathematics' (or 'mathematical') with the phrase "(concerning) a system for dealing with quantitative, relational, or spatial aspects of human experience", or "QRS-system" for short. Thus any system that helps us deal with quantity or measurement, or the relationships between things or ideas, or space, shapes or patterns, can be regarded as mathematics. My translation allows the word 'mathematical' to be used much more widely than just to refer to things in mathematics texts or journals. If I want to talk about the smaller, formal, conventional world of academic mathematics as it is exemplified in schools and universities all over the world, then I will use the words "near-universal, conventional mathematics", or "NUC-mathematics" to refer to it. As an aside, I am told by sailing friends that NUC means "not under control" and refers to ships that have been abandoned at sea. Elements of this idea in NUC-mathematics will be illuminated in the following pages.

The caveats. Although I have taken the advice of many linguists, I do not claim to be a linguist myself. Nor do I claim fluency in any language other than English, despite a little Maori and a smattering of Spanish. I have used at least one first-language speaker of each language amongst my informants. Therefore the linguistic evidence is viewed from outside the discipline of linguistics, and from outside each of the languages used in the examples. This book, however, is about mathematics, so the languages are examined not so much for their linguistic characteristics, but for their mathematical ones.

A second caveat is that this work is written in English. To the extent that mathematical ideas differ between languages, the reflexive principle means that the ideas in this book would be different if they were written in another language. The discussion of other languages is from my point of view as an English speaker. If Euskera was my natural language, for example, then all the linguistic features quoted here would be seen in another way.

The third caveat is about coverage. I am mostly concerned about spoken language. Also there is no comprehensive coverage of all language families. Readers will note the lack of examples from Arabic and Asian languages, in particular Mandarin. Writing this book leaves me with a curiosity about those languages. I am certain that the written form is also important in mathematics, for example, it is significant that written Mandarin is iconographic while written English (and the other languages of my examples) is symbolic. Despite the importance of this issue, I will just acknowledge it and move on, leaving the fundamental influence of written language on mathematics for another time.