BestMasters

RESEARCH

Sophia Denker

Characterizing Multiparticle Entanglement Using the Schmidt Decomposition of Operators

MOREMEDIA

Springer Spektrum

BestMasters

Mit **„BestMasters“** zeichnet Springer die besten Masterarbeiten aus, die an renommierten Hochschulen in Deutschland, Österreich und der Schweiz entstanden sind. Die mit Höchstnote ausgezeichneten Arbeiten wurden durch Gutachter zur Veröffentlichung empfohlen und behandeln aktuelle Themen aus unterschiedlichen Fachgebieten der Naturwissenschaften, Psychologie, Technik und Wirtschaftswissenschaften. Die Reihe wendet sich an Praktiker und Wissenschaftler gleichermaßen und soll insbesondere auch Nachwuchswissenschaftlern Orientierung geben.

Springer awards **“BestMasters”** to the best master’s theses which have been completed at renowned Universities in Germany, Austria, and Switzerland. The studies received highest marks and were recommended for publication by supervisors. They address current issues from various fields of research in natural sciences, psychology, technology, and economics. The series addresses practitioners as well as scientists and, in particular, offers guidance for early stage researchers.

Sophia Denker

Characterizing Multiparticle Entanglement Using the Schmidt Decomposition of Operators

Springer Spektrum

Sophia Denker
Hövels, Germany

ISSN 2625-3577 ISSN 2625-3615 (electronic)
BestMasters
ISBN 978-3-658-43202-7 ISBN 978-3-658-43203-4 (eBook)
https://doi.org/10.1007/978-3-658-43203-4

This Springer Spektrum imprint is published by the registered company Springer Fachmedien Wiesbaden GmbH, part of Springer Nature.
The registered company address is: Abraham-Lincoln-Str. 46, 65189 Wiesbaden, Germany

Abstract

Witness operators are a useful tool to detect and quantify entanglement. A standard way to construct them is based on the fidelity of pure states and mathematically relies on the Schmidt decomposition of vectors [31]. In this thesis a method to build entanglement witnesses using the Schmidt decomposition of operators is presented. One can show that these are strictly stronger than the fidelity witnesses. Moreover, the concept can be generalized easily to the multipartite case and one may use it to quantify the dimensionality of entanglement. Finally, this scheme will be used to provide two algorithms that can be combined in order to improve given witnesses for multiparticle entanglement.

Contents

1 Introduction

In the 20th century it turned out that classical mechanics was not sufficient anymore in order to describe certain phenomena in physics. Hence, in the 1920s the theory of quantum mechanics was developed, which provides a mathematical framework to describe for example physical interactions on small energy scales. In 1935 the phenomenon of entanglement was first described by Einstein, Podolsky and Rosen (EPR) [49] and Schrödinger [52], who questioned the completeness of the theory. The assumption arose that there are hidden variables determining the physical happening. In 1964 Bell found that if there were such hidden variables, the correlations between the results of measurements of composed systems would be upper-bounded by a certain number. However, using entanglement, quantum mechanics violates this bound and thus, the assumption that there are hidden variables completing the theory of quantum mechanics may not be true. This violation could first be shown experimentally by John F. Clauser et al. in 1972 [13]. Later, Alain Aspect et al. performed several experiments closing one of two loopholes in Clauser's experiment [15–17]. Finally, in 2015 the first loophole free experiments were done simultaneously by two groups lead by Anton Zeilinger [27, 56, 61]. For their work, these three physicists received the Nobel Prize in Physics 2022.

With the development of quantum mechanics, also quantum information theory arose, where entanglement is a topic of great interest, too, as it is a resource for many applications such as quantum teleportation, quantum cryptography and quantum metrology. Consequently, entanglement detection is as important. One useful tool to detect and quantify it are entanglement witnesses. These are observables and hence can be implemented experimentally, which motivates their investigation. In this thesis a new type of witness, which is based on the Schmidt decomposition in the operator space (OSD), is introduced. One can show that these witnesses are

S. Denker, *Characterizing Multiparticle Entanglement Using the Schmidt Decomposition of Operators*, BestMasters,
https://doi.org/10.1007/978-3-658-43203-4_1

strictly stronger than those, based on the Schmidt decomposition in the vector space, which are considered as the standard witnesses. Furthermore the concept can be generalized to the multiparticle case and also be used to quantify the dimensionality of entanglement.

In the first part of this thesis the physical and mathematical background is given. We start with putting the concept of states and measurements into mathematical terms. After that, quantum entanglement will be discussed in detail and having defined it for the bipartite case, some examples for detection criteria are given. Moreover the Schmidt rank and Schmidt number as quantity for the dimensionality of entanglement are introduced and further, quantum entanglement in the multipartite case will be discussed. Next, entanglement witnesses are introduced. Their formal definition will be given as well as some examples how to construct and quantify them. Additionally, Schmidt number witnesses and witnesses for multiparticle entanglement will be explained. Lastly, some examples for applications of quantum entanglement are given, which are quantum teleportation, quantum cryptography and quantum metrology.

Having introduced the preliminaries, in the next chapter the new type of witnesses, based on the Schmidt decomposition of operators is discussed. First, we will explain how to construct them and further, we will show that they are indeed strictly stronger than the standard witnesses, which are based on the vector Schmidt decomposition. After that, two algorithms to improve given entanglement witnesses or find an entanglement witness that detects a certain target state are constructed. One is based on optimizing the operator Schmidt coefficients (OSC) and the other on optimizing the Schmidt operators (SO). Further, we will give an example where those algorithms are applied to a PPT entangled state. It is found that, starting with a completely random input, one can only find the best witness if both algorithms are applied. Moreover, we will show that the algorithms behave as expected and therefore can be generalized to multiparticle systems.

The next chapter deals with the generalization of the OSD-based witness to the multipartite case. First, its construction will be addressed. Then, we will adapt the two optimization algorithms to the multiparticle case. Starting with the optimization with respect to the operator Schmidt coefficients, we will show that the algorithm improves the fidelity witness quite well for many states. However, we will find that for W states the optimization does not work that well. Therefore, two ideas to modify the algorithm in order to find better results are introduced and it is found that indeed those modifications lead to better results for the W states. Furthermore, another optimization approach will be discussed. The second part of this chapter focuses on the optimization algorithm with respect to the Schmidt operators. After the adaption, it will be applied to the same example states as from the previous