

Andrew Peterson · Gregory Durgin

Transient Signals on Transmission Lines

An Introduction to Non-Ideal Effects and Signal Integrity Issues in Electrical Systems

Second Edition



Synthesis Lectures on Computational Electromagnetics

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Preface to the First Edition

The material that follows consists of modules on the topic of transient signals on transmission lines. Emphasis has been placed on aspects of the subject that have application to signal integrity and high-speed digital circuit design issues, including proper termination schemes to avoid impedance discontinuities, reactive and nonlinear loads, and an introduction to crosstalk. This material has formed the first part of the core undergraduate electromagnetic fields course at the Georgia Institute of Technology since 1999. Since transmission line transients have been de-emphasized in most current textbooks, including those that have been used at Georgia Tech during this time, this material was prepared to supplement traditional texts. With the exception of the material on crosstalk, the authors typically cover each chapter that follows in one 50 min class period.

Atlanta, GA, USA

Andrew Peterson Gregory Durgin

Preface to the Second Edition

After over a decade more of experience and feedback from teaching *Transient Signals on Time-Domain Transmission Lines*, we have made some substantial upgrades to the text for the second edition. Each chapter has been revised and updated with more examples and additional problem sets. New material was added to many of the chapters to integrate additional applications.

A new chapter on differential transmission lines was also added, as the authors felt this was an important topic for industry engineers that has been largely neglected in textbooks.

There are now video links in the reference section of each chapter that allow the reader to "sit in" on an actual lecture by the authors on the topic of the chapter. Thus, it is now possible to take an entire mini-course on the subject of time domain transmission lines at your own pace and place. Hopefully, all of these upgrades to the second edition allow the concepts and information to be transmitted to the reader with $\tau \approx 1!$

Atlanta, GA, USA July 2023 Andrew Peterson Gregory Durgin

Acknowledgements Over the years, the authors have assimilated ideas and perspectives from many of our teachers and colleagues. During the years that this material was being used at Georgia Tech, homework problems and other suggestions originating from others who taught ECE 3025 have been incorporated. We would especially like to acknowledge the contributions, direct and indirect, of I. M. Besieris, D. A. de Wolf, B. Degnan, P. W. Klock, P. E. Mast, N. N. Rao, W. T. Rhodes, P. G. Steffes, A. Hasan, and M. Swaminathan. We also acknowledge the influence of S. Rosenstark's 1994 book *Transmission Lines in Computer Engineering*, from which the crosstalk development is adapted.

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Introduction to Transmission Lines

1.1 Objectives

- Provide motivation by introducing the finite velocity of electrical signals and the remedy afforded by transmission line theory.
- Derive the transmission line equations using physical reasoning and Kirchhoff's laws.

1.2 Introduction

Greetings! This is a text about electric circuits and how they *really* work. It would be difficult for most of us to imagine life today without the convenience afforded by electric circuits, from household appliances to video entertainment devices to electronic ignitions in automobiles to computers at home and work. Electric circuits have changed in two major respects since the 1950s: They have shrunk dramatically in size, from the typical chassis housing a vacuum tube circuit (25 cm on a side) to the integrated circuits ubiquitous in modern "toys" (nanometers in size). As their size shrunk by many orders of magnitude, the electrical signals existing within them have increased similarly in frequency content by a similar factor. In the 1950s, most consumer goods used no more than kHz frequencies, although FM radio and VHF television involved frequencies on the order of 100 MHz; today personal communication devices such as smartphone handsets involve frequencies exceeding 10 GHz and the clock rates of personal computers exceed several GHz. This revolution in electronics was made possible by a property unique to integrated circuits: as transistor size decreases, performance increases and production cost decreases [4].

Because of these changes, the old approaches (Kirchhoff's laws) for analyzing and designing circuits do not always work: a new approach is needed. Actually, the approach is not

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really new, but its application to many circuit problems is. The approach we speak of is transmission line theory and was developed in the 1800s for telegraph applications! And, it has found continuous use since then for modeling circuits that work at high frequencies, such as those within that VHF TV your grandparents may have owned. It is therefore no surprise that transmission line theory is alive and well for applications involving GHz frequencies in wireless personal communication applications. But there is also a new stomping ground for transmission line concepts: high-performance digital circuits [2, 6]. As clock rates approach the GHz range, digital circuits seldom behave in the manner predicted by models appropriate for lower frequencies. Therefore there is a broader audience for transmission line theory among today's engineering students, and that audience now includes computer engineers as well as those interested in communications, RF and millimeter-wave circuits, and other applications such as power distribution networks, high-speed internet, and cable TV (CATV) systems.

Wait a minute, you ask! What do we mean when we say that Kirchhoff's laws do not "work"? Unfortunately Kirchhoff's laws work for linear, lumped parameter circuits circuits that can be represented with a finite number of elements such as resistors, capacitors, and inductors. In the real world, signals actually propagate through a continuum of space. Furthermore, the finite speed of electrical signal propagation becomes an important consideration. Electrical signals propagate at the speed of light, which has the value c = 299, 792, 458 m/s in a vacuum, and somewhat slower than that on a typical printed wiring board. Kirchhoff's laws neglect the finite velocity of an electrical signal, and therefore fail when the time delay or phase shift due to that finite velocity becomes significant. This is seldom a concern at lower frequencies found in audio amplifiers, household appliances, and many other devices. However, at higher data speeds and signal frequencies the effect can be pronounced.

If we think in the time domain, the critical parameter is the transit time (or time of flight) of a signal relative to the size of the circuit. The velocity of light $(3 \times 10^8 \text{ m/s})$ in air is roughly equivalent to a time delay of 1 ns per foot of travel. A time interval of 1 ns seems small, but not if the clock rate of the circuit is 1 GHz! Under these conditions, it may take an entire clock period for a signal to travel across a six inch printed wiring board, and part of the circuit may be a full clock period behind. This time delay is neglected by lumped-parameter circuit analysis that treats this pathway as a simple short circuit. Early integrated circuits involved TTL devices with internal delays of 15 ns or more, limiting the clock rates to lower levels and making the transit time of the signals negligible in comparison. Modern devices have improved to the point where transit delays are the limiting factor in digital circuit design. These transit delays are also known as latency. Transmission line theory explicitly includes this time delay, and therefore is applicable to these situations.

If we think in terms of the frequency domain, the key parameter is the wavelength of the signal relative to the size of the circuit. For a sinusoidal signal at frequency f, the wavelength λ is given by

$$\lambda = \frac{v}{f} \tag{1.1}$$

where v denotes the velocity of the signal in the medium of concern. For an electrical signal, this velocity is that of light. Note that as the frequency increases, the wavelength decreases. At 1 GHz, the wavelength in air is approximately 30 cm. One wavelength corresponds to 360° of phase shift. If the signal accumulates more than a small fraction of 360° of phase shift in traveling across a circuit, lumped-parameter circuit analysis no longer accurately describes the situation. Transmission line theory does.

Example 1.1: Frequency and Wavelength

Problem: As a general rule of thumb, transmission line effects become important if a circuit component becomes larger than 0.1λ , where λ is the wavelength of the excitation. Using this rule, decide which of the following situations require transmission line theory:

- a. A 60 Hz power line spans the 120-mile distance between Atlanta, GA and Greenville, SC. The velocity of propagation on the line is 8×10^7 m/s.
- b. A residential analog landline telephone line connects a home phone with a public digital network. The distance from the home to the nearest switch is 100 m. The highest major frequency component in transmitted speech is 2 kHz and the velocity of propagation on the wire is 5×10^7 m/s.
- c. A memory chip is connected to a microprocessor on a computer motherboard with 4 cm microstrip lines. The velocity of propagation is 1.2×10^8 m/s and the highest harmonic content of the binary signaling is 3 GHz.

Solution: These are fairly straight-forward calculations based on the relationship in Eq. (1.1). For the three examples given, this leads to the following answers:

- a. Using 1609.3 meters per mile, the 120 mile distance is 193,121 m. At 60 Hz, the wavelength is 1.33×10^6 m. Therefore, the distance is 0.145λ , just within the range where we need to employ transmission line theory. **Yes**.
- b. The 100m telephone line is 0.004λ for 2 kHz frequencies. No.
- d. The 4cm microstrip line is exactly 1λ at 3 GHz. Yes.

1.3 Electrical Properties of Materials

We should digress for a moment to make a couple of things more precise. Most situations that we will encounter will involve dielectric materials, which are described in terms of a parameter known as the relative permittivity ϵ_r (also called the dielectric constant). Magnetic materials (most often encountered in power distribution applications) are described by a relative permeability μ_r . The parameters ϵ_r and μ_r are unitless scale factors. The speed of light, or of any electrical signal, in a general material is given by

1			•	
Material	€r	Material	ϵ_r	
Air/Vacuum	1.0	Alumina	9.4	
A-35 Ceramic	5.6	Glazed Ceramic	7.2	
Gallium Arsenide	13.0	Germanium	13.0	
Glass epoxy	4.0	FR4	4.9	
Lucite	2.6	Mica	6.0	
Nylon	3.5	Plexiglas	2.6	
Polyethylene	2.3	Polyimide	3.5	
Polystyrene	2.6	Quartz	3.5	
Rexolite 1422	2.5	Silicon	11.8	
Silicon dioxide	3.9	Teflon	2.1	

Table 1.1 Relative permativities of assorted materials (ϵ_r at approximately 1 GHz)

$$v = \frac{2.998 \times 10^8}{\sqrt{\epsilon_r \mu_r}} \tag{1.2}$$

This velocity is always slower than that of light in air. Combining Eqs. (1.1) and (1.2), we see that the wavelength of a single-frequency signal in a medium characterized by ϵ_r and μ_r is given by

$$\lambda = \frac{1}{\sqrt{\epsilon_r \mu_r}} \frac{2.998 \times 10^8}{f} \tag{1.3}$$

where f is the frequency in Hertz and λ is the wavelength in meters. Table 1.1 provides dielectric constants for common materials. Note that, as suggested by the table heading, the relative permittivity of a material depends on the frequency of the signal of concern. This also implies that the velocity of propagation depends on the frequency, which contributes to a form of distortion known as dispersion for signals that have energy distributed over a wide band of frequencies.

There are other "non-ideal" effects that can also be taken into account using transmission line theory, including reflections and the associated reflection noise, crosstalk between closely-spaced traces, simultaneous switching noise due to the inductance in the power supply path connecting active drivers, etc. These topics will be considered in the chapters to follow. This aspect of circuit analysis is often referred to as signal integrity, since the desired signal is corrupted by these effects. In digital applications, non-ideal effects are usually thought of as additional noise introduced into the system.

A modern electrical device often involves the construction shown in Fig. 1.1, consisting of a chip located on a chip carrier or *package*, which in turn is attached to a printed wiring board (PWB) or printed circuit board (PCB). Signals must travel from the PWB through the package to the chip and back again, while power and electrical ground circuitry follows a similar meandering path. In a complex system such as a computer, various PWBs are linked

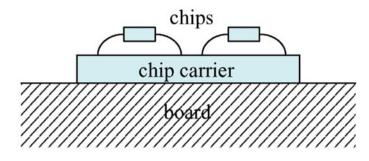


Fig. 1.1 Construction representative of many modern electrical devices. Chips may include lines and capacitors, as well as active devices like logic gates. The chip carrier contains transmission lines; the board may contain both lines and R, C, and L components

together with signals routed between them using cables of some form or a backplane. In some situations, a driver chip is located on one PWB while the receiver is located on another. The chip is much smaller than the PWB, and made from different materials, and therefore limiting factors differ in each of these environments. Furthermore, the chip may be connected to the package and board in one of several ways, including wirebonding (small individual wires connected between the chip pads and package leads) and ball grid arrays (metallic balls connected directly between the chip pads and the PWB). For accurate analysis under a variety of circumstances that do arise in practice, signals in each of these environments may need to be treated using transmission line theory.

Example 1.2: Frequency and Wavelength in Materials

Problem: Using the "10% of wavelength" rule-of-thumb, decide which of the following situations require transmission line theory:

- a. The radio hardware of a cellular tower is located on the ground and uses long coaxial cables to connect with the antennas at the top of 60 m towers. The operating frequency is 1920 MHz and the dielectric parameters of the coaxial cable are $\epsilon_r = 5.3$ and $\mu_r = 1$.
- b. A 25-foot ethernet cable transmits binary signals with 100 MHz of maximum frequency content between a router and a computer. The dielectric parameters of the cable are $\epsilon_r = 2.7$ and $\mu_r = 1$.

Solution: These calculations are also based on the relationship in Eq. (1.1). For the examples given, this leads to the following answers:

- a. Keeping in mind that $v_p = \frac{c}{\sqrt{\epsilon_r \mu_r}}$, the cell-tower coaxial line is 925 λ at 1920 MHz. Yes.
- b. At 100 MHz frequency content, the electromagnetic size of the 25-ft ethernet cable is 4.1λ . Yes.

1.4 Transmission Line Equations

So, what exactly is transmission line theory? To illustrate, consider the two cable crosssections shown in Fig. 1.2. This figure shows a coaxial cable (a very popular type of transmission line widely used in microwave measurements and CATV systems) and a parallel strip transmission line (which might approximate interconnects on certain PWBs or chips). When electricity flows on either of these cables, there is a physical movement of charge carriers (electrons) down one conductor and back on the other. This flow of current involves energy stored in the magnetic field associated with the current. This effect is equivalent to some series inductance. There is also some series resistance since the metal is not a "perfect" conductor of electricity, and therefore some of the electrical energy is converted to heat as the current flows. At the same time, equal and opposite charge is stored instantaneously on the two conductors, giving rise to energy stored in the electric field and some shunt capacitance. If the material separating the conductors is not a perfect insulator, there will also be some leakage current from one conductor to another, which we will model as shunt conductance.

Thus, we can consider an equivalent circuit for a short section of transmission line (say, of length Δz , where Δz is much smaller than a wavelength or the equivalent time of flight) to be that depicted in Fig. 1.3. The inductance and capacitance present in this equivalent circuit provide the time delay and phase shift lacking in the treatment of these transmission lines using conventional circuit theory (which would assume that they provide a direct connection

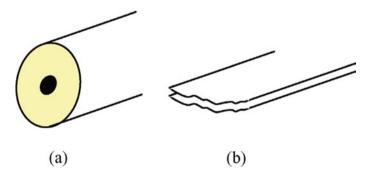


Fig. 1.2 Common transmission lines: a coaxial cable and b parallel conducting strips

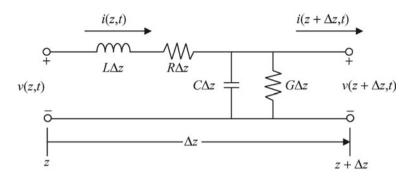


Fig. 1.3 General model for a transmission line segment, appropriate for small lengths Δz

from one end to the other). We can therefore analyze this equivalent lumped-parameter circuit segment using Kirchhoff's voltage and current laws.

In Fig. 1.3, the quantity $L\Delta z$ (H) is the total series inductance of the equivalent circuit, which depends on the inductance per unit length L (H/m). $C\Delta z$ (F) is the total capacitance, which depends on the shunt capacitance per unit length C (F/m). The total series resistance $R\Delta z$ and shunt conductance $G\Delta z$ depend on the resistance per unit length R (Ω /m) and the conductance per unit length G (S/m), respectively.

The application of Kirchhoff's voltage law to the equivalent circuit in Fig. 1.3 yields the equation

$$v(z + \Delta z, t) + L\Delta z \frac{\partial i}{\partial t} + R\Delta z i(z, t) = v(z, t)$$
(1.4)

which can be rewritten in the form

$$\frac{v(z + \Delta z, t) - v(z, t)}{\Delta z} = -Ri(z, t) - L\frac{\partial i}{\partial t}$$
(1.5)

In the limiting case as Δz tends to zero, this equation becomes

$$\frac{\partial v}{\partial z} = -Ri(z,t) - L\frac{\partial i}{\partial t}$$
(1.6)

n

An application of Kirchhoff's current law to the circuit in Fig. 1.3 produces

$$i(z + \Delta z, t) - i(z, t) = -G\Delta zv(z + \Delta z, t) - C\Delta z\frac{\partial v}{\partial t}$$
(1.7)

which, in the limiting case as $\Delta z \rightarrow 0$, yields

$$\frac{\partial i}{\partial z} = -Gv(z,t) - C\frac{\partial v}{\partial t}$$
(1.8)

Equations (1.6) and (1.8) are known as the *Transmission Line Equations*. In the past, these were called the *Telegrapher's Equations* in view of the fact that they were originally derived

in the 1800s for that application. We note in passing that these equations can also be derived directly from Maxwell's equations (the equations describing electromagnetic fields) applied to a specific transmission line geometry [1, 5].

Example 1.3: Transmission Line Effects on Circuit Traces

Problem: A circuit board trace runs through a material with relative permittivity $\epsilon_r = 12$ and relative permeability $\mu_r = 1$.

- a. What is the propagation delay through 1 cm of this material?
- b. If a sinusoidal signal with frequency 3 GHz is sent through the material, what is the phase shift, in degrees, experienced by the wave through 1 cm of this material?

Solution:

a. The propagation velocity is given by

$$v_p = \frac{c}{\sqrt{\epsilon_r \mu_r}} \cong \frac{3 \times 10^8}{\sqrt{12}} = 8.654 \times 10^7 \,\mathrm{ms}^{-1}$$

Therefore, the propagation delay is given by

$$1.155 \times 10^{-8} \text{ sm}^{-1} = 115.5 \text{ pscm}^{-1}$$

It follows that the delay through 1.00 cm is 115.5 ps.

b. The wavelength is given by

$$\lambda \cong \frac{3 \times 10^8}{\sqrt{12}(3 \times 10^9)} = 0.0289 \text{m} = 2.89 \text{ cm}$$

Since 1λ corresponds to a phase shift of 360° , a 1.00 cm distance corresponds to

$$\frac{1.00 \text{ cm}}{2.89 \text{ cm}} \times 360^{\circ} = 124.6^{\circ}$$

of phase shift.

1.5 Conclusion

Transmission lines introduce a number of potential detriments to signal transport. These include latency, reflections, ringing, frequency selectivity (also called dispersion), load transformation (for sinusoidal signals), and loss. Many of these effects will be studied in subsequent chapters.

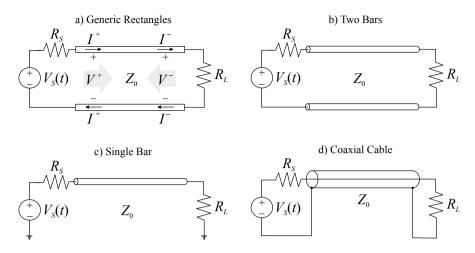


Fig. 1.4 Common conventions for drawing transmission line systems. A single transmission line is often drawn as two parallel bars or wires, as in **a** and **b**. Alternatively, it can be drawn as a single wire with the return path indicated by connections to a common ground (**c**). In some cases, it is drawn as a coaxial cable (**d**). The primary (left) side is usually referred to as the *source* or *generator* and the right side is the *load*

In the following chapter, we will study properties of the solution of the Transmission Line Equations. Before closing, we observe that there are several conventions for sketching transmission lines, and we illustrate those in Fig. 1.4.

Problems

- 1. Short Answer Questions:
 - a. When the transit time of a transmission line causes delays in pulses that threaten the synchronization of a high speed digital circuit, we call this effect ______.
 - b. When transmission line reflections cause data-distorting echos, we call this effect
 - c. In a ______transmission line, there is no per-unit-length resistance or conductance.
 - d. A transmission line comprises per-unit-length series _____ and shunt capacitance.
- 2. A repetitive square wave travels on a transmission line with velocity of propagation 1.3×10^8 m/s. The fundamental frequency of this waveform is at 100 MHz, with significant harmonics at 300 MHz and 500 MHz. What are the wavelengths corresponding to these three frequencies?
- **3.** A trace runs through a material with $\epsilon_r = 4.9$ and $\mu_r = 1$.

- a. What is the propagation delay through 4 cm of this material?
- b. If a sinusoidal signal at frequency f = 100 MHz is sent through this material, how many degrees of phase shift arise over the 4 cm length?
- **4.** A pair of wires is embedded in a solid block of fiberglass, which is non-magnetic and has a relative permittivity of 4.0. Estimate the velocity of propagation for the line.
- 5. Acoustic transmission lines behave in ways that are analogous to electrical transmission lines, including the relationship of Eq. (1.1) relating speed to wavelength and frequency. Use internet research to estimate the velocity of propagation of a vibrating wave on the C-string of a cello assuming that the resonance on the string occurs at $\lambda/2$.
- **6.** A chip company announced a new approach for transmitting data within a computer. By placing the edge of one integrated circuit chip directly in contact with another, and avoiding the connection through a printed circuit board, they proposed to move data at rates up to 100 times faster than by conventional means. Using the principles of transmission line theory, explain why it might be advantageous to connect chips in this fashion.
- 7. One wire in a long, high-speed USB cable connects a 5V digital logic source with 100Ω of source impedance to a 400 Ω -load digital input pin of a printer. The conductive sheath of the cable connects the grounds of the devices at both ends. Draw a circuit schematic of this electrical scenario treating the digital wire and ground sheath as a transmission line pair.
- 8. List 5 detrimental effects that can result from propagation on transmission lines.

References

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