Peter Grundke

Integrated Market and Credit Portfolio Models

nbf neue betriebswirtschaftliche forschung
Band 361

Peter Grundke

Integrated Market and Credit Portfolio Models

Risk Measurement and Computational Aspects

With a foreword by
Univ.-Prof. Dr. Thomas Hartmann-Wendels

GABLER EDITION WISSENSCHAFT

Bibliographic information published by Die Deutsche Nationalbibliothek
Die Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at <http://dnb.d-nb.de>.

Habilitationsschrift Universität zu Köln 2006

1st Edition 2008

Editorial Office: Claudia Jeske

Gabler-Verlag is a company of Springer Science+Business Media.
www.gabler.de

Cover design: Regine Zimmer, Dipl.-Designerin, Frankfurt/Main
Printed on acid-free paper
Printed in Germany

ISBN 978-3-8349-0875-9

Preface

Banks are exposed to various kinds of risks; among them are credit default risks, market price risks and operational risks the most important ones. Aggregating these different risk exposures to a comprehensive risk position is an important, yet challenging and up to now unresolved task. Banks' current state of the art in risk management is still far away from achieving a fully integrated view of the risks they are exposed to. This shortfall traces back to both, to conceptual problems of constructing an appropriate risk model and to the computational burden of calculating a loss distribution.

The approach presented in this book takes credit default risk as a starting point. By integrating market risks, a general credit risk model is constructed that comprises the standard industry credit risk models as special cases. Within the framework of this general credit risk model, the effects of simplifying assumptions that are typical for standard credit risk models can be analyzed. Important insights gained by this analysis are that neglecting market price risks and losses given default correlated to default rates can cause a significant understatement of value at risk figures.

While solving the conceptual problems of designing an integrated risk model has its own merits for scientific purposes, it is of limited use for practical applications as long as the computational problems remain unsolved. As the value at risk of a complex credit risk model cannot be determined analytically, simulation techniques that are both, sufficiently precise and not too time-consuming, are needed. Fourier transformations and importance sampling are two simulation procedures that proved to be successful in cutting down the computational burden in pure credit risk models and pure market risk models, respectively. A natural approach therefore is to analyze the power of these procedures for an integrated risk model. Unfortunately, both procedures loose much of their advantages when they are applied outside of simplified model settings. As a result, the necessity of developing improved simulation techniques becomes evident.

The analysis presented in this book paves the way for the development of more powerful risk models. The results are important for both, for academics interested in designing more satisfying risk models and for practitioners in charge of solving the computational problems associated with the implementation of risk models.

Cologne, December 2007

Thomas Hartmann-Wendels

Acknowledgments

This book was written while I was a scientific assistant of Professor Dr. Thomas Hartmann-Wendels at the Chair of Banking at the University of Cologne. It was accepted as my habilitation thesis by the Faculty of Management, Economics and Social Sciences at the University of Cologne in October 2006.

My special thanks go to Professor Dr. Thomas Hartmann-Wendels for providing an excellent research environment, his permanent support, and for giving me the necessary academic freedom with respect to this work. I also wish to thank Professor Dr. Dieter Hess for serving as a referee for my habilitation thesis.

To finish the project 'Habilitation', the help of many people is needed. Now, it is a pleasure for me to thank them. As already during the work for my PhD thesis, Wolfgang Spörk continued to do an excellent job in helping to preserve the necessary motivation. Stefanie Martens undertook the cumbersome task of checking the manuscript for typos and improving the English writing style. Thanks to both of them.

For the success of this work, it is crucial to have the necessary support and encouragement in the private background. I thank my parents, Renate and Bernward Grundke, as well as Hannelore and Horst Martens for giving me this support and encouragement. Of course, the largest part of the support was provided by my wife Kristina who had to bear periods of my mental (and physical) absence. Thanks for your patience, understanding and permanent confidence that I am on the right way. Hence, this book is dedicated to Kristina and our new-born star Lennard.

Cologne, December 2007

Peter Grundke

Contents

List of Tables

List of Figures

List of Acronyms

AR(2)	second-order autoregressive process
FFT	Fast Fourier Transformation
IRB	internal ratings-based
IS	importance sampling
LGD_n	loss given default of obligor n
OTC	over-the-counter
QRN	quasi random numbers
VaR	Value-at-Risk
wtr	without transition risk

List of Symbols

b	product $\delta^{\mathrm{T}}\mathrm{C}$ in the delta-gamma approximation $L^{wtr,\Delta,\Gamma}(\mathrm{X},H)$ of the credit portfolio loss at the risk horizon H without transition risk
b_m	component of the vector b
$b_{s,c}$	factor weight of factor Z_c in sector s in the Credit Portfolio View model
$\mathbb{C}$	complex numbers
C	matrix product of the matrices $\tilde{\mathrm{C}}$ and U
C	number of systematic credit risk factors
C_n	collateral value of obligor n
$\tilde{\mathrm{C}}$	matrix of the Cholesky decomposition of the covariance matrix Σ_{X}
$C(\cdot)$	price of an European call option with counterparty risk
$\tilde{C}(\cdot)$	price of an European call option without counterparty risk
$C^{\Delta,\Gamma}(\cdot)$	delta-gamma approximation of a price of an European call option with counterparty risk
$\tilde{C}^{\Delta,\Gamma}(\cdot)$	delta-gamma approximation of a price of an European call option without counterparty risk
c	convexity of a bond
c_n	loss incurred by a default of obligor n in a default mode model
D	number of simulation runs (also: number of defaults in the simple contagion model)
d	modified duration of a bond
d_1, d_2	parameters of the price of an European call option
E	number of exposure buckets
F_j	face value of a zero coupon bond issued by an obligor whose initial rating is j
F_n	face value of the zero coupon bond issued by obligor n
$F_y(\mathrm{z},\mathrm{x})$	exponent of the upper boundary of the optimization problem which has to be solved for finding the IS means for the systematic risk factors
$f(\cdot)$	probability density function
$f_{n,i,k}(\cdot)$	conditional probability of obligor n to migrate from rating grade i to k within the risk horizon
G	joint distribution of the systematic credit and market risk factors
G^C	joint distribution of the systematic credit risk factors Z
G_i	group $i \in \{1,2\}$
G^M	joint distribution of the systematic market risk factors X
$g(\cdot)$	probability density function
H	risk horizon
h	step size for the Simpson integration rule
$h(\cdot)$	function of some random variable
$h_{n,i,k}(\cdot)$	conditional IS probability of obligor n to migrate from rating grade i to k within the risk horizon

I	identity matrix
$\mathrm{Im}(\cdot)$	imaginary part of a complex number
i	credit quality (also: imaginary unit of the complex numbers)
i_n	instrument issued by obligor n
K	number of credit qualities (ratings)
k	credit quality
$L(H)$	credit portfolio loss at the risk horizon H
$L_{(d)}(H)$	d^{th} order statistic of the credit portfolio loss $L(H)$
$L^{wtr}(\mathrm{X},H)$	credit portfolio loss at the risk horizon H without transition risk
$L^{wtr,\Delta,\Gamma}(\mathrm{X},H)$	delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ at the risk horizon H without transition risk
$L_n(H)$	loss of instrument n at the risk horizon H
$L_n^{wtr}(\mathrm{X},H)$	loss of instrument n at the risk horizon H without transition risk
$L_{n,k}(H)\vert\mathrm{X}$	conditional loss of instrument n at the risk horizon H when its issuer is in rating class k
l_n	exposure of obligor n in a default mode model
$l(\mathrm{S})$	likelihood ratio
M	number of market risk factors
$\mathbb{N}$	natural numbers
N	number of instruments (obligors, issuers) in the credit portfolio
N_i	number of obligors in group $i \in \{1,2\}$
$N(\mu,\sigma^2)$	normal distribution with mean μ and variance σ^2
n	identification of an obligor (also: number of grid points for the Gauss-Legendre integration rule)
n_j	number of bonds whose issuers have the initial rating j
P	real-world probability measure
$\tilde{P}$	risk-neutralized pricing measure (also: IS probability measure)
$\mathrm{P_n}$	vector of parameters relevant for the instrument issued by obligor n
$\tilde{P}_\theta(\cdot)$	conditional IS transition probability measure with twisting parameter θ
$\mathrm{P}_{\mu(\mathrm{S})}$	IS probability measure for the systematic credit risk factors Z, which depends on the realization of the market risk factors S
p	confidence level of a Value-at-Risk (also: probability of a percentile)
$p_n(\cdot)$	price of a credit-risky instrument issued by obligor n
$\tilde{p}(\cdot)$	price of a default risk-free zero coupon bond
$Q(\mathrm{S})$	quadratic approximation (without constant) of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ at the risk horizon H without transition risk
q_{ik}	unconditional probability to migrate from rating grade i to k within one year
q_K^n	unconditional one-year default probability of obligor n
$\mathbb{R}$	real numbers
$\mathbb{R}_+$	non-negative real numbers
$\mathbb{R}^C$	$C-$dimensional vector of real numbers
R_k^i	asset return threshold for a rating transition from i to k
R_n	asset return of obligor n

$\mathrm{Re}(\cdot)$	real part of a complex number
$R(X_r, H, T)$	stochastic risk-free spot yield for the time interval $[H, T]$
$R(\infty)$	return of a risk-free zero coupon bond with infinite time to maturity
$r(t)$	(risk-free) short rate at time t
$\mathrm{S} = (S_1, \ldots, S_M)$	vector of independent standard normally distributed random variables
$S_k(H, T)$	stochastic credit spread of rating class k for the time interval $[H, T]$
s	segment in the Credit Portfolio View model
T_n	maturity date of the zero coupon bond issued by obligor n (without index n: identical maturity date for all obligors)
T_n^C	expiration date of an European call option issued by counterparty n (without index n: identical expiration date for all obligors)
t	point of time or period of time (also: argument of a characteristic function, grid point)
$t_\nu(\cdot)$	cumulative density function of the $t-$distribution with ν degrees of freedom
$t_\nu^{-1}(\cdot)$	inverse cumulative density function of the $t-$distribution with ν degrees of freedom
U	orthogonal matrix whose columns are the eigenvectors of the matrix $0.5\tilde{\mathrm{C}}^{\mathrm{T}}\Gamma\tilde{\mathrm{C}}$
U_n	random variable which drives the stochastic collateral value of obligor n
W	centrally χ^2-distributed random variable with ν degrees of freedom
$W_r(t)$	standard Brownian motion of the (risk-free) short rate process at time t under the real-world probability measure P
X_n	exercise price of an European call option issued by obligor n
X_r	(risk-free) interest rate factor
$X_r^{(d)}$	realization of the (risk-free) interest rate factor X_r in the d^{th} simulation run
X_{r,η_0}	(risk-free) interest rate factor, which is relevant for the price of the underlying of an European call option issued by a counterparty with initial rating η_0
$\mathrm{X} = (X_1, \ldots, X_M)$	stochastic vector of systematic market risk factors at the risk horizon H
$\mathrm{X}^0 = (X_1^0, \ldots, X_M^0)$	vector of systematic market risk factors at $t = 0$
x_r	realization of the (risk-free) interest rate factor
Y_n	random variable which is independent from the random variable U_n, which drives the stochastic collateral value of obligor n (also: default indicator of obligor n)
Z	systematic credit risk factor
$Z^{(d)}$	realization of the systematic credit risk factor Z in the d^{th} simulation run
$\mathrm{Z} = (Z_1, \ldots, Z_C)$	stochastic vector of systematic credit risk factors
z	realization of the systematic credit risk factor

Greek Symbols

α	parameter of a beta-distributed random variable
α_n	sensitivity in the linear factor representation of the random variable U_n, which drives the stochastic collateral value of obligor n (without index n: identical sensitivity for all obligors)
$\alpha_{p\%}(L(H))$	$p\%-$percentile of the credit portfolio loss variable $L(H)$
$\hat{\alpha}_{p\%}(L(H))$	estimator of the $p\%-$percentile of the credit portfolio loss variable $L(H)$
β	parameter of a beta-distributed random variable
β_n	sensitivity in the linear factor representation of the random variable U_n, which drives the stochastic collateral value of obligor n (without index n: identical sensitivity for all obligors)
$1-\beta$	confidence level of a confidence interval for percentile estimators
Γ	matrix of the second derivatives of $L^{wtr}(\mathrm{X},H)$ with respect to the market risk factors X
$\Gamma_{n,m}$	second derivative of $L^{wtr}(\mathrm{X},H)$ with respect to the market risk factors X_n and X_m
$\Gamma(\cdot)$	Gamma function
γ	probability of the mixing parameter λ of a discrete mixture of normal distributions
γ_n	sensitivity in the linear factor representation of the random variable U_n, which drives the stochastic collateral value of obligor n (without index n: identical sensitivity for all obligors)
δ	vector of the first derivatives of $L^{wtr}(\mathrm{X},H)$ with respect to the market risk factors X
δ_n	recovery rate of obligor (issuer) n (also: first derivative of $L^{wtr}(\mathrm{X},H)$ with respect to the market risk factor X_n)
ε_n	idiosyncratic risk of obligor n
η_n	idiosyncratic collateral value risk of obligor n
η_0^n	rating of obligor n at time $t=0$
η_H^n	rating of obligor n at the risk horizon H
θ	mean level of the short rate process (also: twisting parameter)
θ^{wtr}	twisting parameter of the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk
θ_y	optimal twisting parameter corresponding to the argument y in the excess probability $P(L(H)>y)$
$\theta_{y^*}^{wtr}$	optimal twisting parameter of the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk corresponding to the argument y^* in the excess probability $P(L(\mathrm{X},H)^{wtr}>y^*)$
$\theta_y(\mathrm{Z},\mathrm{X})$	optimal conditional twisting parameter corresponding to the argument y in the excess probability $P(L(H)>y)$
ϑ	angle in the Euler formula for a complex number
ϑ_s	noise term of sector s in the CreditPortfolioView model

κ	mean reversion parameter of the short rate process
Λ	diagonal matrix containing the eigenvalues of the matrix $0.5\tilde{\mathrm{C}}^{\mathrm{T}}\Gamma\tilde{\mathrm{C}}$
λ	market price of interest rate risk (also: mixing parameter of a discrete mixture of normal distributions)
λ_m	eigenvalue of the matrix $0.5\tilde{\mathrm{C}}^{\mathrm{T}}\Gamma\tilde{\mathrm{C}}$
$\lambda(Z)$	sum of the conditional default probabilities
μ	vector of IS means of the systematic risk factors
μ_δ	mean of the beta-distributed recovery rate
$\mu_{h(X)}$	mean of $h(X)$
$\hat{\mu}_{h(X)}$	estimator of the mean of $h(X)$
$\hat{\mu}_{h(X)}^{\tilde{P}}$	IS estimator of the mean of $h(X)$
μ_k	credit spread mean of rating class k
μ_n	mean of the random variable U_n, which drives the stochastic collateral value of obligor n (without index n: identical mean for all obligors)
μ_c^{Z}	IS mean of the systematic credit risk factor Z_c under the IS distribution
μ_m^{X}	IS mean of the market risk factor X_m under the IS distribution
μ_k^{cont}	credit spread mean of rating class k in the simple contagion model
$\mu_{X_r}^{2\,step}$	IS mean of the interest rate factor X_r for the two-step-IS technique
$\mu_{X_r}^{3\,step}$	IS mean of the interest rate factor X_r for the three-step-IS technique
$\mu_Z^{2\,step}$	IS mean of the systematic credit risk factor Z for the two-step-IS technique
$\mu_Z^{3\,step}$	IS mean of the systematic credit risk factor Z for the three-step-IS technique
$\mu_m(\theta^{wtr})$	mean of the market risk factor S_m under the IS distribution for the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk
$\mu(\theta^{wtr})$	mean vector of the market risk factors S under the IS distribution for the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk
$\mu(\mathrm{S})$	vector of IS means of the systematic credit risk factors Z, which depends on the realization of the market risk factors S
$\mu^{\mathrm{IS}}(y)$	mean vector of the systematic risk factors under the IS distribution, which depends on the argument y in the probability term $P(\Pi(H) < y)$ that is estimated
$\mu^{i,\mathrm{IS}}(y)$	mean vector of the systematic risk factors under the IS distribution when the imaginary part of the characteristic function is estimated
$\mu^{r,\mathrm{IS}}(y)$	mean vector of the systematic risk factors under the IS distribution when the real part of the characteristic function is estimated
ν	number of degrees of freedom of a $t-$distribution
$\Pi(H)$	value of the credit portfolio at the risk horizon H
$\Pi(H)^{\Delta,\Gamma}$	delta-gamma approximation of the value of the credit portfolio at the risk horizon H
ρ_R	asset return correlation
$\rho_{R,i}$	asset return correlation in group $i \in \{1,2\}$

$\rho_S^{k,j}$	correlation between the credit spreads of rating class k and j
$\rho_{U,R}$	correlation between the random variable U_n, which drives the stochastic collateral value of obligor n, and the asset return of obligor n
$\rho_{X_r,R}$	correlation between the interest rate factor X_r and the asset returns
$\hat{\rho}_{X_r,R}$	modified correlation between the interest rate factor X_r and the asset returns when the latter ones are modeled by a $t-$distribution
$\rho_{X_r,S}$	correlation between the interest rate factor X_r and the credit spreads
$\rho_{Z,S}$	correlation between the systematic risk factor Z and the credit spreads
$\hat{\rho}_{Z,S}$	modified correlation between the systematic risk factor Z and the credit spreads when the asset returns are modeled by a $t-$distribution
$\rho_{Z_i,S}$	correlation between the systematic risk factor Z_i ($i \in \{1,2\}$), which is part of a discrete mixture of normal distributions, and the credit spreads
Σ	covariance matrix of the systematic risk factors
Σ_{X}	covariance matrix of the market risk factors X
$\Sigma(\theta^{wtr})$	covariance matrix of the market risk factors S under the IS distribution for the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk
σ_k	credit spread volatility of rating class k
σ_n	standard deviation of the random variable U_n, which drives the stochastic collateral value of obligor n (without index n: identical standard deviation for all obligors)
σ_r	volatility parameter of the (risk-free) short rate process
σ_δ^2	variance of the beta-distributed recovery rate
$\sigma^2(Z_i)$	variance of the systematic risk factor Z_i ($i \in \{1,2\}$), which is part of a discrete mixture of normal distributions
$\sigma_{mn}^2(\theta^{wtr})$	covariance of the market risk factors S_m and S_n under the IS distribution for the delta-gamma approximation of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ without transition risk
$\sigma(\Pi(H))$	standard deviation of the credit portfolio value at the risk horizon
τ_n	default time of issuer n
υ	parameter of the price of an European call option
$\Phi(\cdot)$	cumulative density function of the standard normal distribution
$\Phi^{-1}(\cdot)$	inverse cumulative density function of the standard normal distribution
$\phi(\cdot)$	density function of the standard normal distribution
$\varphi_X(\cdot)$	characteristic function of a random variable X
$\varphi_{\Pi(H)\mid \mathrm{Z},\mathrm{X}}(\cdot)$	conditional characteristic function of the credit portfolio value $\Pi(H)$
$\psi_{L(H)}(\theta)$	cumulant generating function of the credit portfolio loss $L(H)$
$\psi_{L(H)\mid \mathrm{Z},\mathrm{X}}(\theta)$	conditional cumulant generating function of the credit portfolio loss $L(H)$
$\psi_Q(\theta^{wtr})$	cumulant generating function of the quadratic approximation $Q(\mathrm{S})$ of the credit portfolio loss $L^{wtr}(\mathrm{X},H)$ at the risk horizon H without transition risk
$\omega_{n,c}$	factor weight of factor Z_c for obligor n in the CreditRisk$^+$ model

Chapter 1

Introduction

1.1 Motivation

Banks are exposed to many different risk types due to their business activities. Among these risk types are credit risk, market risk, operational risk, and business risk. The task of the risk management division is to measure all these risks and to determine the necessary amount of economic capital that is needed as a buffer to absorb losses associated with each of these risks. Most frequently, economic capital is understood as a Value-at-Risk (VaR) number. Thus, it is the amount of capital needed to absorb unexpected losses within a given time period up to a specified probability.

Predominantly, the necessary amount of economic capital is determined for each risk type separately. That is why the problem arises how to combine these various amounts of capital to a single number. Within the so-called building-block approach stipulated by the regulatory authorities, the amount of regulatory capital that banks have to hold for the different risk types are just added. This is a quite conservative approach because it ignores diversification effects between the risk types. Internal models, which can capture the diversification effects between different (market) risk types, are only allowed for determining the regulatory capital for the market risk of the trading book (see *Basel Committee on Banking Supervision* (1996)).[1] As a

[1] FX risks represent an exception in so far as internal models are allowed for determining the regulatory capital of this risk type for positions in the trading book as well as in the banking book.