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Preface

Fractional calculus and fractional control are one of the rapidly developing research areas in current science and engineering. Due to some advantageous properties of fractional calculus itself, fractional-order models have significant research and applications in various fields, such as physics, biology, and control theory. However, like anything else, fractional calculus has its imperfections. So far, fractional calculus lacks the intuitive simplicity of explanation that integer-order calculus offers. Furthermore, at the application level, fractional-order models often exhibit complex dynamic behaviors, which can be stable, unstable, or even chaotic. This complexity poses challenges for the further development of fractional-order theory.

Therefore, in the face of these challenges, there is an urgent need for research into control theories and methodologies for fractional-order systems to ensure their stability and facilitate practical applications. This is the primary motivation behind the publication of this book. This book primarily discusses several classes of typical fractional-order ordinary differential systems and fractional-order partial differential systems, such as fractional-order chaotic system, fractional-order mathematical model in biology, and fractional-order reaction-diffusion system. Furthermore, this book also explores the application of control methods such as sliding mode control and feedback control for these fractional-order systems. This book can be used by researchers to carry out studies on stability and control theory of fractional systems.

Here, I would like to express my gratitude to the other co-authors of this book. Without their contributions and involvement, this book would not have come to fruition. I would also like to thank Dr Na Zhang, Dr Hui Fu, and Ying Li for their contributions to the research outcomes of this book. Thanks to all the editors at Springer Publishing for their hard work.

The contents of the book are divided into three parts, and they are organized as follows. Chapter 1 provides a brief introduction to the research background of this book, as well as several types of fractional-order systems, including fractional-order ordinary differential systems, fractional-order partial differential systems, and fractional-order mathematical models in biology. Chapters 2–5 investigate synchronization and stability issues of various fractional-order ordinary differential systems, along with sliding mode control design. Chapters 6–8 address the stability and synchronization problems of fractional-order reaction-diffusion control systems using sliding mode control and feedback control methods. Chapters 9 and 10 research the dynamic behavior of fractional-order mathematical models in biology with functional response functions.

Due to the limitations of the author's knowledge, there may be some shortcomings and errors in this book. We sincerely hope and welcome readers to provide criticism and suggestions for this book.

Weihai, SD, China Yonggui Kao
April, 2023

Contents

Chapter 1
Introduction

1.1 Background

Fractional-order (FO) calculus has developed for more than 300 years, but it has not attracted much attention until recent decades. Compared with the classical integer-order derivative, it is known that the fractional derivative has better performance in simulating long-memory processes and materials, abnormal diffusion, long-range interactions, long-term behaviors, power laws, allometric scaling laws, and so on [1]. Up to now, fractional calculus has become a hot research topic due to its ability to describe fractal geometry, power law phenomena, memory characteristics, and other related processes [2, 3]. Fractional calculus theory and methods are widely employed in fields such as physics [4], viscoelastic theory [5], control science [6–16], diffusion phenomenon [17–20], biology [21–25], neural networks [26–31], engineering and technology [32–36] as well as scientific computing [37–39].

While fractional calculus has begun to make its mark in several domains, showcasing distinct advantages over integer-order models, its applications still face significant challenges. On one hand, fractional-order models are often employed to simulate complex nonlinear phenomena, which are frequently accompanied by modeling uncertainties and various external disturbances in real-world scenarios. Consequently, fractional-order models tend to be exceedingly complex nonlinear systems. On the other hand, fractional-order operators themselves exhibit weak singularity, long-term behaviors, nonlocality, among other characteristics, and considering that fractional-order systems are infinite-dimensional systems, these often lead to the manifestation of intricate dynamical behaviors, making the utilization of such systems, which are difficult to predict and may be unstable, a daunting task in practical applications. Considering that most real-world applications require a stable and reliable system, it is essential to develop and study the control of fractional-order systems in the face of the challenges mentioned above.

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Next, we will provide a brief introduction to the main types of fractional-order systems studied in this book.

1.2 Fractional Chaos Systems

Chaos is a complex phenomenon that is unique to nonlinear systems. It is a kind of motion with internal randomness, local instability, and overall nonperiodicity generated by deterministic systems that are extremely sensitive to the initial conditions. Chaotic motion blurs the boundary between deterministic and stochastic motion and provides a new way of thinking for analyzing various natural phenomena and even sheds insight on some basic concepts of human understanding of nature. The existence of chaos in nature and human society has been widely accepted, and how to use the results of chaos theory to solve practical problems in the real world has become one of the important issues in the development of nonlinear science. In recent years, chaos science has become a popular research topic at home and abroad, and its research and applications are rapidly expanding in many fields, such as nuclear energy systems, biomedical engineering, laser research, secure communication, and information technology.

With the emergence of fractional electronic circuits and the development of fractional calculus theory, the study of fractional chaos has become a hot research topic in the field of fractional calculus [40, 41]. Fractional systems provide a better intrinsic nature and accuracy than traditional integer systems in describing real-world physical phenomena. Fractional systems are dynamic systems containing fractional derivatives and fractional integrals, and many systems in physics and engineering can be effectively modeled as fractional systems. Compared with integer chaotic systems, the chaotic attractors of fractional chaotic systems are more complex, and past research on the stability and control of fractional differential equations and fractional nonlinear systems has resulted in a large number of control strategies for fractional chaotic systems. Chaos has been found in numerous fractional nonlinear systems, such as fractional Chua circuits [42], fractional Lorenz systems [43], fractional Chen systems [44], fractional Liu systems [45], etc. Fractional systems are beneficial for the use of chaotic systems in image encryption and promoting the steady development of chaos theory. Therefore, it is important to study fractional chaotic systems effectively. For fractional chaotic systems, the control problem has gradually received extensive interest from researchers in the field of fractional control [46, 47], and various control methods have been proposed based on the stability theory of fractional differential equations [48, 49].

1.3 Fractional Reaction–Diffusion Systems

Since the concept of reaction–diffusion equations was proposed, the research of reaction–diffusion equations has been paid increasing attention by scholars in several fields because a large number of physical, biological, and chemical models in the real world can be summarized as reaction–diffusion equations. Therefore, it is of great practical significance to study the reaction–diffusion equation [50–53].

Fractional-order reaction–diffusion systems are distributed parameter systems. In practical applications, the dynamical characteristics of many systems are not only time-dependent but also space-dependent, so they cannot be modeled by centralized parametric systems but must be described by fractional parametric systems. A distributed parameter system is an infinite-dimensional dynamical system described by a partial differential equation, an integral equation, a generalized differential equation, or a differential equation in an abstract space. The inputs, outputs, states, and parameters of a distributed parameter system are functions of time and space variables and can therefore more accurately describe objects whose state space is spatiotemporally distributed. For example, distributed parameter systems are widely used in engineering control processes such as large heating furnaces and elastic vibration systems, in chemical processes in nuclear reactor control systems, and physical processes such as electromagnetic fields. Partial differential equations are an effective tool for describing distributed parameter systems, and the control methods for such systems can be divided into distributed control and boundary control. Distributed control is a control method in the spatial domain, which requires a large number of actuators and has good control performance. Boundary control only applies control signals at the system boundary, which is easier to implement in some application scenarios. Currently, scholars have proposed many control methods for integer-order distributed parameter systems, such as sliding mode control [54, 55], pinning control [56, 57], event-triggered control [58, 59], H_∞ control [60], etc. However, due to the weak singularity and nonlocality of fractional-order derivatives themselves, the stability studies on fractional-order distributed parameter systems are far more complicated and challenging than the stability and control theory in the integer-order sense. This has led to the fact that some control theory for integer-order systems cannot be directly extended to fractional-order systems. For this reason, fractional-order control has attracted much attention from scholars. The research on the stability and control of fractional-order distributed parameter systems is still in the early stage. In particular, academic results on control methods, stability, and synchronization of fractional-order reaction–diffusion systems are still very limited, and many issues need to be explored in more depth.

1.4 Fractional Biomathematical Models

With the development of fractional calculus theory, fractional differential equations are widely used in many fields, such as biomathematics [61–63], cybernetics [64–66], physics [67–69], engineering [70, 71], hydrology [72, 73], medicine [74–76], and so on. The introduction of fractional differential equation solves the problem of time memory, which makes the fractional ecosystem more reasonable than the ordinary differential ecosystem. On most occasions, fractional differential equation models seem to be more in line with the actual phenomenon than integer-order models. This is because fractional derivative and integral can describe the inherent memory and genetic characteristics of various materials and processes in most biological systems.

When we use biomathematical models to study the survival status of the biological population, the relationship between two related biological populations was summarized as follows: predator–prey [77–79], competitive [80], reciprocal cooperative [81, 82], and so on. The research on the ecosystem of two species mainly focuses on the existence and uniqueness of solutions, uniform boundedness of solutions, global attractiveness of solutions [83, 84], stability of equilibrium points [85], periodic solution [86], and so on. Many scholars have done a great deal of research on the ecosystem of two species [87, 88]. However, when there are more than two species in the ecosystem, the relationship between populations becomes complex and diverse. And the study of the three species ecological model becomes relatively complex. In reference [89], there listed 34 kinds of reasonable Volterra models of three species. Many scholars have been concerned about the study of three species food chain model[90–92] for a long time because the food chain relationship is widespread and important in the ecosystem. In addition to the existence and uniqueness of solutions, uniform boundedness of solutions, and stability of equilibrium point, the ecosystem of three species often has complex dynamic behaviors such as bifurcation [93–95] and chaos [96]. The study on fractional-order biological systems involves the existence of solutions [97, 98], boundedness of solutions, stability of equilibrium point [99, 100], bifurcation [101], and chaos [102]. It is different from the integer-order ecosystem that the chaos may occur in ecosystem with orders less than 3 [103].

1.5 Organization of the Book

This book focuses on the control, stability, and synchronization of fractional-order systems. The structure of the book can be summarized as follows:

Chapter 1 introduces the system description, background knowledge, and the motivation for the book.

Chapter 2 discusses the Lyapunov stability analysis of general fractional differential systems (GFDSs) and the adaptive sliding mode control (ASMC) of uncertain general fractional chaotic systems (UGFCSs) with uncertainty and external disturbances. The chapter presents the existence and uniqueness of solutions, the Lyapunov stability criterion for GFDSs, and establishes general fractional integral type sliding surfaces and reaching law. Based on the proposed stability criterion, the chapter demonstrates that the states of UGFCSs can reach the sliding surface in finite time and asymptotically converge to zero along the sliding surface.

Chapter 3 employs two ASMC strategies to achieve finite-time synchronization of uncertain general fractional unified chaotic systems (UGFUCSs) when uncertainty and external disturbances exist. The chapter introduces the general fractional unified chaotic system (GFUCS), which can transition from the general Lorenz system to the general Chen system, and the general kernel function could compress and extend the time domain. The chapter applies two ASMC methods to achieve finite-time synchronization of UGFUCSs, where the system states arrive at sliding surfaces in finite time.

Chapter 4 is dedicated to exploring the finite-time (FET) synchronization problem of time-varying delay fractional-order (FO) coupled heterogeneous complex networks (TFCHCNs) with external interference via a discontinuous feedback controller.

Chapter 5 focuses on the Mittag–Leffler synchronization problem of fractional-order memristor-based neural networks (FOMNNs) with time delays via a feedback controller and an adaptive controller, respectively.

Chapter 6 mainly studies the existence of solutions and global Mittag–Leffler stability of delayed fractional-order coupled reaction–diffusion neural networks without strong connectedness. In this chapter, we provided the proof of existence and uniqueness of solutions for the studied system. Subsequently, we explored the sufficient conditions for global Mittag–Leffler stability of the system solutions using the hierarchy method from graph theory.

Chapter 7 develops the sliding mode control method for coupled delayed fractional reaction–diffusion Cohen–Grossberg neural networks on a directed non-strongly connected topology. In this chapter, we designed a new fractional-order integral type sliding mode switching function, which leads to the sliding mode functional. By designing the sliding mode control law, we achieved global Mittag–Leffler synchronization between two different CGNN systems.

Chapter 8 discusses the projective synchronization of uncertain fractional-order reaction–diffusion systems via the fractional adaptive sliding mode control method. The prevalence of uncertainties is pervasive in real-world systems. We combine the theory of fractional calculus, sliding mode control, and adaptive control methods to design a fractional-order adaptive sliding mode control strategy to counteract uncertain disturbances. Moreover, we improve the constructed control law based on the properties of fractional derivatives, suppressing unnecessary oscillations of the control input while ensuring that the control performance is maintained or even improved.

Chapter 9 investigates a fractional-order prey–predator system with Beddington–DeAngelis functional response, considering predator avoidance and prey shelter. It establishes conditions for solution existence and well-posedness and analyzes stability with Hopf bifurcation concerning fear coefficient, shelter rate, and fractional derivative order. The study shows that stronger memory enhances species coexistence, while fading memory diminishes stability in the predator–prey system.

Chapter 10 examines a fractional-order three species food chain with prey refuge and Holling-II functional response. It establishes solution existence, boundedness, and asymptotic behavior conditions, analyzing stability and bifurcation. Numerical simulations illustrate the impact of parameters like half-saturation constant, prey refuge coefficient, and fractional-order derivative on system stability.

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Part I
Control and Synchronization of Several Classes of General Fractional Systems

Chapter 2
Adaptive Sliding Mode Control for Uncertain General Fractional Chaotic Systems

This chapter focuses on the Lyapunov stability analysis of general fractional differential systems (GFDSs), as well as adaptive sliding mode control (ASMC) of uncertain general fractional chaotic systems (UGFCSs) with uncertainty and external disturbances. Initially, the existence and uniqueness of solutions and the Lyapunov stability criterion for GFDSs are presented and verified. Furthermore, general fractional integral type sliding surfaces and reaching law are established. Based on the proposed stability criterion, it is shown that the states of UGFCSs can reach the sliding surface and asymptotically converge to zero along the sliding surface. Lastly, the efficacy and efficiency of the proposed fractional controllers are demonstrated by numerical simulations.

2.1 Introduction

Fractional calculus with kernel functions is ideally suited to describe materials and physical processes with memory and genetic properties [1, 2]. Currently, fractional calculus has been used in a wide range of interesting and novel applications in a number of disciplines, including biomathematics [3, 4], reaction–diffusion [5, 6], neural network [7, 8], and has become a powerful mathematical modeling tool and mathematical analysis tool of great interest [9–11].

Exploring fractional calculus with new memory and genetic properties is of great importance for fractional modeling of nonlinear phenomena. Fractional calculus with general kernel functions has been proposed in some research works [12–15]. The general fractional calculus can not only reduce to the known fractional calculus, but also generate new memory effects. Recently, the physical meaning, function space, and boundedness are discussed in [16] and [17]. Using general fractional calculus, the kernel functions may be selected flexibly according to the actual phenomena. Furthermore, as the general kernel functions may compress and

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extend the time domain, the suitable choice of kernel functions in the numerical simulation can shorten the simulation time.

With the emergence of fractional electronic circuits and the development of fractional calculus theory, the study of fractional chaos has become a hot research topic in the field of fractional calculus [18, 19]. Chaos has been found in numerous fractional nonlinear systems, such as fractional Chua circuits [20], fractional Lorenz systems [21], fractional Chen systems [22], fractional Liu systems [23], etc.

For fractional chaotic systems, the control problem has gradually received extensive interest from researchers in the field of fractional control [24, 25], and various control methods have been proposed based on the stability theory of fractional differential equations [26, 27]. In practical applications, it is often necessary to consider the uncertainty of the systems modeling and the interference of external disturbance. ASMC is the main control methods for dealing with systems uncertainty and external disturbances [28, 29]. ASMC techniques are able to estimate and compensate for systems uncertainties and unknown parameters in the controlled model [30]. The fractional ASMC method combines the advantages of both control techniques, ensuring robustness of the controlled systems while dealing with the variation or uncertainty of the parameters.

In this chapter, ASMC methods are applied to UGFCSs to achieve chaotic control. Our principal contributions may be summarized as follows:

(1) The existence and uniqueness of solutions for GFDSs and the general Lyapunov stability theorem are presented.
(2) A general fractional integral sliding surface and reaching law are developed to ensure that the states of the systems can reach the sliding surface.
(3) For UGFCSs, the ASMC is designed to stabilize the UGFCSs by estimating the unknown parameters with adaptive techniques.
(4) Numerical simulations of UGFCSs under different kernel functions are carried out using a nonequidistant partition approach.

This work is arranged as follows: Sect. 2.2 explores the fundamental definition of general fractional calculus and presents the existence and uniqueness of solutions and Lyapunov stability for GFDSs. In Sect. 2.3, ASMC designs for UGFCSs are investigated when uncertainty and external disturbance appear. Finally, numerical simulations are carried out to demonstrate the feasibility and efficiency of the proposed controller in Sect. 2.4.

2.2 Preliminaries

This section describes the space and definition of the general fractional calculus. Then, the general Lyapunov stability analysis is discussed. Consider the space

$X_c^p(a,b)(c \in \mathbf{R}, 1 \le p \le \infty)$ on $[a,b]$ for which $\|f\|_{X_c^p} < \infty$ [12], where the norm is given by

$$\|f\|_{X_c^p} = \left(\int_a^b \left|t^c f(t)\right|^p \frac{dt}{t}\right)^{1/p},$$

for $1 \le p < \infty,\ c \in \mathbb{R}$, and

$$\|f\|_{X_c^\infty} = \text{ess} \sup_{a \le t \le b} [t^c |f(t)|],$$

for $p = \infty,\ c \in \mathbb{R}$. X_c^p is identical to $L^p(a,b)$ as $c = 1/p$ $(1 \le p \le \infty)$.

Definition 2.1 ([12, 13]) Let $f \in X_c^p(a,b)$, $g \in C^1[a,b]$, $g(a) \ge 0$, and $g'(t) > 0$. For $0 \le a \le t \le b$ and $\mu > 0$, the general fractional integral of $f(t)$ is defined by

$$ {}_a^g I_t^\mu f(t) = \frac{1}{\Gamma(\mu)} \int_a^t (g(t) - g(\tau))^{\mu-1} f(\tau) g'(\tau) d\tau. \tag{2.1}$$

Following that, $AC_\delta^n[a,b]$ space is introduced, which is dependent on the general kernel functions $g(t)$.

$$AC_\delta^n[a,b] = \left\{ r(t) : [a,b] \to \mathbb{C} : \delta^{n-1}[r(t)] \in AC[a,b] \right\},$$

where $\delta = \frac{1}{g'(t)} \frac{d}{dt}$.

Definition 2.2 ([14]) Let $\mu > 0$, $n = [\mu] + 1$ and $r \in AC_\delta^n[a,b]$. The general Caputo derivative of $r(t)$ can be defined as

$$ {}_a^{Cg} D_t^\mu r(t) = \frac{1}{\Gamma(n-\mu)} \int_a^t (g(t) - g(\tau))^{n-\mu-1} g'(\tau) \delta^n r(\tau) d\tau. \tag{2.2}$$

For $\mu = n$, then ${}_a^{Cg} D_t^n r(t) = \delta^n r(t)$.

Next, analyze the initial value problem of GFDSs

$$\begin{cases} {}_a^{Cg} D_t^\mu x(t) = f(t, x(t)), \\ x(a) = x_a, \end{cases} \tag{2.3}$$

where $\mu \in (0,1)$, $t \in [a,T]$, $f \in C(G,R)$, $G = [a,T] \times D$, and $D \subset R$.

Theorem 2.1 *Suppose $f(t,x(t)) \in C(G,R)$, and for $t \in [a,T]$ and $x_1(t), x_2(t) \in D$, there exists a constant $L > 0$, such that $|f(t,x_1(t)) - f(t,x_2(t))| \le L\|x_1(t) - x_2(t)\|$. And there exists $M > 0$, such that $|f(t,x(t))| \le M < +\infty$. Then, existence*