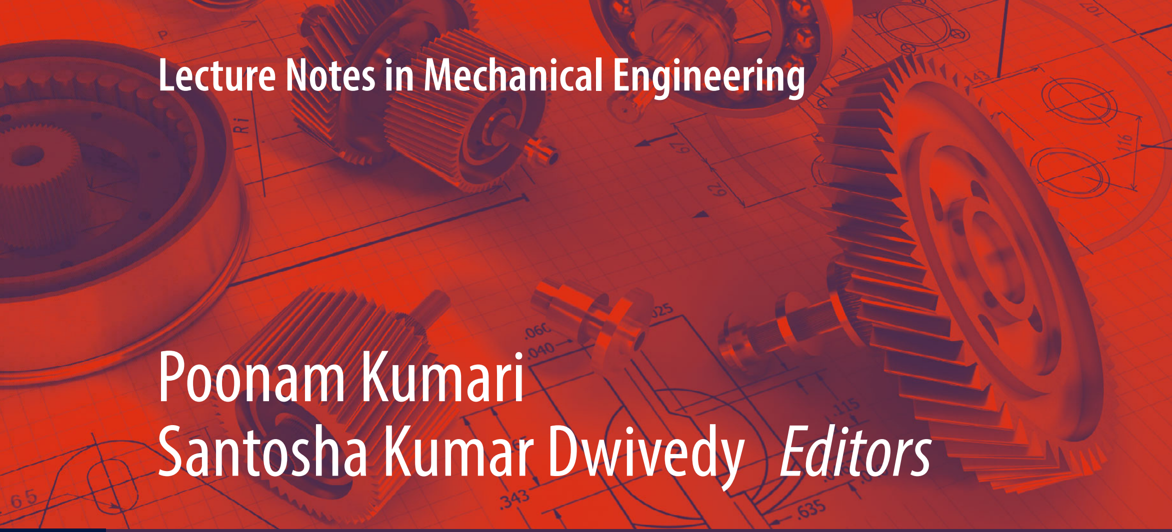

Lecture Notes in Mechanical Engineering

Poonam Kumari
Santosha Kumar Dwivedy *Editors*

Recent Advances in Mechanics of Functional Materials and Structures

Proceedings of the 8th Asian Conference on Mechanics of Functional Materials and Structures 2022

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Lecture Notes in Mechanical Engineering

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Editors

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Preface

We proudly introduce in this book the selected and peer-reviewed papers presented in the Eighth Asian Conference on Mechanics of Functional Materials and Structures (ACMFMS 2022) which was held in the Indian Institute of Technology, Guwahati, from December 11 to 14, 2022. I would like to present in brief the history of the series of this conference.

The first conference was hosted in 2008 by Prof. Fumihiro Ashida (Shimane University, Japan) in Matsue, Japan. Subsequently, it has been held in Nanjing, China (2010); 3rd in New Delhi, India (2012); 4th in Nara, Japan (2014); 5th in Shanghai, China (2016); and 6th in Tainan, Taiwan (2018). The 7th conference was to be held in Sendai, Japan, on October 2–5, 2020. However, due to the ongoing COVID-19 pandemic, the decision was made to hold the conference via online mode from March 12 to 15, 2021. The 7th Asian Conference on Mechanics of Functional Materials and Structures (ACMFMS 2020) was chaired by Fumio Narita (Tohoku University, Japan).

This multidisciplinary conference had attracted participation from prestigious academic institute, e.g., IIT Delhi, IIT Bombay, IIT Madras, IIT Mandi, IIT Kanpur, IIT Kharagpur, and many more and research institutes from India and abroad (China, Japan, Bangladesh, Dubai, Canada, and Singapore). Participants have attended, interacted, and exchanged innovative ideas. Many Ph.D. research scholars also attended, and this has enabled them to acquire international exposure and encourage them to undertake research activities in their areas of interest. The conference has brought many prominent scientists, technologists, and young researchers from different parts of the world together for showcasing their achievements and discussing new research trends and the emerging field of mechanics. The scientific program included plenary sessions, invited lectures, and oral presentations. The conference has 4 plenary sessions, 2 semi-plenary sessions, 8 keynotes, 26 invited lectures, and 100 presentations.

This book entitled *Recent Advances in Mechanics of Functional Materials and Structures* consists of 53 chapters and includes different topics related to the fields of Mechanics of Functional and Intelligent Materials, Mechanics of Functional and Smart materials, Structural Health Monitoring, Elasticity (Mathematical, Thermo, Electro, Electromagneto, Photo), Plasticity (Mathematical, Multiscale, Thermo, Visco), Fracture and Damage Mechanics, Impact Mechanics and Dynamic Material Behavior, Contact Mechanics, Solid-Fluid Interaction, Bio-mechanics, Biomaterials, etc. We believe that these selected papers will guide the future direction of research.

We would like to acknowledge the financial support provided by sponsors to make the event financially viable for hosting a huge number of student participants. I would like to thank BRNS, CSIR, DRDO, SERB, TIH IITG, ADMECA Design and Engineering solutions LLP, and EDS Technologies. It was impossible to make the conference successful without their support.

We wish to thank all the authors, reviewers, sponsors, invited speakers, members of the advisory board, the organizing team, student volunteers, and all others who have contributed to the successful organization of the conference. We are very grateful to Prof. T. G. Sitharam, Director, Head of the Department of Mechanical Engineering, and Prof. K. S. R. Krishna Murthy, IIT Guwahati, for their encouragement and providing the necessary infrastructure. We sincerely thank the team of research scholars Mr. Abir Saha, Mr. Vaibhav R. Partap, Mr. Mukesh Kumar, Mr. Nikhil D. Kulkarni, Miss Mridusmita Bora, Mr. Viwek Kumar, and Ms. Akanksha for their help in bringing out this book.

A large number of members in the steering committee, international advisory committee, local organizing committee, and manuscript review committee are highly acknowledged for their time-to-time help and support for these technical events.

Last but not least, the editors sincerely appreciate the significant contribution of Mrs. Vaishnavi O. Bichkar for bringing out this book. She has contributed significantly in managing all these chapters and communicated with the authors for shaping these books. Her efforts are sincerely appreciated. Before concluding this preface, we sincerely thank the team from Springer Nature for their contributions in making this book an archived technical reference documents for the next-generation readers/researchers.

Guwahati, India

Dr. Poonam Kumari
Conference Secretary

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About the Editors

Poonam Kumari is currently Associate Professor in the Department of Mechanical Engineering at the Indian Institute of Technology Guwahati. She received her Ph.D. from Indian Institute of Technology Delhi in 2012. She did her Post-doctoral Fellowship at Simon Fraser University, Canada. She works in continuum mechanics and smart material and structures. She has developed three-dimensional as well as two-dimensional solutions for composite and piezolaminated plates. Currently, she is working in the field of wearable devices and nanofiber reinforced PVDF flexible mats for energy harvesting. She has published 41 international journal publications and 45 international conference publications. She has received Young Engineer Award-2017 from Indian National Academy of Engineers. In 2019, she also received SERB Women Excellence Award. She got DUO-India Professor Fellowship 2020 Award for conducting research at UK.

Dr. Santosha Kumar Dwivedy completed his Ph.D. in Mechanical Engineering from Indian Institute of Technology Kharagpur in 2000, and he is working at Mechanical Engineering Department of IIT Guwahati since October 1999. He is working in the field of nonlinear vibration, robotics, and biomechanics. He guided 15 Ph.D. students, more than 60 M.Tech. students, and around 50 B.Tech. projects. He has published more than 200 papers in international journals, conferences, and book chapters. He has been the editor and guest editor for several international publications. He is also one of the editors of the Springer Books on *Advances in Structural Vibration* Springer Publication, *Advances in Rotor Dynamics, Control and Structural Health Monitoring* and *Mechanical Sciences: The Way Forward*.

Fatigue Crack Growth in Fiber-Reinforced Polymer Composite Laminate Using Higher-Order XFEM

Kishan Dwivedi and Himanshu Pathak

Abstract Fiber-reinforced polymer composite laminate has a wide range of applications due to its high strength-to-weight ratio. Fatigue loading is the most common cause of failure for these materials. Fatigue life estimation in symmetric ply composite laminate under cyclic mechanical loading is performed in this work. The analysis is performed for various laminate schemes such as $[0°/15°]_s$, $[0°/30°]_s$, $[0°/45°]_s$, and $[0°/60°]_s$. For each laminate scheme, several discontinuities, such as edge cracks, multiple holes, and multiple minor cracks, are considered for detailed numerical analysis. Higher-order XFEM is used to analyze fracture behavior. Higher-order XFEM employs a tenfold higher-order crack tip enrichment function instead of fourfold enrichment function for improving solution accuracy. The performance of higher-order XFEM is demonstrated in few numerical examples in which the fatigue life curve is compared for different symmetric ply composite laminates.

Keywords Fiber-reinforced polymer composite laminate · SIFs · Fatigue · Higher-order XFEM

1 Introduction

Composite materials have appealing properties, such as high strength-to-weight ratio. Due to this property, composite materials exhibit a high level of resistance to the growth of unstable cracks when stretched in the direction of the fibers. Composite material is mainly used for lightweight applications. As a result, composite materials are widely used in aerospace, automotive, marine, and biomedical engineering, among others [1]. Many factors can cause cracks to form and grow in composite

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materials. Crack propagation reduces the material's stiffness and may result in catastrophic failure. Numerous numerical crack analysis methods are available, but only extended finite element method (XFEM) [2–4] provides more accurate results. Several researchers [5–8] have tried XFEM to determine the fracture response of crack subjected to various loadings. Numerous studies on XFEM fracture models have been conducted recently. The following are a few of them: Refs. [9, 10] for three-dimensional cracks, Refs. [11, 12] for dynamic fracture, and Refs. [13–17] for computing fracture parameters and crack growth analysis. A great introduction to XFEM may be found in [18].

With the introduction of enrichment by higher-order terms, Liu et al. [19] estimated SIFs with the XFEM framework for the first time in 2004. The findings demonstrate that high-order terms may reach outstanding accuracy after examining several benchmarks. The researcher studied the enrichment with higher-order functions using h-p clouds [20, 21]. With this technique, Zamani et al. [22, 23] combined an overlapping domain decomposition scheme with acceptable enrichment scheme and obtained outstanding accuracy. Zamani et al. [24] utilize higher-order terms to enrich the displacement and temperature field in the thermo-elastic crack problems and achieve more accuracy in SIF. Xiao et al. [25] updated the original XFEM by substituting the four branch functions with a predetermined number of higher-order terms at the crack tip asymptotic field. Cheng et al. [26] used higher-order XFEM to solve issues with curved discontinuities. Saxby et al. [27] employed a higher-order XFEM approach to attain the best convergence rate for Poisson and linear elasticity problems with curved discontinuities. Mousavi et al. [28] studied the numerous intersecting and branching cracks in the elastic domain using a higher-order extended finite element approach.

Mechanical fatigue is mainly blamed for laminated composites' failure in practical applications. The fatigue behavior of laminated composite structures and their failure mechanisms under tension–compression [29] and impact [30] have been extensively studied in the literature. Putic et al. [31] performed high-frequency fatigue on carbon fibe0072-reinforced composites. In the experiment, he considered 0°/90° cross-ply and 45° angle ply laminates with and without notches. Spearing and Beaumont [32] created a novel method for post-fatigue modeling of the strength of composite laminates with notches. Spearing et al. [33] investigated a fatigue model to track the growth of notch tip damage in a carbon fiber epoxy laminate. Shokrieh and Lessard [34] developed progressive fatigue damage modeling for predicting fatigue behavior of composite laminates with and without stress concentration. Degriech and Paepegem [35] present an overview of the main fatigue models and methodologies for predicting the lifetime of laminates.

The literature study shows that geometrical discontinuity simulation frequently uses the XFEM approach. Practically every structural material's fracture behavior has been studied using this method. To increase the computational accuracy of the XFEM technique, higher-order terms are added at the crack tip asymptotic field. This study uses a higher-order XFEM for fatigue analysis of composite laminate material.

2 Mathematical Formulation with XFEM Approach

Figure 1 depicts that the laminate domain (Ω) with various discontinuities ($\Gamma_{\mathrm{c/h}}$) is subjected to displacement (Γ_{v}) and traction (Γ_{t}) boundary conditions. The problem's equilibrium and boundary conditions are as follows [36]:

$$\nabla : T + f_{\mathrm{b}} = 0 \text{ in } \Omega, \tag{1}$$

$$T = D : \gamma$$

$$T \cdot n = \overline{T} \text{ on } \Pi_{\mathrm{t}}, \tag{2}$$

$$T \cdot n = 0 \text{ on } \Pi_{\mathrm{c}}, \tag{3}$$

In above equations, ∇ is represent to gradient operator, T is Cauchy stress tensor, D is a fourth-order material tensor, γ is a strain tensor, f_{b} is body force per unit volume, and n is outward unit normal.

Weak form of equilibrium equation can be expressed as below [37],

$$\int\limits_{\Omega} \gamma\,(\boldsymbol{u}){:}\boldsymbol{D}{:}\gamma\;(\boldsymbol{v})\,\mathrm{d}\Omega - \int\limits_{\Omega} f_{\mathrm{b}} \cdot \boldsymbol{v}\;\mathrm{d}\Omega - \int\limits_{\Pi_{\mathrm{t}}} \overline{\boldsymbol{T}} \cdot \boldsymbol{v}\,\mathrm{d}\Gamma = 0. \tag{4}$$

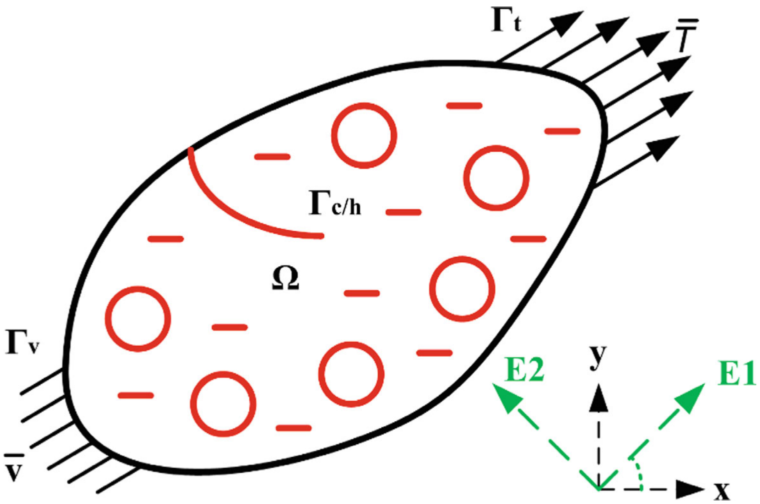

Fig. 1 Laminate domain with multiple discontinuities

Displacement-based approximation [38] for modeling the crack with XFEM is written as,

$$v^h(x) = \sum_{\alpha=1}^{n} \Psi_\alpha(x) v_\alpha + \sum_{\beta=1}^{n} \Psi_\beta(x)[\Theta(x) - \Theta(x_\beta)] l_\beta + \sum_{\delta=1}^{n} \Psi_\delta(x)[\zeta(x) - \zeta(x_\delta)] m_\delta. \tag{5}$$

Here, $\Psi(x)$ is the shape function. $\Theta(x)$ is Heaviside function defined for elements completely cut by the crack surface. $\zeta(x)$ is tip enrichment function defined for those elements that is partially cut by crack surface. The variables l_β and m_δ represent an additional degree of freedom.

Crack tip enrichment functions [36] for the orthotropic laminate material are given as,

$$\zeta(x) = \left[\sqrt{d} \sin \tfrac{\lambda_1}{2} \sqrt{\omega_1(\varphi)} \;\; \sqrt{d} \cos \tfrac{\lambda_2}{2} \sqrt{\omega_2(\varphi)} \;\; \sqrt{d} \cos \tfrac{\lambda_1}{2} \sqrt{\omega_1(\varphi)} \;\; \sqrt{d} \sin \tfrac{\lambda_2}{2} \sqrt{\omega_2(\varphi)} \right]. \tag{6}$$

Furthermore, the higher-order terms can be added to the crack tip enrichment functions.

$$\zeta(x) = \begin{bmatrix} \sqrt{d} \sin \frac{\lambda_1}{2} \sqrt{\omega_1(\varphi)} & \sqrt{d} \cos \frac{\lambda_2}{2} \sqrt{\omega_2(\varphi)} & \sqrt{d} \cos \frac{\lambda_1}{2} \sqrt{\omega_1(\varphi)} & \sqrt{d} \sin \frac{\lambda_2}{2} \sqrt{\omega_2(\varphi)} \\ & d^2 \sqrt{\omega_1(\varphi)} & d^2 \sqrt{\omega_2(\varphi)} & \\ d^2 \cos 2\lambda_1 \sqrt{\omega_1(\varphi)} & d^2 \sin 2\lambda_1 \sqrt{\omega_1(\varphi)} & d^2 \cos 2\lambda_2 \sqrt{\omega_2(\varphi)} & d^2 \sin 2\lambda_2 \sqrt{\omega_2(\varphi)} \end{bmatrix}. \tag{7}$$

After the weak formulation, we obtained discrete equation for an element can be written as [37],

$$\left[\boldsymbol{K}_{ij}^e\right]\left\{\boldsymbol{v}^e\right\} = \left\{\boldsymbol{F}^e\right\}. \tag{8}$$

The elemental contributions of stiffness matrix $\boldsymbol{K}_{ij}^e$ for extrinsically enriched approximation are given as [38]

$$\boldsymbol{K}_{ij}^e = \begin{bmatrix} \kappa_{ij}^{vv} & \kappa_{ij}^{vl} & \kappa_{ij}^{vm} \\ \kappa_{ij}^{lv} & \kappa_{ij}^{ll} & \kappa_{ij}^{lm} \\ \kappa_{ij}^{mv} & \kappa_{ij}^{ml} & \kappa_{ij}^{mm} \end{bmatrix}, \tag{9}$$

$$\kappa_{ij}^{rs} = \int_{\Omega^e} \left(\boldsymbol{B}_i^r\right)^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}_j^s \mathrm{d}\Omega \text{ and } r, s = v, l, m, \tag{10}$$

$$\boldsymbol{F}^{e} = \left\{ \boldsymbol{F}_{i}^{v} \ \boldsymbol{F}_{i}^{l} \ \boldsymbol{F}_{i}^{m} \right\}, \tag{11}$$

$$\boldsymbol{U}^{e} = \left\{ \boldsymbol{U}_{i}^{v} \ \boldsymbol{U}_{i}^{l} \ \boldsymbol{U}_{i}^{m} \right\}. \tag{12}$$

Here, $\boldsymbol{B}$ is derivative matrix of shape function and $\boldsymbol{D}$ is two-dimensional fourth-order material stiffness matrix.

For composite laminate, material matrix [39] can be defined as

$$D = \sum_{k=1}^{\mathrm{NL}} \int_{z_{k-1}}^{k} \overline{Q_{ij}}(1 z_k)\mathrm{d}z. \tag{13}$$

The domain-based interaction integral method [40, 41] was used to determine stress intensity factors. This method is accurate and effective in extracting individual SIFs under mixed mode loading conditions.

3 Fatigue Crack Growth Analysis

The crack propagation is tracked and modeled using the level set technique. This technique defines normal and tangential level set function values at each node to find the moving crack tip. In orthotropic laminate media, the maximum circumferential tensile stress hypothesis [42] is employed as failure criteria. The equivalent stress intensity factor and crack propagation angle are calculated using the equation below.

$$\theta_{\mathrm{c}} = 2 \arctan \frac{1}{4}\left(\frac{K_I}{K_{II}} + \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8} \right), \tag{14}$$

$$K_{\mathrm{eq}} = \frac{1}{2}\cos\left(\frac{\theta_{\mathrm{c}}}{2}\right)(K_{\mathrm{I}}(1 + \cos(\theta_{\mathrm{c}})) - 3K_{II}\sin(\theta_{\mathrm{c}})). \tag{15}$$

Here, K_{eq} is the equivalent stress intensity factor and θ_{c} is the crack propagation angle. The formula below illustrates the Paris relationship between the SIF range and the crack growth rate.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{\mathrm{eq}})^m. \tag{16}$$

In the above expression, da represents crack growth, dN is the change in number of cycles, ΔK_{eq} is represented to change in the equivalent stress intensity factor, and C and m are material constants.

4 Results and Discussion

4.1 Edge Crack Composite Laminate

Figure 2a depicts the validation model of an edge crack composite laminate with dimensions of $a = 9$ mm, $b = 101.6$ mm, $c = 18$ mm, and $d = 203.2$ mm. The total thickness of the laminated plate is 1.235 mm. The lower surface of the laminate is limited to moving in the y-direction, while the upper surface is used for tensile traction load. Φ represents to laminar angle orientation of each lamina. For computation, the domain is discretized with 7500 nodes. Table 1 compares the SIF value for edge crack composite laminate obtained from both computational (XFEM, HO-XFEM) approaches with the literature.

For fatigue analysis purpose, we consider edge crack composite laminates plate with dimensions of $a = 5$ mm, $b = 30$ mm, $c = 50$ mm, and $d = 60$ mm, as shown in Fig. 2a. The laminate has a total thickness of 1.235 mm. The plate's bottom surface is constrained to move in the y-direction, while the upper surface is subjected to cyclic mechanical loading with maximum stress (1 MPa) and zero stress ratio. The Paris constants [43] $C = 2.29 \times 10^{-13}$, and $m = 4.867$ is used to estimate fatigue life in composite laminate. A total of 6096 DOF are employed for computing from higher-order XFEM approach. A linear crack section used a 1 mm crack increment in each step of the simulation. Figure 2c shows that $[0°/15°]_s$ laminate has the highest fatigue life and $[0°/60°]_s$ laminate has the lowest fatigue life then compared to another symmetric ply composite laminate.

4.2 Edge Crack with Multiple Holes'/Cracks' Composite Laminate

The dimensions of edge crack composite laminate are $a = 5$ mm, $b = 15$ mm, $c = 50$ mm, $d = 60$ mm, $e = 15$ mm, $f = 15$ mm, $g = 15$ mm, $h = 12.5$ mm, $i = 11.5$ mm, $j = 13.5$ mm as shown in Fig. 3a. Multiple holes of radius 4 mm and multiple small crack of 2 mm length are spread over the surface of composite laminate plate. The total thickness of the composite laminate plate is 1.235 mm. The plate's bottom surface is constrained to move in the y-direction, while the upper surface is subjected to cyclic mechanical loading with maximum stress (1 MPa) and zero stress ratio. The Paris parameter [43] $C = 2.29 \times 10^{-13}$, $m = 4.867$ estimates fatigue life in composite laminate. A total of 6624 DOF are used to discretize the domain. A linear crack section used a 1 mm crack increment in each step of the simulation. Figure 3c shows that $[0°/15°]_s$ laminate has the highest fatigue life and $[0°/60°]_s$ laminate has the lowest fatigue life when compared to other symmetric ply composite laminate.

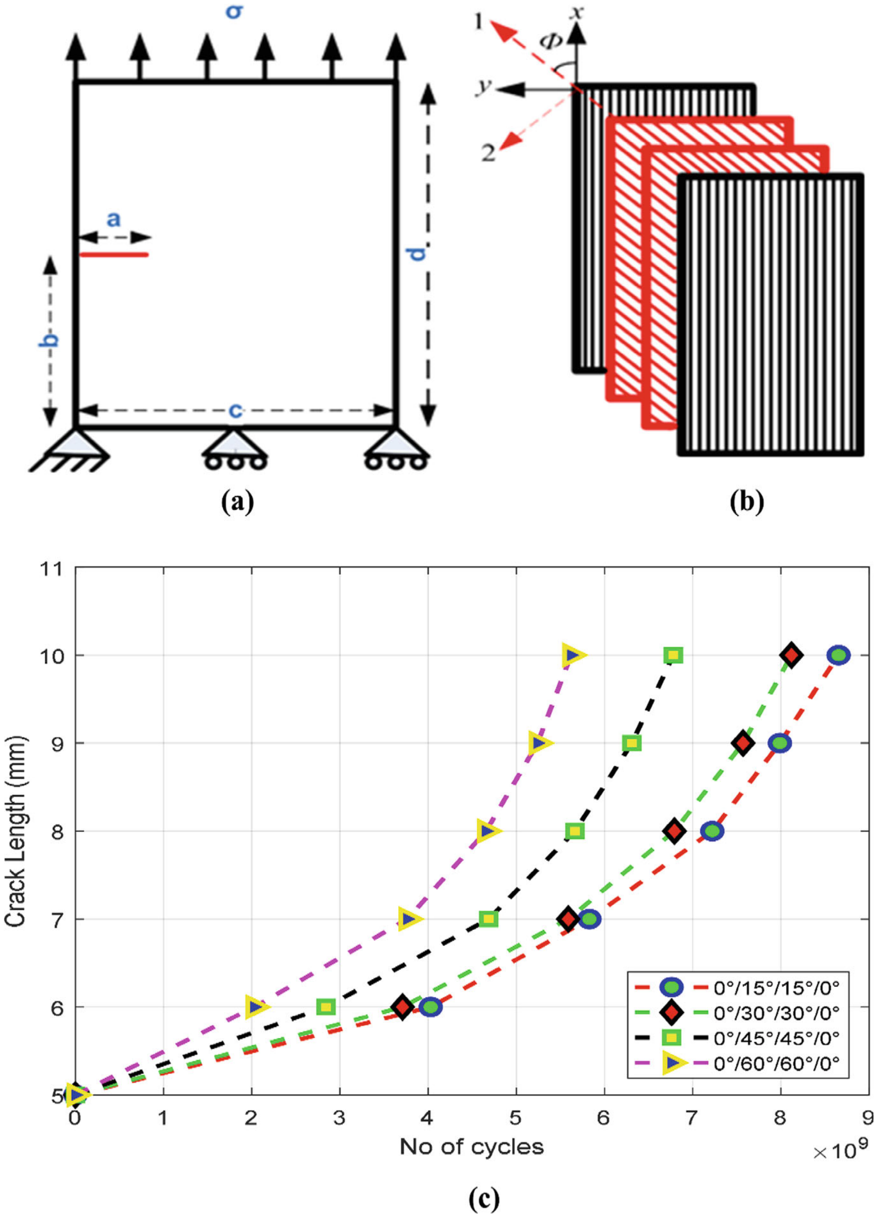

Fig. 2 **a** Composite laminate plate with edge crack, **b** ply sequence in symmetric ply composite laminate (0°/Φ°/Φ°/0°), **c** comparison of fatigue life curve for edge crack symmetric ply composite laminate at zero stress ratio, simulated from higher-order XFEM method

Table 1 SIF obtained from different computational approaches for single-edge crack symmetric ply composite laminate under uniform tensile traction load was compared to the literature

Lamination schemes	Stress (MPa) [44]	SIF (K_{ic}) [44] MPa $\sqrt{mm}$	SIF (K1) MPa $\sqrt{mm}$ (XFEM)	SIF (K1) MPa $\sqrt{mm}$ (HO-XFEM)	Error (%) with XFEM	Error (%) with HO-XFEM
0°/15°/15°/0°	293.077	4404.62	3971.7	4171.7	9.83	5.3
0°/30°/30°/0°	231.48	3478.88	3096.1	3264.8	11	6.15
0°/45°/45°/0°	212.442	3192.76	3053.8	3089.6	4.35	3.23
0°/60°/60°/0°	235.224	3535.14	3235	3377.9	8.5	4.44

5 Conclusions

This paper used higher-order extended finite element method for estimation the fatigue life of different symmetric ply composite laminates under cyclic mechanical load environment. Few numerical examples are introduced for demonstrating the performance of the proposed computational approach. From the numerical results, we obtained the following observation, illustrated below.

- Use of higher-order enrichment function at the crack tip improves the accuracy of solution.
- The fatigue life of symmetric ply composite laminate decreases as we increase the ply angle from 15° to 60° for symmetric ply composite laminate.
- As we increase the discontinuity in the domain of laminate, then fatigue life is also decreasing.

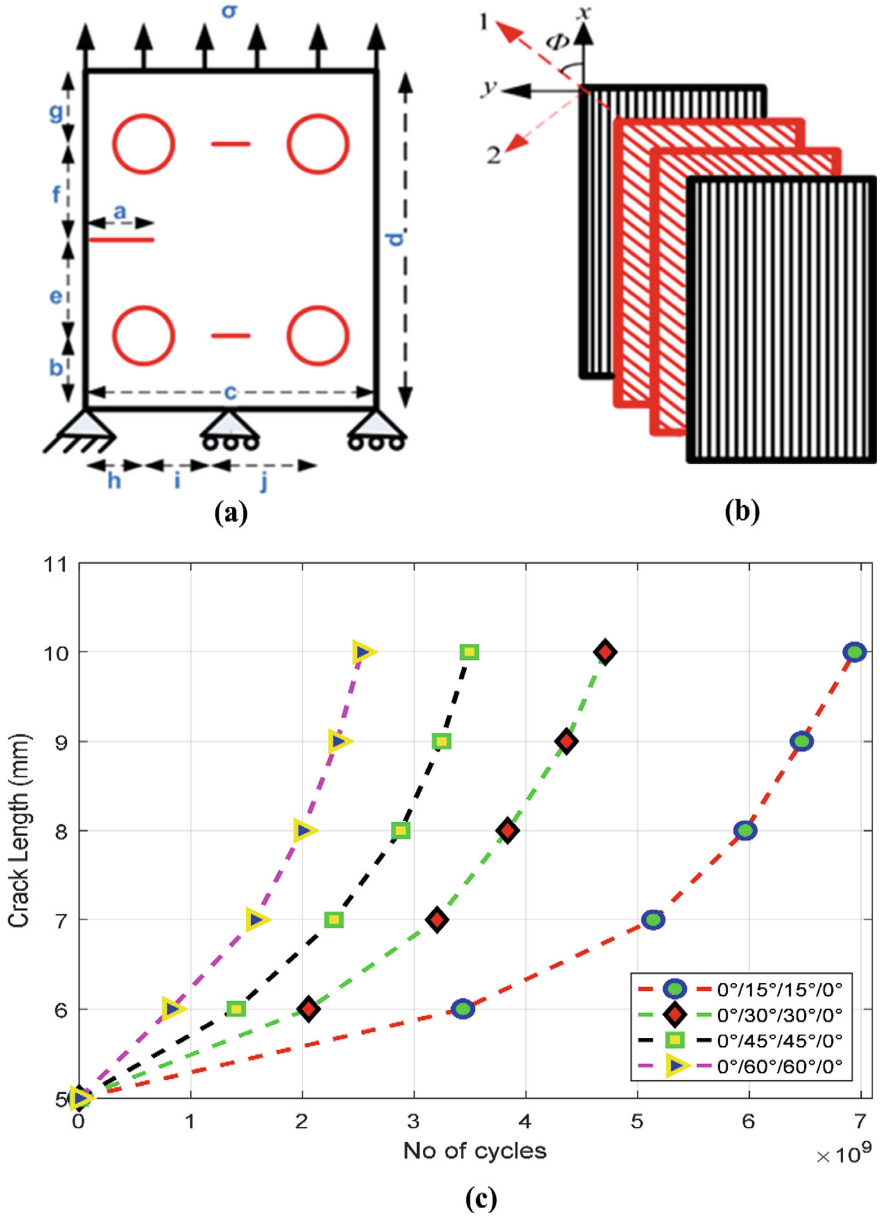

Fig. 3 **a** Composite laminate plate with multiple holes and cracks, **b** ply sequence in symmetric ply composite laminate, **c** comparison of fatigue life curve for multiple holes and cracks symmetric ply composite laminate at zero stress ratio, simulated from higher-order XFEM method

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An Eshelby–Mori–Tanaka-Based Micromechanical Model for Piezoelectric Polymer Composites with Spatially and Randomly-Oriented Inclusions in Orthotropic Matrix

Neelam Mishra and Kaushik Das

Abstract A mean-field micromechanics model based on the Eshelby–Mori–Tanaka approach is used to investigate the effect of spatial orientation of reinforcements on the effective elastic, dielectric, and piezoelectric properties of a polymer composite, with an orthotropic matrix and a transversely isotropic reinforcement. The analysis is also performed for composites with random orientation of the piezoelectric reinforcements and with different shapes of spheroidal reinforcements ranging from aspect ratios of 2–1000. The effect of orientation on the effective axial and transverse Young's moduli is not prominent at a low aspect ratio of 2, while the effective properties do not change significantly as functions of volume fraction for aspect ratios of 100 and above. The results also confirm that the random composites show significantly poorer properties than their perfectly-aligned counterparts. The results also allow us to obtain the specific orientation angles that would provide the highest elastic, piezoelectric, and dielectric properties for a given reinforcement volume fraction.

Keywords Piezoelectric polymer composites · Orthotropic matrix · Eshelby–Mori–Tanaka micromechanics · Random orientation · Effective electro-elastic properties

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1 Introduction

Piezoelectric ceramics have high electromechanical coupling constants and have been routinely used as the major functional component in a large variety of sensors and actuators [1, 2]. However, since ceramics are stiff and brittle, they cannot be used in applications demanding high flexibility and conformability to curved surfaces. These drawbacks can be overcome by using the piezoelectric ceramic as reinforcement in a piezoelectric polymer matrix. These piezoelectric polymer composites combine the properties of polymers, such as being lightweight and flexible, with stiffness and high electromechanical coupling of piezoelectric ceramics. Hence, these piezoelectric polymer composites are excellent candidates for applications in flexible electronics and microelectromechanical systems (MEMSs) [3]. Judicious selection of such piezoelectric composites for the design and development of novel microdevices can be hastened if experimental efforts are aided by computational prediction of the effective properties of these piezoelectric composites. In most of Eshelby–Mori–Tanaka-based approaches extended to linear piezoelectric domain [4], the primary assumption is that the local coordinate axes of the reinforcements coincide with the global coordinate axes of the matrix. However, it is extremely difficult to get unidirectionally-oriented reinforcements in practice, and most real cases of particle-reinforced composites involve randomly-oriented particles. In addition, it is desirable to give a certain spatial orientation to piezoelectric fibers for a specific application in some cases. Hence, a detailed analysis of piezoelectric composites with spatially-oriented fibers is necessary, especially for cases when the matrix and reinforcement are both anisotropic. In terms of computation of effective properties of composites with randomly-oriented reinforcements, Odegard et al. [5] have established a protocol for evaluating the effective elastic properties of composites with randomly-oriented reinforcements. However, micromechanical model(s) to study the effect of spatially-oriented as well as randomly-oriented transversely isotropic piezoelectric particles and fibers (such as PZT) in an orthotropic piezoelectric matrix (such as PVDF) have not been explored fully. Therefore, in this work, we have attempted to address the issue of micromechanics-based computational estimation of overall properties of piezoelectric composites for two cases, viz. (i) composites having spatially-oriented reinforcements and (ii) composites with randomly-oriented reinforcements. The composites considered in this work have transversely isotropic piezoelectric spheroidal lead zirconate titanate (PZT-7A) inclusions in orthotropic piezoelectric polyvinylidene fluoride (PVDF) polymer matrix.

2 Constituent Material Properties and Reinforcement Geometry

In this section, the constituent material properties and the reinforcement geometry types are discussed. The composite considered here has PVDF as the matrix and PZT-7A as the reinforcement. The electro-elastic properties such as the stiffness, piezoelectric stress constants, and relative dielectric constant of both the materials are available in literature [6]. PVDF is a piezoelectrically active polymer, and PZT, on the other hand, is a ferroelectric ceramic having a perovskite structure and is composed of Lead Zirconate (PZ) and Lead Titanate (PT) having a very high electromechanical coupling constant. The constituent materials considered here are assumed to be poled in the three direction.

In this work, the reinforcement geometry is considered to be spheroid. The shape of the reinforcement is changed by changing the spheroid aspect ratio. And finally, composites with four different aspect ratios: 2, 10, 100, and 1000 are considered in this work.

3 Constitutive Equations of a Linear Piezoelectric Material

The stress-charge form of the constitutive equations of a linear piezoelectric material relates the independent variables elastic strain ε_{mn} and electric potential E_i to dependent variables stress σ_{ij} and electric displacement D_i given by Eq. (1) as:

$$\begin{aligned} \sigma_{ij} &= C^{E}_{ijmn}\varepsilon_{mn} - e_{nij}E_n, \\ D_i &= e_{imn}\varepsilon_{mn} + \kappa^{\varepsilon}_{in}E_n, \end{aligned} \tag{1}$$

where C_{ijmn}, κ_{in}, and e_{nij} are the components of the stiffness tensor, components of the permittivity tensor, and components of the piezoelectric stress coefficient tensor, respectively. The above equations can be expressed in Barnett and Lothe notation [7], where the lower case subscripts take integral values in the range of 1–3, while uppercase subscripts take integral values in the range of 1–4.

$$Z_{Mn} = \begin{cases} \varepsilon_{mn} & \text{for } M(=m) = 1, 2, 3 \\ -E_n & \text{for } M = 4 \end{cases}, \tag{2}$$

$$\Sigma_{iJ} = \begin{cases} \sigma_{ij} & \text{for } J(=j) = 1, 2, 3 \\ D_i & \text{for } J = 4 \end{cases}, \tag{3}$$

$$F_{iJMn} = \begin{cases} C_{ijmn} & \text{for } J, M = 1, 2, 3 \\ \mathrm{e}_{nij} & \text{for } J = 1, 2, 3;\ M = 4 \\ \mathrm{e}_{nij} & \text{for } J = 4;\ M = 1, 2, 3 \\ -\kappa_{in} & \text{for } J = M = 4 \end{cases}, \tag{4}$$

where Z_{Mn}, Σ_{iJ}, and F_{iJMn} are the strain-electric field, stress-electric displacement, and piezoelectric or electro-elastic moduli, respectively. The constitutive equation in matrix form for an orthotropic material is given as

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \\ D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & 0 & 0 & e_{31} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 & 0 & 0 & e_{32} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 & 0 & 0 & e_{33} \\ 0 & 0 & 0 & C_{44} & 0 & 0 & 0 & e_{24} & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 & e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & -\kappa_{11} & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 & 0 & -\kappa_{22} & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 & 0 & 0 & -\kappa_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \\ -E_1 \\ -E_2 \\ -E_3 \end{bmatrix}. \tag{5}$$

Here, the subscripts are written in Voigt notation.

3.1 Piezoelectric Reinforcement Problem

Consider a general two-phase piezoelectric inclusion problem of a spheroidal piezoelectric reinforcement which is transversely isotropic, embedded in a piezoelectric orthotropic matrix. The system is subjected to homogeneous electro-elastic boundary conditions Z^0 such that

$$u_i(\Omega) = \varepsilon^0_{ij} x_j \text{ and } \phi_i(\Omega) = E^0_i x_i. \tag{6}$$

Here Ω is the surface of the composite. The effective electro-elastic moduli F is calculated by evaluation of the volume-averaged piezoelectric fields as

$$\Sigma^{\text{eff}} = F^{\text{eff}} Z^{\text{eff}}, \tag{7}$$

where F is the overall electro-elastic modulus of the homogenous composite system. F^{eff} is given by [4]

$$F^{\text{eff}} = F_m + v_f \big(F_f - F_m\big) T^{\text{piezo}}, \tag{8}$$

where T^{piezo} is the concentration tensor (strain-electric displacement) that relates the average strain and potential gradient in the reinforcement phase to that in the composite. Estimating the strain-electric displacement concentration tensor is one of the key steps in evaluating the electro-elastic properties of piezoelectric composites. The strain-potential gradient concentration tensor derived using the modified EMT model is given as [8]

$$T_{\mathrm{EMT}}^{\mathrm{piezo}} = T_{\mathrm{dil}}^{\mathrm{piezo}}[v_{\mathrm{m}}\mathbb{I} + v_{\mathrm{f}}T_{\mathrm{dil}}^{\mathrm{piezo}}]^{-1}, \tag{9}$$

where

$$T_{\mathrm{dil}}^{\mathrm{piezo}} = [\mathbb{I} + S^{\mathrm{piezo}}F_{\mathrm{m}}^{-1}(F_{\mathrm{r}} - F_{\mathrm{m}})]^{-1}. \tag{10}$$

Here, S^{piezo} is the piezoelectric Eshelby tensor and $\mathbb{I}$ is the identity tensor. v_{m} and v_{f} are the volume fraction of matrix and reinforcement, respectively. Finally, the effective electro-elastic modulus of a binary composite [9] is given by

$$F_{\mathrm{EMT}}^{\mathrm{eff}} = F_{\mathrm{m}} + v_{\mathrm{r}}(F_{\mathrm{r}} - F_{\mathrm{m}})T_{\mathrm{EMT}}^{\mathrm{piezo}}. \tag{11}$$

3.2 The Piezoelectric Eshelby Tensor

The geometry of the reinforcement is considered as an ellipsoid that satisfies the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \tag{12}$$

where a, b, and c are the semi-axes of the reinforcement. The ellipsoidal reinforcement can be considered as spheroidal inclusion when $a = b$ and its shape can be changed from particle-like short fibers to very long fibers by changing the aspect ratio $\frac{c}{a}$.

The estimation of the overall properties of a piezoelectric composite depends on the determination of the strain-potential concentration tensor. The determination of the strain-potential concentration tensor, in turn, depends on the determination of the piezoelectric Eshelby tensor. The piezoelectric Eshelby tensor is a function of the reinforcement geometry and the electro-elastic properties of the matrix as given by [9].

$$S_{MnAb} = \begin{cases} \frac{1}{8\pi}F_{iJAb}(I_{inmJ} + I_{imnJ}) & \text{for } M = 1, 2, 3 \\ \frac{1}{4\pi}F_{iJAb}I_{in4J} & \text{for } M = 4 \end{cases}. \tag{13}$$

Here, F_{iJAb} is the component of the electro-elastic matrix of the polymer (matrix), and for 0–3 composites, the nonzero components of I_{inMJ} are given as