Applied and Numerical Harmonic Analysis

Stephen D. Casey
M. Maurice Dodson
Paulo J. S. G. Ferreira
Ahmed Zayed
Editors

Sampling, Approximation, and Signal Analysis

Harmonic Analysis in the Spirit of J. Rowland Higgins

Birkhäuser

Applied and Numerical Harmonic Analysis

Applied and Numerical Harmonic Analysis (ANHA) publishes works ranging from abstract harmonic analysis to engineering and scientific subjects having significant applicable harmonic analysis components. The interface between mathematics, science, and engineering is the overriding theme.

Harmonic analysis is a wellspring of ideas and applicability in mathematics, engineering, and the sciences that has flourished, evolved, and deepened with continued research and exploration. The ANHA series reflects the intricate and fundamental relationship between harmonic analysis and general disciplines such as signal and image processing, partial differential equations, machine learning, and data science.

This series provides a means of disseminating important, current information along with computational tools for harmonic analysis. The following topics are covered:
* Analytic Number theory * Antenna Theory * Artificial Intelligence * Biomedical Signal Processing * Classical Fourier Analysis * Coding Theory * Communications Theory * Compressed Sensing * Crystallography and Quasi-Crystals * Data Mining * Data Science * Deep Learning * Digital Signal Processing * Dimension Reduction and Classification * Fast Algorithms * Frame Theory and Applications * Gabor Theory and Applications * Geophysics * Image Processing * Machine Learning * Manifold Learning * Numerical Partial Differential Equations * Neural Networks * Phaseless Reconstruction * Prediction Theory * Quantum Information Theory * Radar Applications * Sampling Theory (Uniform and Non-uniform) and Applications * Spectral Estimation * Speech Processing * Statistical Signal Processing * Super-resolution * Time Series * Time-Frequency and Time-Scale Analysis * Tomography * Turbulence * Uncertainty Principles * Waveform design * Wavelet Theory and Applications.

The series includes professional monographs, advanced textbooks, and cohesive and carefully edited contributed works.

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ANHA Series Preface

The *Applied and Numerical Harmonic Analysis* (ANHA) book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art ANHA series.

Our vision of modern harmonic analysis includes a broad array of mathematical areas, e.g., wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, deep learning, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods.

The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key

role of ANHA. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

*Analytic Number Theory * Antenna Theory * Artificial Intelligence * Biomedical Signal Processing * Classical Fourier Analysis * Coding Theory * Communications Theory * Compressed Sensing * Crystallography and Quasi-Crystals * Data Mining * Data Science * Deep Learning * Digital Signal Processing * Dimension Reduction and Classification * Fast Algorithms * Frame Theory and Applications * Gabor Theory and Applications * Geophysics * Image Processing * Machine Learning * Manifold Learning * Numerical Partial Differential Equations * Neural Networks * Phaseless Reconstruction * Prediction Theory * Quantum Information Theory * Radar Applications * Sampling Theory (Uniform and Non-uniform) and Applications * Spectral Estimation * Speech Processing * Statistical Signal Processing * Super-resolution * Time Series * Time-Frequency and Time-Scale Analysis * Tomography * Turbulence * Uncertainty Principles *Waveform design * Wavelet Theory and Applications

The above point of view for the ANHA book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of “function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor’s set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener’s Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but is a fundamental tool for analyzing the ideal structures of Banach algebras. It also provides the proper notion of spectrum for phenomena such as white light. This latter process leads to the Fourier analysis associated with correlation functions in

filtering and prediction problems. These problems, in turn, deal naturally with Hardy spaces in complex analysis, as well as inspiring Wiener to consider communications engineering in terms of feedback and stability, his cybernetics. This latter theory develops concepts to understand complex systems such as learning and cognition and neural networks; and it is arguably a precursor of deep learning and its spectacular interactions with data science and AI.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory.

The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'etre of the ANHA series!

College Park, MD, USA — John Benedetto
Wojciech Czaja
Boston, MA, USA — Kasso Okoudjou

John Rowland Higgins

Biography

John Rowland Higgins was born on 17 August 1935 in Hitchin, a market town 40 miles north of London. During the Battle of Britain in 1940, bombers and fighters frequently flew overhead to or from London. By the age of 5, Rowland, always known by his second name, far from being frightened, could identify the planes and would stand in the garden calmly naming them. He had also become interested in bird watching; his interest in flying, be it of planes or birds, remained with him all his life. His father, who had volunteered for the Royal Navy, died in 1940 and life became harder for the family. Rowland and his older sister Nancy both attended a local infants school. When 8, Rowland left and went to a nearby boys preparatory boarding school. From there, at 13, he went on to Aldenham, an independent secondary boarding school for boys.

After passing School Certificate (the forerunner of the current General Certificate of Secondary Education), Rowland left school, having just turned 16, and following the family tradition and his interest in planes, joined the Fleet Air Arm, the naval arm of the Royal Air Force, to be a pilot. He completed his cadet training for the Royal Navy and then did his flying training in the United States. However, after four years in the Navy, Rowland made a remarkable career change. He decided that mathematics was more worthwhile than flying jet fighters off and onto aircraft carriers and, with characteristic determination, left the Fleet Air Arm and began a completely different career in mathematics by preparing for university as a mature student at Sir John Cass College, which was associated with the University of London. Having qualified for university entrance, he went on to graduate in 1961 with a BSc in Mathematics from the University of London. The next step was graduate school in the United States, with a Teaching Assistantship at the University of Minnesota Institute of Technology at Minneapolis while working for a master's degree. He stayed on there to study for a PhD in Mathematics. He showed independence in choosing his topic and was described as "going against the flow." Characteristically, he continued in the direction he had chosen, was successful, and

was awarded a PhD in 1970. While at the University he met Nancy Gustafson, a librarian at the University Library. They were married in Minneapolis, Minnesota and moved to England in 1970.

By now in his mid-30s, Rowland took up an academic post in the Mathematics and Statistics Division at the Cambridgeshire College of Arts and Technology. He and Nan bought a house with a large vegetable garden in Bourn, a village outside Cambridge, where they brought up their two daughters. At that time, the College provided technical and academic qualifications for professions and universities. For Rowland it had the huge advantage of being in the same city as the Cambridge University Mathematics Department and the famous University Library, an invaluable facility for academic and scholarly research. Through his research, Rowland made and maintained contact with mathematicians and engineers in the UK and abroad, often travelling with Nan when visiting them. In 1995 he and Nan went to an international conference/workshop on Signal Theory at the University of Riga, where the participants included Professor P. L. Butzer, Professor H. Feichtinger and Professor A. J. Jerri. This was a seminal meeting, as the participants appreciated the important role of sampling both in signal processing and in mathematics, with the prospect of exciting advances arising from the overlap between mathematical and engineering research. The idea of regular international conference on sampling theory and its applications for mathematicians and engineers with interdisciplinary interests to be held all over the world, combined with establishing an international journal, was clearly desirable, so the Sampling Theory and Applications (SAMPTA) Committee was established to organize them.

This was a very successful venture, starting from a relatively small workshop/conference to the present fully-fledged biennial conferences held all over the world and attracting an unusually even mix of mathematicians and engineers as well as publishing an international journal. Friendly, modest and approachable, an advocate of collegiality and collaboration, with sound judgement and an encyclopaedic knowledge of sampling that he was always happy to share, Rowland played an active part in the development of SAMPTA from its inception at Riga. From serving on the original Steering Committee of the SAMPTA meetings, he became a valued mainstay of the SAMPTA Community, encouraging and advising younger members and, drawing on his extensive mathematical knowledge, pointing out new interconnections in the theory. His university was pleased by the international recognition of his research and had provided him with considerable financial support.

The latter half of the twentieth century was a period of university expansion in the UK and in 1992 the College merged with other regional academic institutions to form part of the Anglia Polytechnic University, which in 2005 was renamed Anglia Ruskin University. However, in this process, Mathematics was absorbed into Computing and Statistics, an all too common fate for small departments. After nearly 30 years of service, Rowland felt it time to retire and left in 1999 as a Professor Emeritus, in recognition of his outstanding contribution to mathematics.

On 31 December 1999, Rowland, who was also a bell-ringer at the Bourn village church, rang in the new Millennium until 3 am, 1 January 2000. On the first day

of the new Millennium, he and Nan drove to France, moving to the village of Montclar, near Carcassonne, for a last adventure and lived happily there for 20 years. They enjoyed living in France, exploring the Languedoc and trying the local cuisine and wines. They renovated their house and organised the garden, which had been part of an old vineyard. Hospitable and welcoming company, they had many visits from family and friends. In addition, Rowland, who had remained a keen birdwatcher, continued his birdwatching in the surrounding countryside, often with Paulo Ferreira, a SAMPTA friend from Portugal. Rowland never lost his early interest in aviation and for his 70th birthday was taken for a spin over Carcassonne in a small 2-seater, briefly taking over the controls.

Mathematically, he had a fruitful two decades of remarkably varied and collaborative research, from meticulously tracing the sometimes obscure origins of key developments in sampling (including an account of interpolation by Babylonian mathematicians to solve a compound interest problem 4000 years ago!) to important advances in the current theory. He continued to do mathematics, producing a paper almost annually, attending conferences and lecturing and maintaining his close links with SAMPTA.

They returned to Cambridge early in 2020 to be closer to their children. Sadly Rowland died on 24 February 2020, leaving his widow Nan, two daughters Jenny and Emily, each with a grandson, Solomon and Joe, respectively, and his sister Mrs. Nancy Hughes.

We are grateful to Mrs. Nan Higgins and Mrs. Nancy Hughes for information and help with preparing this brief account of his life.

Publications

Higgins' paper *Five short stories about the cardinal series*, Bulletin of the AMS, Volume 12, Number 1, pp. 45–89 (1985) is one of the most referenced articles about Sampling Theory. It is "required reading" for all in the field, and has been frequently passed from teacher to student. The paper begins with a short table of contents.

CONTENTS

Many of Higgins' papers, appropriately, fit into the categories established by these "short stories."

- An interpolation series associated with the Bessel-Hankel transform.
  J. London Math. Soc., **5**, (2), 707–714, (1972).
- A sampling theorem for irregularly spaced sample points.
  IEEE Trans. Inform. Theory, **IT- 22**, (5), 621–622, (1976).
- *Completeness and basis properties of sets of special functions.*
  Cambridge Tracts in Mathematics, No. 72. Cambridge University Press, Cambridge-New York-Melbourne, (1977).
- Some orthogonal and complete sets of Bessel functions associated with the vibrating plate.
  Math. Proc. Cambridge Philos. Soc., **91**, (3), 503–515, (1982).
- Five short stories about the cardinal series.
  Bull. AMS, **12**, (1), 45–89, (1985).
- A fresh approach to the derivative sampling theorem.
  Mathematics in signal processing, Inst. Math. Appl. Conf. Ser. New Ser. Bath, 12, Oxford Sci. Pub., (1985), 25–31, Oxford Univ. Press, New York, (1987).
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  Applied Analysis., **41**, (1–4), 155–169, (1991).
- Sampling and aliasing for functions band-limited to a thin shell.
  Numer. Function Analysis Optimization, **12**, (3–4), 327–337, (1991).
- with Beaty, M. G., Dodson, M. M.,
  Approximating Paley-Wiener functions by smoothed step functions.
  J. Approx. Theory **78**, (no. 3), 433–445, (1994).
- Sampling theory for Paley-Wiener spaces in the Riesz basis setting.
  Proc. Royal Irish Acadamy, **94**, (2) Sect. A, 219–236, (1994).
- with Beaty, M. G.,
  Aliasing and Poisson summation in the sampling theory of Paley-Wiener spaces.
  J. Fourier Anal. Appl., **1**, no. 1, 67–85, (1994).
- Sampling for multi-band functions.
  Mathematical analysis, wavelets, and signal processing (Cairo, 1994),
  Contemp. Math., (190), Amer. Math. Soc., Providence, RI, 165–170, (1995).
- An appreciation of Paul Butzer's work in signal theory.
  Results Math., **34**, (1–2), 3–19, (1998).
- *Sampling Theory in Fourier and Signal Analysis: Foundations*, Clarendon Press, Oxford, (1996).

- with Stens, R. L. (Eds.),
  Sampling Theory in Fourier and Signal analysis: Advanced Topics,
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- with Schmeisser, G., Voss, J. J.,
  The sampling theorem and several equivalent results in analysis.
  J. Comput. Anal. Appl., **2**, no. 4, 333–371, (2000).
- with Butzer, P. L., Stens, R. L.,
  Sampling theory of signal analysis.
  Development of mathematics 1950-2000, 193–234, Birkhäuser, Basel, (2000).
- Sampling theorems from the iteration of low order differential operators.
  Modern sampling theory, 219–227, *Appl. Numer. Harmon. Anal.*, Birkhäuser Boston, Boston, MA, (2001).
- A sampling principle associated with Saitoh's fundamental theory of linear transformations.
  Analytic extension formulas and their applications (Fukuoka, 1999/Kyoto, 2000), 73–86, *Int. Soc. Anal. Appl. Comput.*, 9, Kluwer Acad. Publ., Dordrecht, (2001).
- The 'Riesz basis method' for deriving sampling series: an overview and some applications.
  Trends in industrial and applied mathematics (Amritsar, 2001), 63–76, *Appl. Optim.*, 72, Kluwer Acad. Publ., Dordrecht, (2002).
- Historical origins of interpolation and sampling, up to 1841.
  Sampl. Theory Signal Image Process, **2**, no. 2, 117–128, (2003).
- with Butzer, P. L., Stens, R. L.,
  Classical and approximate sampling theorems; studies in the $L^p(\mathbb{R})$ and the uniform norm.
  J. Approx. Theory, **137**, no. 2, 250–263, (2005).
- Some groupings of equivalent results in analysis that include sampling principles.
  Sampl. Theory Signal Image Process., **4**, no. 1, 19–31, (2005).
- Integral transforms and sampling theorems.
  Integral Transforms Spec. Funct., **17**, no. 1, 45–52, (2006).
- Linear interpolation and a clay tablet of the Old Babylonian period.
  Sampl. Theory Signal Image Process, **6**, no. 3, 243–247, (2007).
- The Riemann zeta function and the sampling theorem.
  Sampl. Theory Signal Image Process., **8**, no. 1, 1–12, (2009).
- with Beaty, M. G., Dodson, M. M., Eveson, S. P.,
  On the approximate form of Kluvánek's theorem.
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- with Butzer, P. L., Nashed, M. Z.,
  Abdul Jerri—an appreciation on his seventy-seventh birthday.
  Sampling Theory Signal Image Processes, **8**, no. 3, 209–223, (2009).
- Paley-Wiener spaces and their reproducing formulae.
  Progress in analysis and its applications, World Sci. Publ., Hackensack, NJ, 273–279, (2010).
- with Butzer, P. L., Ferreira, P. J. S. G., Schmeisser, G., Stens, R. L.,
  Interpolation and sampling: E. T. Whittaker, K. Ogura and their followers. *J. Fourier Anal. Appl.*, **17**, no. 2, 320–354, (2011).
- with Butzer, P. L., Ferreira, P. J. S. G., Schmeisser, G., Stens, R. L.,
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  Applied Analysis, **90**, no. 3–4, 431–461, (2011).
- with Butzer, P. L., Dodson, M. M., Ferreira, P. J. S. G., Lange, O., Stens, R. L.,
  Multiplex signal transmission and the development of sampling techniques: the work of Herbert Raabe in contrast to that of Claude Shannon.
  Appl. Anal., **90**, no. 3–4, 643–688, (2011).
- with Ferreira, P. J. S. G.,
  The Establishment of Sampling as a Scientific Principle—A Striking Case of Multiple Discovery.
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- Sampling in reproducing kernel Hilbert space.
  New perspectives on approximation and sampling theory, 23–38, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, Cham, (2014).
- Converse sampling and interpolation.
  Sampl. Theory Signal Image Process, **14**, no. 2, 145–152, (2015).
- with Dodson, M. M.,
  Sampling theory in a Fourier algebra setting.
  Sampling: Theory and Applications, 51–91, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, Cham, (2020).
- Claude Shannon: American genius.
  Sampling: theory and applications, 1–7, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, Cham, (2020).

Rowland was a gift to our community. Through his deep interest in the rich mathematics of sampling theory and his wide knowledge of its fascinating history, he provided for all of us an informed perspective and a sense of the history of the subject. We were able to see the deep roots and interconnections of sampling and appreciate the tremendous potential of this theory, as it branched out to exciting new areas, even beyond the genius of Shannon.

However, even more brilliant than the excellence of Rowland as a mathematician and lecturer, was his role to our community as a friend and mentor. With an encyclopaedic knowledge of sampling which he wore lightly, he was in a real sense a scholar and a gentleman. His kindness, generosity and courtesy brought out, in many ways, our best. And for that gift we will always be grateful.

Washington, DC, USA — Stephen D. Casey
York, UK — M. Maurice Dodson

Sampling: Theory and Applications – A History of the SAMPTA Meetings

Abstract SAMPTA (**Sampl**ing : **T**heory and **A**pplications) is a biennial interdisciplinary international conference for mathematicians, engineers, and applied scientists. The main purpose of SampTA is to exchange recent advances in sampling theory and to explore new trends and directions in the related areas of application. The SampTA conferences are a bridge between the mathematical and engineering signal processing communities. There have been 13 official meetings and a preliminary "pre-meeting," at which the concept of the conference was developed. The conferences included talks and papers on signal and image processing, compressed sensing, frames, geometry, wavelets, non-uniform and weighted sampling, finite rate of innovation, universal sampling, time-frequency analysis, operator theory, and, of course, traditional sampling from both a mathematical and engineering perspective (A-to-D conversion). The purpose of this chapter is to give the reader an overview of SampTA and its contribution to the mathematical and engineering communities. Professor J. R. Higgins, to whom this volume is dedicated, was one of the founding members of SampTA and whose work inspired many researchers in the field.

Introduction: Overview of SAMPTA

The genesis of the SAMPTA meetings occurred at the *International Conference on Mathematical Analysis and Signal Processing*, held at Cairo University in Egypt, January 3–9, 1994. The conference was organized by Profs. A. Zayed, M. Ismail, Z. Nahed, and A. Ghaleb. Participants included Prof. J. R. Higgins, Prof. P. L. Butzer, Prof. Z. Nashed, Prof. A. Jerri, and Prof. G. Walter. At the Cairo conference, a meeting was planned—the first SAMPTA meeting. The prospect of exciting advances arising from the overlap between mathematical and engineering research led to the idea of regular international conference on sampling theory and its applications for mathematicians and engineers with interdisciplinary interests to be held all over the world.

The first official SAMPTA meeting occurred during 1995 in Riga, Latvia, an international conference/workshop on Signal Theory at the Institute of Electronics and Computer Science in Riga, Latvia. Participants included Prof. J. Rowland Higgins, Prof. P. L. Butzer, Prof. H. Feichtinger, Prof. A. Jerri, Prof. F. Marvasti, and Prof. A. Zayed, the core group that organized and sustained the SAMPTA meetings. The Riga meeting was particularly fruitful. A Sampling Theory and Applications (SAMPTA) Committee was established to organize the meetings, combined with establishing an international journal—*Sampling Theory in Signal and Image Processing (STSIP)*. In a spirit true to the SAMPTA meetings, *STSIP* was proposed during a walk along the coast by some conference attendees. *STSIP* continues today as *Sampling Theory, Signal Processing, and Data Analysis*, and focuses on the mathematics relating to sampling theory, signal processing, data analysis, and associated recovery problems from partial or indirect information. This journal continues the SAMPTA mission of inducing interactions leading to cross-disciplinary advances.

The second SAMPTA meeting was two years later, in the University of Aveiro in Portugal, and was chaired by Prof. Paulo J. S. G. Ferreira. These first two meetings established the SAMPTA meetings as a regular event in the harmonic analysis/signal processing communities.

The participants appreciated the important role played by sampling both in engineering and in mathematics. This role was envisioned in Claude Shannon's seminal papers. Shannon placed what we commonly refer to as the Classical (a.k.a., Whittaker-Kotel'nikov-Shannon (WKS)) Sampling Theorem as a cornerstone of information theory in his paper *Communications in the presence of noise*.[1]

SAMPTA is a very successful venture, starting from a relatively small workshop/conference to the present fully-fledged biennial conferences held all over the world and attracting an unusually balanced mix of mathematicians and engineers and publishing an international journal. The conference is now a biennial inter-

[1] Shannon himself was careful to note that the Sampling Theorem did not originate with him. The history of the theorem is rich, and includes the names Cauchy, Raabe, and Ogura, among others. Prof. Higgins wrote extensively on this.

disciplinary international conference for mathematicians, engineers, and applied scientists. The main purpose of SAMPTA is to exchange recent advances in sampling theory and to explore new trends and directions in the related areas of application. SAMPTA has focused on such fields as signal processing and image processing, coding theory, control theory, real analysis and complex analysis, harmonic analysis, and the theory of differential equations. The conference has always featured plenary talks by prominent speakers, special sessions on selected topics reflecting the current trends in sampling theory and its applications to the engineering sciences, as well as regular sessions about traditional topics in sampling theory, and poster sessions.

• **The intellectual merit of** SAMPTA**:** The SAMPTA conferences are a bridge between the mathematical and engineering signal processing communities. The mix between mathematicians and engineers is unique, and leads to extremely useful and constructive dialog between the two communities. For example, SAMPTA conferences have had sessions on theory—compressed sensing, frames, geometry, wavelets, non-uniform and weighted sampling, finite rate of innovation, universal sampling, time-frequency analysis, operator theory, and application—A-to-D conversion, computational neuroscience, mobile sampling issues, and biomedical applications. The interaction at SAMPTA pushed the envelopes on these topics forward in both communities. The meetings bring together world-renowned mathematicians and engineers to work on these subjects.

• **The broader impacts resulting from** SAMPTA **:** The SAMPTA conferences have and will continue to serve as a meeting ground for harmonic analysts and electrical engineers, and give graduate students and junior investigators a chance to learn about the developments of the subjects. SAMPTA gives the community an opportunity to interact with some of the leaders in the field in a relaxed and yet very constructive environment. The plenary talks are delivered by top researchers in the fields. The conference papers were reviewed and presented via the *EDAS* system, and then uploaded into *IEEE Xplore*. Longer papers were published as journal papers in *Sampling Theory in Signal and Image Processing (STSIP)* and contributed chapters in Springer-Birkhäuser books in the Applied and Numerical Harmonic Analysis Series.

SAMPTA Meetings

- SAMPTA 2019—University of Bordeaux, Bordeaux, France, July 8–12, 2019
- SAMPTA 2017—Tallinn University, Tallinn, Estonia, July 3–7, 2017
- SAMPTA 2015—American University, Washington, DC, USA, May 25–29, 2015
- SAMPTA 2013—Jacobs University, Bremen, Germany, July 1–5, 2013
- SAMPTA 2011—Nanyang Technical University, Singapore, May 2–6, 2011
- SAMPTA 2009—CIRM, Marseilles, France, May 18–22, 2009
- SAMPTA 2007—Aristotle University, Thessaloniki, Greece, June 1–5, 2007
- SAMPTA 2005—Samsun, Turkey, July 10–15, 2005
- SAMPTA 2003—Strobl, Austria, May 26–30, 2003
- SAMPTA 2001—U. of Central Florida, Orlando FL, USA, May 13–17, 2001
- SAMPTA 1999—Loen, Norway, August 11–14, 1999

- SAMPTA 1997—University of Aveiro, Aveiro, Portugal, July 16–19, 1997
- SAMPTA 1995—Riga Technical Institute, Riga, Latvia, September 20–22, 1995

We also mention "SAMPTA Zero," the *International Conference on Mathematical Analysis and Signal Processing*, held at Cairo University in Egypt, January 3–9, 1994.

Plenary Talks at SAMPTA Conferences

The SAMPTA conferences have and will serve as a meeting ground for mathematicians and engineers. They have also been a strong mix of junior and senior scientists, providing graduate students and junior investigators a chance to learn about the latest developments of the subjects. SAMPTA gives the community an opportunity to interact with some of the leaders in the field in a relaxed and yet very constructive environment. The plenary talks were delivered by top researchers in the fields.

Plenary Speakers for SAMPTA 2019 – Bordeaux

Name	Affiliation
Alexandre d'Aspremont	ENS Paris, France
Bubacarr Bah	AIMS, South Africa
Marcin Bownik	University of Oregon, USA
Massimo Fornasier	T.U. Munich, Germany
Anna Gilbert	University of Michigan, USA
Rémi Gribonval	INRIA Rennes, France
Urbashi Mitra	University of South California, USA
Ursula Molter	University of Buenos Aires, Argentina
Pierre Vandergheynst	EPF Lausanne, Switzerland

Plenary Speakers for SAMPTA 2017 – Tallinn

Name	Affiliation
David Gross	University of Cologne, Germany
Stéphane Jaffard	Université Paris-Est, France
José Príncipe	University of Florida, USA
Justin Romberg	Georgia Institute of Technology, USA
Amit Singer	Princeton University, USA
Rene Vidal	Johns Hopkins University, USA
Laura Waller	University of California, Berkeley, USA
Rachel Ward	University of Texas at Austin, USA

Plenary Speakers for SAMPTA 2015 – Washington, DC

Name	Affiliation
Richard G. Baraniuk	Rice University, USA
Robert Calderbank	Duke University, USA
Laurent Demanet	Massachusetts Institute of Technology, USA
Yonina Eldar	Technion, Israel
Pascal Frossard	EPF Lausanne, Switzerland
Stanley Osher	University of California Los Angeles, USA
Thomas Strohmer	University of California Davis, USA
Alexander Ulanovskii	University of Stavanger, Norway

Plenary Speakers for SAMPTA 2013 – Bremen

Name	Affiliation
Emmanuel Candès	Stanford University, USA
Wolfgang Dahmen	RWTH Aachen, Germany
Hans Feichtinger	University of Vienna, Austria
Piotr Indyk	Massachusetts Institute of Technology, USA
Michal Irani	Weizmann Institute, Israel
Nikolai Nikolski	Université Bordeaux, France
Jan Rabey	UC Berkeley, USA
Yannis Tsividi	Columbia University, USA
Roman Vershynin	University of Michigan, USA

Plenary Speakers for SAMPTA 2011 – Singapore

Name	Affiliation
Martin Vetterli	EPF Lausanne, Switzerland
Stéphane Mallat	École Polytechnique, France
John Benedetto	University of Maryland, USA
Graham Goodwin	University of Newcastle, Australia
Albert Cohen	Université Pierre et Marie Curie, France
Shen Zouwei	Nanyang Technical University, Singapore
Willy Sansen	Katholieke Universiteit Leuven, Belgium
Steven Smale	University of California, Berkeley, USA

Plenary Speakers for SAMPTA 2009 – Marseilles

Name	Affiliation
Yurii Lyubarskii	NTNU, Trondheim, Norway
Mauro Maggioni	Johns Hopkins University, USA
Ron DeVore	Texas A&M University, USA
Jean-Luc Starck	CEA-Saclay, France

Plenary Speakers for SAMPTA 2007 – Thessaloniki

Name	Affiliation
Michael Unser	EPF Lausanne, Switzerland
Sergei Avdoni	University of Alaska, USA
Antonios Melas	University of Athens, Greece
Sinan Güntürk	NYU-Courant, USA
Martin Vetterli	EPF Lausanne, Switzerland
Ognyan Kounchev	Bulgarian Academy of Sciences, Bulgaria
Joel Tropp	California Institute of Technology, USA
Ioannis Pitas	Aristotle University, Thessaloniki, Greece

Plenary Speakers for SAMPTA 2005 – Samsun

Name	Affiliation
Yoram Bresler	University of Illinois, USA
Ingrid Daubechies	Princeton University, USA
M. Maurice Dodson	University of York, UK
Hans Feichtinger	University of Vienna, Austria
Paulo Ferreira	Universidade de Aveiro, Portugal
Karlheinz Gröchenig	University of Connecticut, USA
Abdul Jerri	Clarkson University, USA
Yurii Lyubarskii	NTNU, Trondheim, Norway
Steven Smale	University of Chicago, USA
Jared Tanner	Stanford University, USA
Ahmed Tewfik	University Of Minnesota, USA

Plenary Speakers for SAMPTA 2003 – Strobl

Name	Affiliation
Vadim Olshevsky	University of Connecticut, USA
Michael Lacey	University of Texas, USA
J. Rowland Higgins	Anglia Ruskin, UK
Amir Averbuch	Tel Aviv, Israel
Jeremy Levesley	University of Leicester, UK

Plenary Speakers for SAMPTA 2001 – Orlando, FL

Name	Affiliation
Stéphane Jaffard	University of Paris, France
N. K. Bose	Pennsylvania State University, USA
John Benedetto	University of Maryland, USA
Michael Unser	EPF Lausanne, Switzerland
P. P. Vaidyanathan	California Institute of Technology, USA
Alberto Grunbaum	University of California- Berkeley, USA
Dennis M. Healy	University of Maryland, USA
Wasfy Mikhael	University of Central Florida, USA

Plenary Speakers for SAMPTA 1999 – Loen

Name	Affiliation
Alexander Petrovskii	Belarusian State University, Belarus
William J. Fitzgerald	University of Cambridge, UK

Plenary Speakers for SAMPTA 1997 – Aveiro

Name	Affiliation
Abdul Jerri	Clarkson University, USA
Michael Unser	EPF Lausanne, Switzerland
John Benedetto	University of Maryland, USA

Plenary Speakers for SAMPTA 1995 – Riga

Name	Affiliation
Hans Feichtinger	University of Vienna, Austria
Paul Butzer	RWTH Aachen, Germany
Ahmed Zayed	University of Central Florida, USA
Ivars Bilinskis	Riga Technical Institute, Latvia
Farokh Marvasti	King's College London, UK

Plenary Speakers for "SAMPTA ZERO" – Cairo

Name	Affiliation
Paul Butzer	RWTH, Germany
Zuhair Nashed	University of Delaware, USA
Carlos Kenig	University of Chicago, USA
W. N. Everitt	University of Birmingham, UK
Charles Chui	Texas A&M, USA
Victor Wickerhauser	Washington University, USA

A Sampling of Special Session Topics at SAMPTA Conferences

Special Sessions for SAMPTA

Compressed sensing and low rank matrix recovery	F. Krahmer, R. Kueng
Frame theory	J. Jasper, D. Mixon
Time-frequency analysis	A. Haimi, J. Romero
Deep learning	M. Belkin, M. Soltanolkotab
Graph signal processing	K. Gröchenig, I. Pesenson
Quantization	S. Dirksen, R. Saab
Mathematical data processing with optimization	A. Bandeira, D. Mixon, D.Needell
Bilinear inverse problems	F. Krahmer, M. Soltanolkotabi
Mathematics of deep learning	H. Boelcskei, P. Grohs, M. Rodrigues
Mathematical theory of quantization	S. Dirksen, R. Saab
Low rank matrix recovery and phase retrieval	R. Balan, D. Gross, X. Li
Shannon sampling 100 years after his birth	G. Pfander, A. Zayed
Time-frequency analysis	K. Gröchenig, J. Romero
Dynamical sampling	A. Aldroubi, J. Bouchot
Sampling and geometry	S. Casey, G. Olafsson, J. Christensen
Frames, nonlinear approximations and the Hilbert transform	A. Aldroubi, S. Kaushik, S. Sharma
Spectral estimation and acoustics	L. Abreu, P. Balasz
Frame Theory	G. Kutyniok, G. Pfander
Dynamic Mobile and Nonlinear Sampling	R. Aceksa, J. Romero, Q. Sun
Sampling in Non-Euclidean Spaces	G. Olafsson
Low Rank Matrix and Tensor Recovery	H. Rauhut
Universal Sampling, Fourier Frames and Riesz Bases of Exponentials	J. Antezana, J. Marzo
Compressed Sensing and Sparsity Based Regularizations	B. Adcock, F. Krahmer
Phase Retrieval	B. Bodmann
A to D Algorithms and Chip Design	L. Fesquet, S. Hoyos, B. Sadler
Sampling Signals with Finite Rate of Innovation in Biomedical Applications	P. Marziliano
Sampling and Stochastic Processes	M. Unser

SAMPTA Organizing Committees

Steering Committee for SAMPTA

Name	Affiliation
Ahmed Zayed	DePaul University, USA
Akram Aldroubi	Vanderbilt University, USA
John Benedetto	University of Maryland, USA
Yonina Eldar	Weizmann, Israel
Paulo Ferreira	Universidade de Aveiro, Portugal
Gitta Kutyniok	TU Berlin, Germany
Farokh Marvasti	Sharif University of Technology, Iran
Götz Pfander	Katholische Universität Eichstätt, Germany
Bruno Torrésani	Aix-Marseille Université, France
Michael Unser	EPF Lausanne, Switzerland

Founding Members for SAMPTA

Name	Affiliation
Paul Butzer	RWTH Aachen, Germany
Hans Feichtinger	University of Vienna, Austria
Karlheinz Gröchenig	University of Vienna, Austria
Rowland Higgins	Anglia Polytechnic University, Cambridge, England
Abdul Jerri	Clarkson University, USA
Andi Kivinuuk	Tallinn University, Estonia
Yurii Lyubarskii	NTNU, Trondheim, Norway
Farokh Marvasti	Sharif University of Technology, Iran
Gerhard Schmeisser	Erlangen-Nürnberg University, Germany
Ahmed Zayed	DePaul University, USA

Chairs of SAMPTA Meetings

Meeting	Name	Affiliation
SAMPTA 2019	Phillipe Jamming	University of Bordeaux
SAMPTA 2017	Andi Kivanuuk	Tallinn University
SAMPTA 2015	Stephen Casey	American University
SAMPTA 2013	Götz Pfander	Jacobs University
SAMPTA 2011	Pina Marziliano	Nanyang Technical University
SAMPTA 2009	Bruno Torresani	Aix-Marseille Université, France
SAMPTA 2007	Nikos Atreas	Aristotle University
SAMPTA 2005	A.Turan Gürkanlı	Samsun, Turkey
SAMPTA 2003	Hans Feichtinger	University of Vienna, Austria
SAMPTA 2001	Ahmed Zayed	University of Central Florida
SAMPTA 1999	Yurii Lyubarskii	NTNU, Norway
SAMPTA 1997	Paulo Ferreira	Universidade de Aveiro, Portugal
SAMPTA 1995	Ivars Bilinskis	Riga Technical Institute, Latvia

We again also mention "SAMPTA Zero," the *International Conference on Mathematical Analysis and Signal Processing*, held at Cairo University in Egypt, January 3–9, 1994. The conference was organized by Prof. A. Zayed *et al*.

Publications

Each presenter at the conference submitted a brief, yet informative paper outlining their main results and justifying their inclusion in the conference. To aid attendees in selecting appropriate talks, the conference papers were available at first by a volume of printed papers, and then later, electronically, at registration. Following a long-standing tradition in the engineering literature, each conference paper for SAMPTA were generally four to five pages. This length was sufficient to contain a careful explanation of the problem to be addressed in the presenter's talk and some of the technical highlights of the solution, without going into extensive detail.

The conference papers were peer-reviewed for technical accuracy and topical relevance by the session organizers. Following on its effective use during SAMPTA 2013, the conference papers were reviewed and published via the *EDAS* system. Starting with SAMPTA 2015, accepted papers were published in *IEEE Xplore*. Longer papers were published in special issues of *Sampling Theory in Signal and Image Processing (STSIP)*, and plenary and special sessions speakers were invited to contribute chapters in books published in the Springer-Birkhäuser Applied and Numerical Harmonic Analysis Series.

Proceedings of SAMPTA Conferences

Meeting affiliation	Number of papers
SAMPTA 2019—University of Bordeaux, Bordeaux, France	133
SAMPTA 2017—Tallinn University, Tallinn, Estonia	144
SAMPTA 2015—American University, Washington, DC, USA	145
SAMPTA 2013—Jacobs University, Bremen, Germany	148
SAMPTA 2011—Nanyang Technical University, Singapore	102
SAMPTA 2009—CIRM, Marseilles, France	108
SAMPTA 2007—Aristotle University, Thessaloniki, Greece	85
SAMPTA 2005—Samsun, Turkey	86
SAMPTA 2003—Strobl, Austria	91
SAMPTA 2001—UCF, Orlando Florida, USA	86
SAMPTA 1999—Loen, Norway	80
SAMPTA 1997—University of Aveiro, Aveiro, Portugal	86
SAMPTA 1995—Riga, Latvia	60

Special Issues in *STSIP* for SAMPTA Meetings

Volume number	Meeting affiliation	Number of papers
Volumes 17	SAMPTA 2017	8
Volumes 15 and 16	SAMPTA 2015	15
Volumes 13 and 14	SAMPTA 2013	20
Volumes 11 and 12	SAMPTA 2011	10
Volume 10	SAMPTA 2009	10
Volume 8	SAMPTA 2007	11
Volume 6	SAMPTA 2005	12
Volume 3	SAMPTA 2003	14

Books Generated by Previous SampTA Meetings

Title	Editors	Meeting
Sampling: Theory and Applications	Casey, Okoudjou Robinson, Sadler	SampTA 2015
Sampling Theory: A Renaissance	Pfander	SampTA 2013
New Perspectives on Approximation and Sampling	Zayed and Schmeisser	SampTA 2013
Proceedings of SampTA 2007	Atreas and Karanikas	SampTA 2007
Sampling, Wavelets, and Tomography	Benedetto and Zayed	SampTA 2001
Modern Sampling Theory	Benedetto and Ferreira	SampTA 1997

SampTA 2023

The SampTA conferences have and will continue to serve as a meeting ground for harmonic analysts and electrical engineers, and will give graduate students and junior investigators a chance to learn about the developments of the subjects. SampTA gives the community an opportunity to interact with some of the leaders in the field in a relaxed and yet very constructive environment. The plenary talks are delivered by top researchers in the fields. The conference papers are reviewed and cataloged. The biennial scheduling of the meetings allows for an opportunity to establish new milestones in the harmonic analysis and signal processing communities.

SampTA 2023 was organized while this volume was written, and was held at Yale University, New Haven, Connecticut during the week of July 10–14.

Washington, DC, USA — Stephen D. Casey
Aveiro, Portugal — Paulo J. S. G. Ferreira
Chicago, IL, USA — Ahmed Zayed