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Physics of Energy and Energy Efficiency

# **Refrigerators, Heat Pumps and Reverse Cycle Engines**

*Principles, State of the Art and Trends*

**Coordinated by  
Jocelyn Bonjour**

ISTE

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## Refrigerators, Heat Pumps and Reverse Cycle Engines



SCIENCES

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Subject Head – Michel Feidt

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# Foreword

**Michel FEIDT**

*Université de Lorraine, LEMTA, CNRS, Nancy, France*

The book you have in your hands is one of the books from the “Physics of Energy and Energy Efficiency” subject in the Engineering and Systems department.

The subject of “Physics of Energy and Energy Efficiency”, albeit recent, is not new. It is particularly underpinned by a thermodynamic approach, whatever the scale.

The selected aspect will be phenomenological and characterized explicitly in order to emphasize the key concept of “efficiency”, essential for any system or process.

The characterization chosen for the development of this subject has been arranged into four successive books, each strongly correlated with each other, and also with other series within the department:

- *Fundamental Physics of Energy*;
- *Thermodynamics of Heat Engines*;
- *Heat Engines with Inverse Cycles*;
- *Efficiency in Practice*.

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I would like to thank ISTE, the various coordinators, and the authors for their contributions and effective actions, despite the very particular conditions of the moment. We are awaiting and would like to encourage comments, suggestions and questions from readers.

# Preface

**Jocelyn BONJOUR**

*CETHIL, INSA Lyon, Villeurbanne, France*

For thousands of years, the quest for energy has been focused on a single purpose: the production of heat (thermal energy), mainly for domestic heating and cooking. Combustion, which practically used to be the only energy conversion technology, was gradually mastered and improved.

Combustion was still at the heart of the Industrial Revolution, which was triggered when heat could be used to produce mechanical work. The title of Sadi Carnot's book is unambiguous: the founder of thermodynamics shared in 1824 his *Reflections on the Motive Power of Fire and on Machines Fitted to Develop that Power*. It was during this period that many thermo-mechanical energy conversion systems were developed. These systems had in common that they were based on cyclic transformations of fluids, then called "engine cycles" or "direct cycles". Thermodynamicists then considered reversing the engine cycle: reverse cycle engines were born, used as refrigerators, heat pumps or even double-function heat pumps.

Climate change and the energy crisis have undoubtedly called for real changes in energy paradigms. Engine cycles will still be widely used to produce mechanical and electrical work, provided that the thermal energy sources are powered less and less by the combustion of fossil fuels. Similarly, the share of thermal energy production derived directly from combustion is bound to decrease over time. Finally, due to global warming, the need for cooling is bound to increase, whether for refrigeration for food safety and sanitary purposes, or for the cooling of premises. It is therefore clear that, more than ever, reverse cycle thermal engines will play an important role in the energy landscape of the coming decades.

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The objective of this book is to offer readers (graduate students, PhD students, engineers, researchers) a state of the art on reverse cycle engines, in order to prepare them for their future positions, as well as to outline the research trends on still emerging technologies.

Thus, Chapter 1 presents a scientific and technical state of the art concerning heating and cooling by the most common reverse cycle engines: vapor compression, ejection, absorption or adsorption engines.

The energy crisis invites us to improve the energy efficiency of systems, whose performances must be evaluated with rigor and precision. This is the purpose of Chapter 2, which develops entropy and exergy analysis methods applied to reverse cycle engines and to the scale of their components. Chapter 3 completes the approach by proposing optimization methods of systems, by way of finite time/finite speed thermodynamics.

The development of mechanical compression engines during the 20th century has made this technology very mature, so that the margins for progress are less than for thermal compression engines (absorption, adsorption), which are therefore the subject of various research projects. There are still some scientific and technical obstacles which are discussed in Chapter 4, as well as the avenues envisaged to overcome them.

Magnetic refrigeration is an emerging technology. It is based on a reverse cycle like the systems mentioned above, but it is a magnetic material (and not a fluid) that undergoes a set of cyclic transformations. Chapter 5 presents the principle of this technology and different current or future applications.

Finally, Chapter 6 presents the thermoelectric effect as an alternative to reverse cycle engines. A good understanding of this physical phenomenon allows us to analyze the performance of thermoelectric refrigeration systems and to identify some applications for which they could be particularly relevant.

We hope that this book will enlighten the reader on the operation and future evolution of all the reverse cycle engines used for heating and cooling, as well as on their essential role in the decades to come.

Lyon  
March 2023

# 1

## Heating and Cooling by Reverse Cycle Engines: State of the Art

**Philippe HABERSCHILL and Rémi REVELLIN**

*CETHIL, INSA Lyon, Villeurbanne, France*

Heat pumps are, from a thermodynamic point of view, no different from refrigerators: in both cases, they are a thermal generator which, thanks to energy consumption, passes heat from a cold source to a hot source (heat sink). On the one hand, the purpose of heat pumps is to provide heat to the hot source (air of a building, domestic hot water, swimming pool, etc.). On the other hand, refrigerating machines make it possible to obtain and maintain a system at a temperature lower than the ambient temperature. To do this, it is necessary to remove heat from this system or even “produce cold”. There are two main types of reverse cycle thermal generators: vapor compression systems (two-heat-source systems) and systems driven by thermal energy (three-heat-source systems). This chapter presents different configurations of vapor compression systems to describe two common systems driven by thermal energy: absorption systems and ejection systems.

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For a color version of all the figures in this chapter, see [www.iste.co.uk/bonjour/refrigerators.zip](http://www.iste.co.uk/bonjour/refrigerators.zip).

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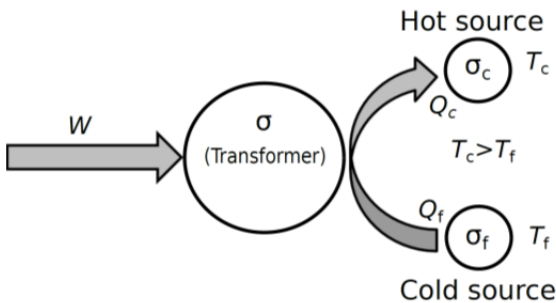
## 1.1. Vapor compression refrigerators and heat pumps

Air refrigeration systems were the first compression refrigerators used. They are increasingly being abandoned (except in particular in the field of very low temperatures: cryogenics) in favor of vapor compression systems, and therefore of condensable fluids under the conditions of use. Such machines, thanks to the use of the refrigerant latent heat of change of state, make it possible to obtain refrigerating effects per unit mass of fluid that are clearly superior to those of gas systems. The systems are thus smaller in size.

### 1.1.1. Operation principle of closed-circuit refrigeration installation: definitions

As in heat engines, the system considered is a fluid in cyclic evolution. This fluid, which is intended to exchange heat with the sources, is called the refrigerant.

If the fluid absorbs the amount of heat  $Q_f$  from the cold source (CS), it therefore releases  $Q_c$  to the hot source (HS) (Figure 1.1):



**Figure 1.1.** Thermal generator

$$|Q_c| = Q_f + W \quad [1.1]$$

where  $W$  is the mechanical (or other) energy received.

NOTE.— The purpose of a refrigerator is to extract  $Q_f$  from the cold source, whereas the purpose of a heat pump is to deliver  $|Q_c|$  to the hot source. Essentially, these two systems are no different.



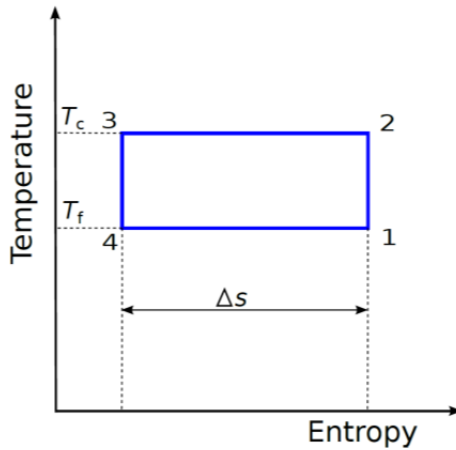
The amount of heat  $Q_f$  taken at the cold source is called the refrigeration effect or cooling capacity.

The ratio  $\varepsilon = \frac{Q_f}{W}$  is called the energy efficiency ratio or coefficient of performance.

Let us look for the expression of  $\varepsilon$  in two different cases: reversible operation and irreversible operation.

#### 1.1.1.1. Reversible operation

Like a heat engine, a thermodynamic generator can operate reversibly (internal and external reversibilities) between two thermal sources only if the evolution cycle of the refrigerant is a Carnot cycle.  $T_c$  is the maximum temperature and  $T_f$  is the minimum temperature (Figure 1.2).



**Figure 1.2.** Carnot engine

Heat and work per unit mass of fluid will be denoted by  $q$  and  $w$ , respectively, expressed in J/kg

$$q_f = T_f \Delta s, q_c = T_c \Delta s \quad [1.2]$$

and

$$w = |q_c| - q_f = (T_c - T_f) \Delta s \quad [1.3]$$

Therefore:

$$\varepsilon_{Carnot} = \frac{T_c}{T_c - T_f} \quad [1.4]$$

with  $s$  being the specific entropy of the fluid expressed in J/kg.K. Moreover,  $\varepsilon_{Carnot}$  is the Carnot energy efficiency ratio. Thus,  $\varepsilon_{Carnot}$  can be greater than 1 depending on the value of  $T_f$  with respect to  $T_c - T_f$ :

$\varepsilon_{Carnot} > 1$  if  $T_f > T_c - T_f \leftrightarrow T_f > \frac{T_c}{2}$  is the most frequent case

$\varepsilon_{Carnot} < 1$  if  $T_f < T_c - T_f \leftrightarrow T_f < \frac{T_c}{2}$ ; this is the case for the liquefaction of certain gases.

### 1.1.1.2. Irreversible operation

For any vapor compression refrigerating machine, the quantities of heat are given by:

$$q_f = T_f |\Delta s_f| \text{ and } q_c = T_c |\Delta s_c| \text{ with } |\Delta s_c| = |\Delta s_f| + s'$$

where  $s'$  represents the entropy generation.

The mechanical energy exchanged is therefore:  $w = (T_c - T_f) |\Delta s_f| + T_c s'$

So

$$\varepsilon = \frac{T_c}{(T_c - T_f) + T_c \frac{s'}{\Delta s_f}} \quad [1.5]$$

For refrigeration machines, as for engines, the Carnot cycle leads to the highest energy efficiency ratio. The difference between any machine and a Carnot machine is measured by the cooling efficiency  $\eta_f$ , which by definition is:

$$\eta_f = \frac{\varepsilon}{\varepsilon_{Carnot}} \quad [1.6]$$

$\eta_f$  also corresponds to the exergy efficiency  $\eta_{ex}$  of the refrigeration cycle if the reference temperature  $T_{ref}$  is considered equal to that of the hot source  $T_c$ . The exergy efficiency is therefore a better indicator of the quality of the thermodynamic cycle than the energy efficiency ratio, in that it is immediately interpreted as a ratio of the actual performance to the ideal performance.

A heat pump is, from a thermodynamic point of view, no different from a refrigerator: in both cases, it is a thermal generator which, thanks to energy consumption, transports heat from a cold source to a hot source. Thus, the previous systems, qualified as “refrigerators”, can all be used as a heat pump. The difference between these two types of machines lies in how they are used. What is interesting about a heat pump is the quantity of heat  $q_c$  which will be supplied to the hot source. This difference of interest gives the definition of the coefficient of performance (COP):

$$COP = \frac{|q_c|}{w} \quad [1.7]$$

This relation is, according to the first law ( $|q_c| = q_f + w$ ), always greater than 1, meaning that these systems are of great theoretical and practical interest. Indeed, unlike other heating processes, this one makes it possible to obtain thermal energy greater than the energy expended to obtain it. The difference of course comes from the energy “pumped” into the cold source.

With the theoretical and technological development of heat pumps being modeled on that of refrigerating systems, there is no need to repeat it here. Emphasis will simply be placed on the difference between the definitions of the energy efficiency ratio  $\varepsilon$ , on the one hand, and of the COP, on the other hand, which of course leads to differences in expressions. For example, the COP of a heat pump operating according to the Carnot cycle is:

$$COP_{Carnot} = \frac{T_c}{T_c - T_f} \quad [1.8]$$

The relative coefficient of performance corresponds to the exergy efficiency  $\eta_{ex}$  (if the reference temperature is equal to the temperature of the cold source) of a heat pump and is given by:

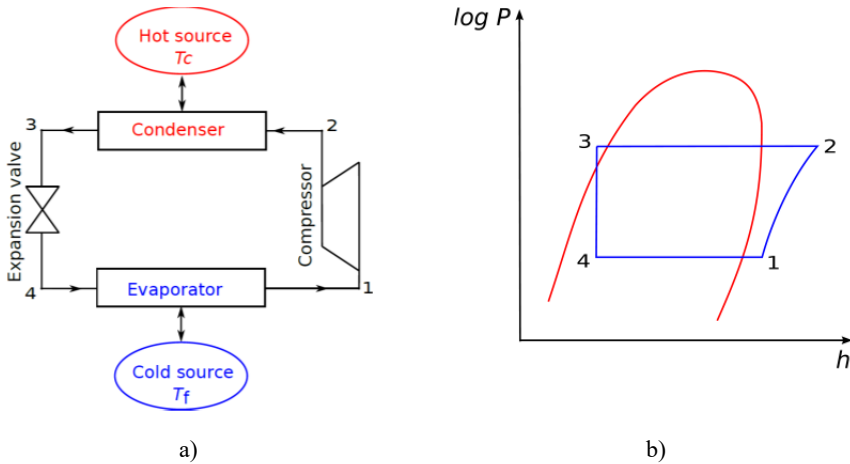
$$COP_{relative} = \frac{COP}{COP_{Carnot}} \quad [1.9]$$

### 1.1.2. Actual cycle with superheating and subcooling

Figure 1.3 shows a diagram of a refrigerator (or heat pump), as well as the evolution cycle of the associated refrigerant. At state 1, the vapor, at low pressure, is either saturated (vapor quality of 1) or superheated (as in the example). The vapor is then compressed in a compressor where its pressure and temperature are increased (point 2). The superheated vapor is then condensed in a condenser at the outlet of which the fluid is in a liquid state, either saturated (vapor quality equal to 0) or

subcooled (point 3). The liquid is then expanded in an expansion valve and partial vaporization is observed (vapor quality of around 0.2–0.3 at point 4). This two-phase fluid is then vaporized in an evaporator to reach state 1.

It should be noted that for a domestic refrigerator, the cold source corresponds to the refrigerated enclosure, while the hot source is represented by the air in the kitchen. Conversely, for a residential heat pump, the cold source and the hot source correspond respectively to the outside air and to the fluid to be heated (air, domestic hot water (DHW), etc.) inside the building.



**Figure 1.3.** Classic cycle of a vapor compression refrigerator or heat pump. a) Schematic of the installation. b) Cycle in an enthalpy diagram

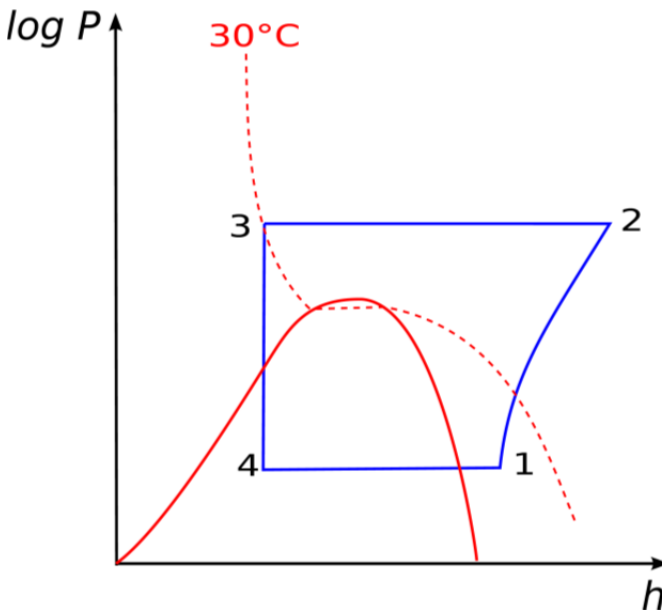
### 1.1.3. Special cycles

#### 1.1.3.1. Transcritical cycles

A pressure higher than the critical pressure of the fluid in the high-temperature exchanger can be a particularity encountered in certain vapor compression cycles. Such cycles are called transcritical. Refrigeration machines using  $\text{CO}_2$  as a refrigerant are often transcritical when the ambient temperature is higher than the critical temperature of the fluid. There is then no more condensation in the “hot” exchanger, but a cooling of the gas. An example of a transcritical cycle is shown in Figure 1.4, which represents a refrigeration cycle. In this figure, there is a strong change in temperature in the high-pressure exchanger: from approximately  $120^\circ\text{C}$  to  $30^\circ\text{C}$ . There are strong thermal differences between the source, whose temperature

varies little, and the refrigerant in the exchanger, which leads to strong transfer irreversibility and contributes to a deterioration in the efficiency in this type of system.

Nevertheless, this strong temperature gradient on the refrigerant can become an advantage in the case of strong variations in the temperature of the hot source, such as in heat pumps intended to produce DHW. In this case, the “hot source”, which is the water to be heated, has a temperature which must change from the network temperature (generally below 20°C) to a temperature above 60°C. The temperature glide of the refrigerant is then partly compensated by the source temperature glide, which reduces irreversibility.



**Figure 1.4.** Example of transcritical cycle (case of  $\text{CO}_2$ )

Note that the HP (high pressure) is no longer conditioned by the condensing temperature, but is left to the discretion of the designer. However, at a constant outlet temperature, there is an optimum pressure:

- below the critical pressure, the refrigeration production is zero because the entire cycle is in the vapor phase;
- at the critical pressure, the cooling capacity is low due to a minimum enthalpy variation in the evaporator;
- for pressures above the critical pressure, the cooling production increases;
- if the pressure is too high, the gain in cooling production is less than the additional cost of compression and the coefficient of performance deteriorates.

The optimum pressure depends on the operating conditions, but is approximately 90 bar for an air-cooled gas cooler with an outlet temperature of 35°C and an evaporation temperature of 0°C.

### 1.1.3.2. Multistage cycles

In certain thermal situations, or for certain refrigerants, the overall compression ratio requires compression to be carried out in two stages, or even more. The fluid passes from one stage to another to cool it in an intermediate exchanger (direct contact or mixer).

Several machine construction schemes are then possible. In all these examples, the compressions will be considered adiabatic. As a result, the compression works will be expressed simply from a difference in enthalpy.

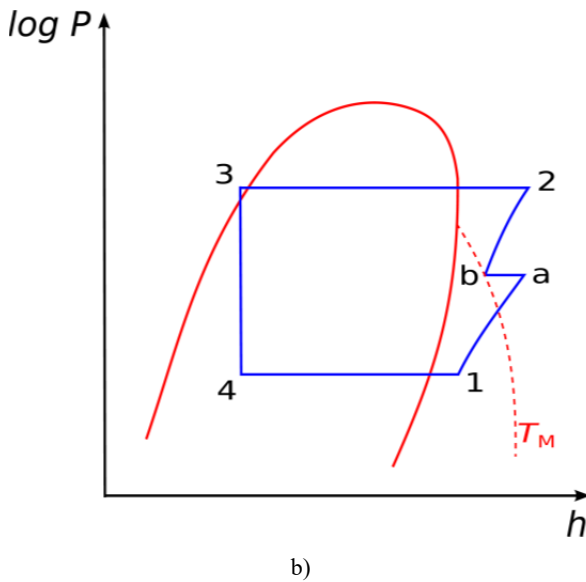
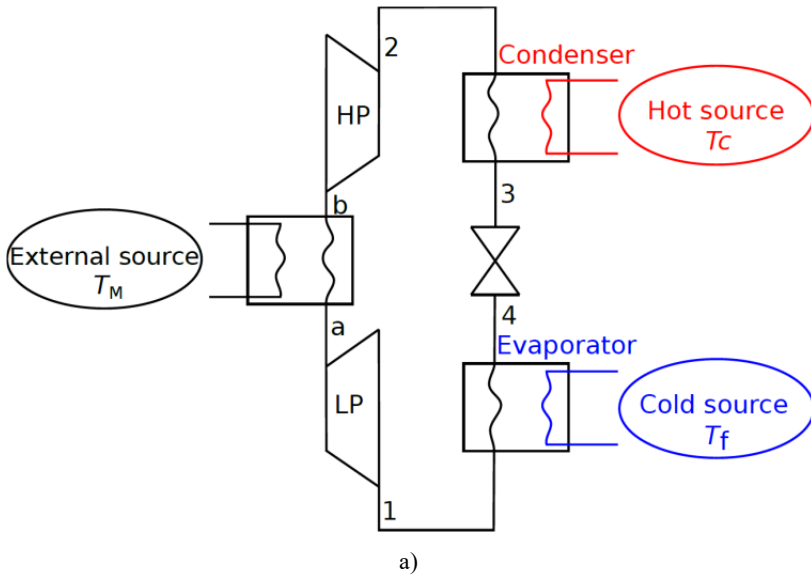
#### 1.1.3.2.1. Heat exchanges with an external source

This system is represented in Figure 1.5(a) with its associated cycle (Figure 1.5(b)). The fluid leaving the LP stage passes through an exchanger connected to the external source at temperature  $T_M$  before entering the HP stage. The sum of the specific work of both stages is clearly lower than the specific work for a single stage. The energy efficiency ratio is expressed by:

$$\varepsilon = \frac{h_1 - h_4}{h_a - h_1 + h_2 - h_b} \quad [1.10]$$

The COP of the installation, meanwhile, is written as:

$$COP = \frac{h_3 - h_2}{h_a - h_1 + h_2 - h_b} \quad [1.11]$$



**Figure 1.5.** Two-stage compression: heat exchange with an external source. a) Diagram of the installation. b) Cycle in an enthalpy diagram

1.1.3.2.2. Heat exchanges with the evaporating fluid

In Figure 1.6(a), the fluid leaving the LP stage is cooled by the cold fluid coming from the expansion of a fraction ( $f$ ) of the fluid leaving the condenser. The energy efficiency ratio is expressed by:

$$\varepsilon = \frac{(1-f)(h_1-h_4)}{h_a-h_1+h_2-h_b} \tag{1.12}$$

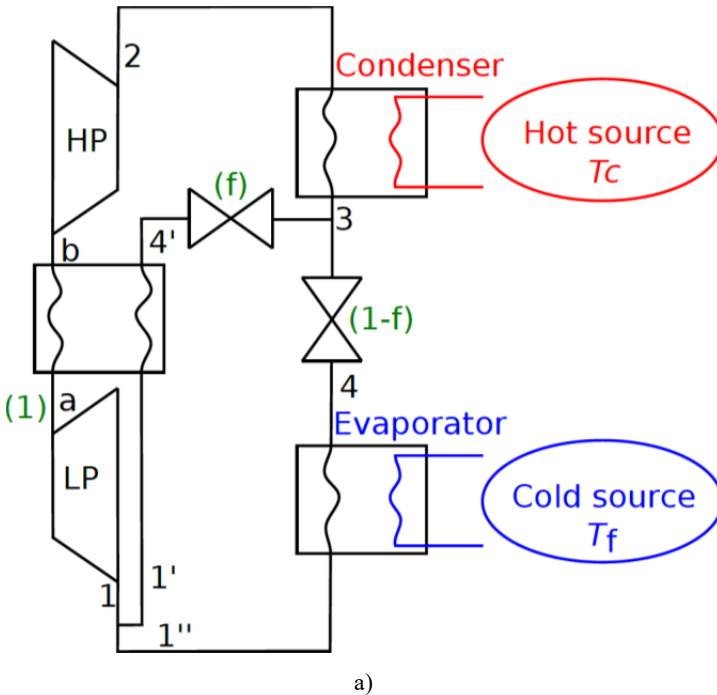
with the fraction  $f$  of fluid removed, which can be determined thanks to an enthalpy balance on the exchanger:

$$f = \frac{h_a-h_b}{h_1-h_4} \tag{1.13}$$

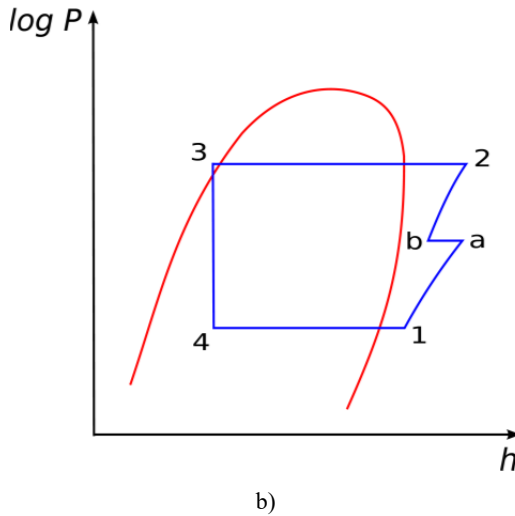
Note that in this case  $h_1 \approx h_{1'} \approx h_{1''}$ , and that  $h_4 \approx h_{4'}$ .

The COP of the installation, meanwhile, is written as:

$$COP = \frac{(h_3-h_2)}{h_a-h_1+h_2-h_b} \tag{1.14}$$







**Figure 1.6.** Two-stage compression: heat exchange with the evaporating fluid. a) Diagram of the installation. b) Evolution cycle of the fluid in an enthalpy diagram. Note that in this case  $h_1 \approx h_{1'}$ ,  $h_{1''}$ , and  $h_4 \approx h_{4'}$

### 1.1.3.2.3. Injection into an intermediate reservoir

In Figure 1.7(a), the fluid leaving the LP stage is cooled by the evaporation (endothermic reaction) of the fraction of the fluid ( $f$ ) leaving 4'. At point b, the fluid is in the form of saturated vapor because it is removed from the upper part of the reservoir containing the two-phase refrigerant in the liquid-vapor state. The evolution cycle in the enthalpy diagram is given in Figure 1.7(b). The energy efficiency ratio is expressed by:

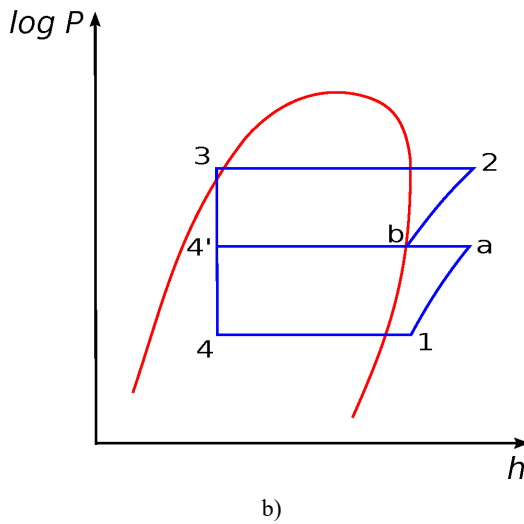
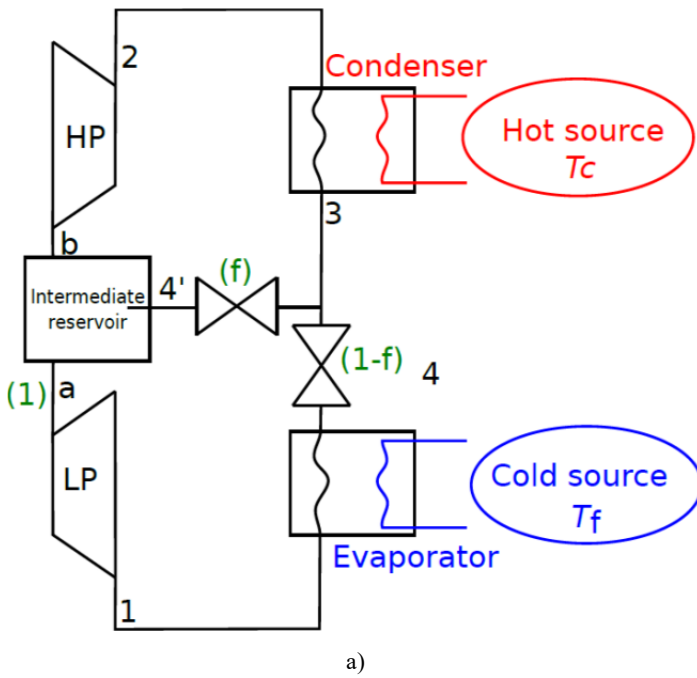
$$\varepsilon = \frac{(1-f)(h_1-h_4)}{(1-f)(h_a-h_1)+h_2-h_b} \quad [1.15]$$

The fraction  $f$  of fluid removed can be determined thanks to an enthalpy balance on the intermediate reservoir:

$$f = \frac{h_a-h_b}{h_a-h_3} \quad [1.16]$$

The COP of the installation, meanwhile, is written as:

$$COP = \frac{(h_3-h_2)}{(1-f)(h_a-h_1)+h_2-h_b} \quad [1.17]$$



**Figure 1.7.** Two-stage compression: injection into an intermediate reservoir.  
 a) Diagram of the installation. b) Evolution cycle of the fluid in enthalpy diagram

#### 1.1.3.2.4. Injection into an intermediate reservoir with staged expansion

An intermediate *mixer*-type heat exchanger (Figure 1.8(a)) at intermediate pressure  $P_i$  receives the superheated vapor, which leaves the low-pressure (LP) stage of the compressor in state a (Figures 1.8(b)), and the fluid from the high-pressure (HP) expansion device in a two-phase state c. The dry saturated vapor (state b) leaves the mixer to enter the HP stage of the compressor. The saturated liquid, which leaves the bottom of the mixer in state d, feeds the LP expansion valve which supplies the evaporator with a fluid in state 4.

For such a refrigerating system, the energy efficiency ratio is given by:

$$\varepsilon = \frac{(1-f)(h_1-h_4)}{(1-f)(h_a-h_1)+h_2-h_b} \quad [1.18]$$

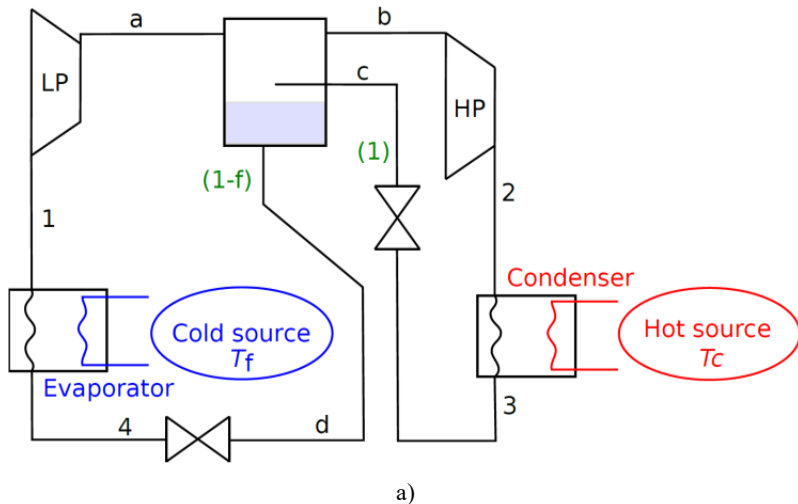
with the fraction  $f$  of fluid removed, which can be determined thanks to an enthalpy balance on the mixer:

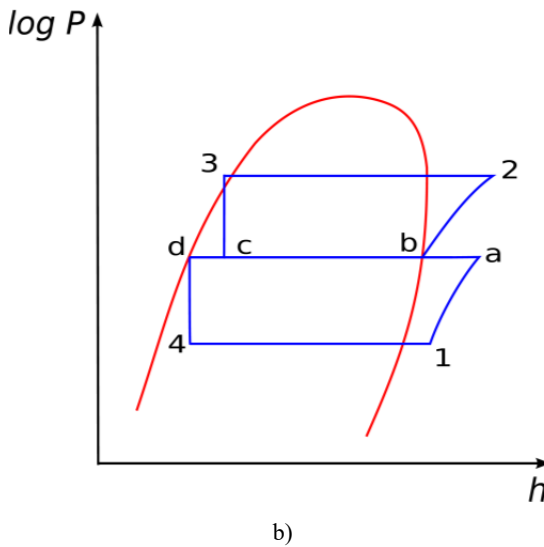
$$f = \frac{h_d-h_a-h_c+h_b}{h_d-h_a} \quad [1.19]$$

The COP of the installation, meanwhile, is written as:

$$COP = \frac{(h_3-h_2)}{(1-f)(h_a-h_1)+h_2-h_b} \quad [1.20]$$

A possible improvement of this system in terms of energy consists of precooling the fluid, leaving the LP stage (state a) by the hot fluid.





**Figure 1.8.** Diagram of a two-stage compression and expansion refrigerating machine and cycles of evolution of the refrigerant. a) Diagram of the installation. b) Evolution cycle of the fluid in an enthalpy diagram

#### 1.1.3.2.5. Cascade cycles

The use of a pure phase change refrigerant remains limited to the temperature interval between the critical point temperature, at which the latent heat of transformation is canceled out, and the triple point temperature, below which any simple mechanical cycling disappears. Moreover, this same temperature difference would cause excessive technical constraints to appear, mainly linked to the volume of the fluid at low pressure, to the difference between high and low pressures, hence correlatively amplified irreversibility.

A judiciously chosen cascade of fluids is then naturally used to ensure lower and lower temperature levels under reasonable pressures. Figure 1.9 shows the block diagram of such an installation in the case of a two-stage cascade. The extension to a higher number of stages is done by iteration.

Currently, two-stage cascade machines are used with  $\text{CO}_2$  in commercial refrigeration, especially in supermarkets as units that can be combined in two different ways:

- $\text{CO}_2$  for both stages (low and high temperatures). In this case, the high-temperature stage is very often transcritical;