

New Frontiers in Regional Science: Asian Perspectives 74

Yasuhiro Sakai

# Games, Decisions, and Markets

 Springer

# **New Frontiers in Regional Science: Asian Perspectives**

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# Games, Decisions, and Markets

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*This book is dedicated to the memory of  
Professor Oskar Morgenstern  
Professor Asatoshi Maeshiro  
Shiro, Toko, and Tora*

## Preface

Ze-a-mi (1363–1443), a legendary Japanese Noh drama writer-performer, once wrote the following maxim:

“Never forget the ideals with which you stated out.”

Carefully looking back at my long career, it seems that I have never forgotten Ze-a-mi’s maxim mentioned above. I was born in Osaka, one of the biggest cities in Japan, just before the Second World War broke out. Between 1943 and 1945, Osaka was bombed more than 30 times, and its damage and casualties caused by repeated bombing were very serious and widely extensive. Although a great number of buildings and private houses were destroyed by bombing, it was a miracle of miracles that my family barely escaped from the large-scale air raid. Then, when I was still a schoolboy, one of my respected teachers gave me lessons about the importance of holding fast to the lofty ideals of my youth. Since Japan was so poor and badly off, I determined to find a way to escape from misery, strongly wishing to become economics professor someday.

In my young student days at Kobe University, there existed two popular yet opposing textbooks in economics. They were nicknamed the “red text” and the “blue text.” In hindsight, this was really a strange start for my student life. The red text was synonymous with the authoritative *Economics Textbook* by the Russian Academy Economics Institute. The issue of “socialism versus capitalism” constituted the central theme of the red text, declaring the proposition that “capitalism is destined to collapse whereas socialism is marching for its final victory over capitalism.” In contrast to the powerful red text mentioned above, the blue text, represented by the popular book *Economics* written by the young American economist Paul A. Samuelson, seemed to be rather modest and even hesitant.

In those days, the red text seemed to give me more impact than the blue text. After some hesitation, however, I boldly decided to go over to United States to continue my graduate study without political and psychological interruptions. I just wanted to get out of the Japanese university disturbances in the 1960s, thus daring to jump into the very core of the capitalist economy.

Very fortunately, in 1968, I was admitted to the Graduate School of Economics, the University of Rochester. Professor Lionel W. McKenzie, a leading professor at Rochester, was one of those pioneers who together with Kenneth J. Arrow and Gerard Debreu greatly contributed to general equilibrium theory. In 1972, I finished my Ph.D. thesis in *Axiomatic Foundations of Consumption and Production Theories*, with McKenzie as my main advisor. Besides, thanks again to McKenzie's recommendation, I could get a teaching job at the University of Pittsburgh in 1971.

At Pittsburgh, I got acquainted with many helpful friends and hard-working students. Professor Asatoshi Maeshiro and his lovely wife Kazuko generously gave material and moral support to me and my wife Tokuko, i.e. the young couple starting a new life in America. After staying in Pittsburgh nicknamed Iron City, however, I gradually began to feel a considerable gap between Economic Theory and Economic Reality, finding increasing interest in Applied Economics. When Oskar Morgenstern, a famous pioneer of the theory of games, happened to give an inspiring lecture on the history of economic theories, he gave me kind advice for researching Games and Uncertainty.

In 1978, Toko and I safely returned to the Land of Rising Sun. We lived in the Tsukuba Academic Town, 100 km north of Tokyo. The striking contrast between New Citizens and Old Farmers gave us not only new and exciting experiences but also a series of unexpected frictions. We learned a lot from the old proverb *Adversity is the parent of virtue*. Then, we eventually settled down in Hikone, a small castle town near Kyoto, which is actually Toko's native place. Speaking of my life full of ups and downs for so many years, I have so frequently changed my place of residence in Japan and the United States, first from Osaka to Kobe, and next to Denver, Rochester, Pittsburgh, Hiroshima, Tsukuba, Tsuchiura, and finally to Hikone.

History sometimes does something cruel. A sequence of so many incredible things and events may happen beyond all expectations. In fact, the seemingly invincible Berlin Wall turned down all of a sudden in 1989, being followed by the total collapse of the mighty Soviet Union into so many independent countries in 1991. Until quite recently, I did not expect at all to return to Hikone and teach *Theory and History* at Shiga University and also at Ryukoku University, old private universities in Kyoto.

This time, when I was generously asked by Mr. Yutaka Hirachi and other staff members of Springer to write my third English book on economics, I racked my brains about what it should be all about. While the first book was entitled *J.M. Keynes Versus F.H. Knight: Risk, Probability, and Uncertainty* and published in 2019, the second book coauthored with my former student Dr. Keisuke Sasaki, Toyo University, was entitled *Information and Distribution: The Role of Merchants in the Market Economy under Uncertainty*, and safely published in 2021, just 2 years ago. In my opinion, the third book should be somehow related to, yet completely independent of, the first and second books. Upon mature consideration, I have determined that the title of the third new book should be *Games, Decisions, and Markets*, which reflects kind and consistent advices from so many scholars including Oskar Morgenstern, Asatoshi Maeshiro, Lionel W. McKenzie, James W. Friedman,



Paul A. Samuelson, Hirofumi Uzawa, Masayoshi Hirota, Edmand Malinvaud, John Hicks, Michio Morishima, Ronald Jones, and Murray Kemp.

This book consists of three parts. Broadly speaking, Part I deals with Games, containing the first three chapters, namely Chaps. 1, 2, and 3. Part II turns to Decisions, consisting of the following four chapters, that is, Chaps. 4, 5, 6, and 7. And Part III is concerned with Markets, being composed of the final two chapters, i.e., Chaps. 8 and 9. In what follows, more detailed discussions of each chapter will be given.

Part I is expected to have a threefold purpose. First of all, it aims to serve as a comprehensive introduction to the whole structure of the Theory of Games, which was first established by the joint work of the famous mathematician John von Neumann and the noted economist Oskar Morgenstern. Second, it systematically discusses the Theory of Games with the help of many graphical illustrations. Third, it specifically deals with zero-sum and non-zero-sum games, with special reference to literary works of Conan Doyle, Edgar Allan Poe, and many others.

To begin with, Chap. 1 gives a historical approach to the Theory of Games. There are two memorial years for the theory—1928 as the Year of Birth and 1944 as the Year of Maturity. It is an extremely important matter to distinguish accurately between “a game under risk” discussed in Conan Doyle’s detective story *The Final Problem* and “a game under uncertainty” shown by Doyle’s story *The Adventure of the Empty House*. I believe that the second kind of games, in which “animal spirits” à la John M. Keynes play a key role in decision-making, remain to be fairly unexplored, requiring future research.

Chapter 2 continues to critically reassess the significance and limitations of zero-sum games. First, I intensively discuss several zero-sum two-person games, with special reference to the Games of Matching Pennies. New graphical illustrations are attempted for clarification of the matter. Then, I turn to novelist Edgar Allan Poe’s best story *The Purloined Letter*, showing how the Poe story may challenge the validity of Neumann-Morgenstern Approach. Next, I shed new light on Conan Doyle’s story *The Final Case* as a variant of the Games of Matching Pennies. Finally, I pick up the Game of Stone, Paper, Scissors, which constitutes another interesting example of zero-sum two-person games.

Chapter 3 critically discusses non-zero-sum games with special reference to Nash equilibrium, which can be regarded as a successor of Cournot equilibrium in the theory of duopoly and oligopoly. I first focus on the “Residence Game,” whose fictional counterpart has often been written in contemporary novels. I pick up the old and the young couples, who have to decide whether they live together or separately, and whether they live in the country or in the city. Now I shed light on “Generation Gaps Problems,” with many other applications. I next turn to the “Battle of the Sexes,” or plainly the “Dilemma of Lovers.” The male and the female have to find good dating spots. While the male prefers to see boxing rather than ballet, the female’s preference is just the opposite. I propose that contrary to the conventional way of reasoning, the introduction of a *third opinion* such as “going to see movies” is good enough to give each couple a *second-best solution*. Finally, comparison between “Econs” and “Humans” is carefully examined.

Part II carefully explores the foundations of consumer and producer decision theories, which constitute the “two keystones” in the grand structure of micro-economic theory. Although its earliest version was my Ph.D. thesis a long time ago, its completely revised draft toward new policy orientation is presented here. Besides, many instructive figures are newly drawn for better understanding.

Chapters 4 and 5 argue the foundations of consumer decision. First, Chap. 4 discusses consumer decision and revealed preference from many angles, thus carefully reexamining the foundations of economic analysis in modern times. It is Paul A. Samuelson’s contribution who boldly introduced to micro-economic theory the brand-new concept of *revealed preference* against the then current doctrine of ordinal utility. The main result of this chapter is that Houthaker’s *strong axiom of revealed preference* holds if and only if Samuelson’s *weak axiom of revealed preference* and Sakai’s *regularity condition* both hold. This new equivalence result distinguishes itself from all the previous works in that neither continuity nor Lipschitz assumptions are made on the demand function. Finally, in the light of uncertainty and behavioral economics in the present times, the significance and limitations of Samuelson-type economics are also discussed.

Chapter 5 studies the foundations of the *indirect utility function*, based on a revealed preference approach à la Paul A. Samuelson. I look at a chain of comparisons of budgets as if it gives a *relation on the normalized-price space* (namely, a *revealed favorability relation*) rather than a relation on the commodity space (namely, a revealed preference relation). In analogy to the weak and strong axioms of revealed preference, the weak and strong axioms of revealed favorability are newly introduced, and a fundamental theorem concerning the relationship between the latter two axioms is skillfully established. Then, the indirect and direct utility functions are effectively derived on the basis of the strong axiom of revealed favorability. It is noted that *neither the continuity of the demand function nor the convexity of its range is required for the approach taken here*.

Chapters 6 and 7 discuss the foundations of producer decision. The purpose of Chap. 6 is to shed new light on input demand theory, which has unfortunately been paid less attention than consumer demand theory. I start our discussion with general production possibility sets rather than specific production functions, deriving what we may call *decomposition equations in input demand theory*. The total effect of a change in the price of a certain input on the demand for another input can be divided into the following two separate effects. They are a *substitution effect along the old isoquant* and an *expansion effect along the new expansion path*. Such a decomposition equation in input demand theory appears to correspond well to the famous Slutsky equation in consumer demand theory. The correspondence between the producer’s decomposition and the consumer’s decomposition, however, is not perfect. How and to what extent they are different should be actually the target of my investigation. In particular, I pay special attention to comparison between the producer’s expansion effect and the consumer’s income effect. In this connection, the effectiveness of LeChatelier-Samuelson principle is also confirmed.

Chapter 7 aims to extend our new approach to input demand theory with a single output to the more general case of *joint production*. Although such an extension

from one output to several outputs seems to be straightforward, it nevertheless requires special care and careful interpretation. More specifically, we are concerned with a systematic examination of the question what happens when the producer that has been in equilibrium at certain prices of inputs and prices of outputs experiences a change in one of those prices. *The Mutual Effects between inputs and between outputs and the Cross Effects between inputs and outputs* are newly explored by means of *various kinds of decomposition equations in production theory*. A definition of the *normal technology* is given to show that inputs are not gross substitutes, nor are outputs, and that the input–output relations are not regressive. This is actually an extension of the Trout Rader result to the case of joint production.

Part III shifts again my investigation from micro-economics to general equilibrium theory. More specifically, close interdependence of several markets at one country level, and international trade model in the presence of risk are explored both intensively and extensively.

Chapter 8 aims to shed new light on the Hicks-Morishima approach to the interdependence of several markets. In spite of its rather simple and ambitious framework, it is quite unfortunate that this approach has been rather neglected in the academic circle. I think that there must exist several reasons for such underestimation. First, the standard general equilibrium approach developed by Lionel W. McKenzie, Gerald Debreu, and Kenneth W. Arrow exclusively works with the commodity space rather than the price space. In contrast, the Hicks-Morishima approach based on Hicks' classical book *Value and Capital* uniquely operates on the price space, thus against the current mainstream of economic theory. Next, the majority of economics readers are usually familiar with straightforward notion of demand and supply curves, but not with the *twisted concept of excess demand curves*. It is one of my main purposes to mend such unlucky tendency, thus proceeding toward the establishment of a *new grand system of social science*.

The risk-free, two-sector, constant-returns-to-scale model has long served as a standard model of international trade, with Ronald W. Jones and Murray C. Kemp being its key promoters. The purpose of Chap. 9 is to make an effort to extend the basic theorems of the standard model to *cover new situations with price risk*. The question to ask is whether and to what extent those results are still applicable to the *new world under risk*. It is shown that when firms in the stochastic sector exhibit decreasing absolute risk aversion, the Rybczynski theorem and the Stolper-Samuelson theorem may fail to hold for some important cases, whereas the factor-price equalization theorem cannot be carried over to the stochastic world at all. The basic reason for such annoying results is actually the presence of *risk-bearing fee* associated with price risk. In short, when any form of risk or uncertainty is introduced into our model, many of the traditional results may lose their validity. In the real world, uncertainty really matters.

The preparation of this book has been made possible by the Japanese Ministry of Education, Culture, Sports, Science and Technology through Grand-in-Aid for Scientific Research. Once again, I would like to say my special thanks to Professor Yoshiro Higano (Editor-in-Chief), Professor Hirotsada Kono (Honorary Editor), Professor Makoto Tawada (Managing Editor), Professor Peter Nijkamp (Chair of

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Yasuhiro Sakai

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# **Part I**

## **Games**

# Chapter 1

## Von Neumann, Morgenstern, and Theory of Games: Critical Reassessment of Zero-Sum Games



**Abstract** This chapter gives a brief yet critical account of the theory of games as jointly developed by the two superstars in the twentieth century; namely, gifted mathematician John von Neumann and brilliant economist Oskar Morgenstern. There are two memorial years for game theory: 1928 as the Year of Birth for Game Theory and 1944 as the Year of Maturity for Game Theory. Even after the basic mathematical skeleton for game theory was provided by von Neumann in his 1928 paper, giving its real body and substance to the skeleton was a difficult job for Morgenstern. In both his 1928 paper and 1935 paper, Morgenstern paid a special attention on the duel between Sherlock Holmes and Professor Moriarty in Conan Doyle's famous detective story, finding an unsolved puzzle between an infinite chain of "out-thinking" and a finite concept of general equilibrium. In 1944, the collaboration of von Neumann and Morgenstern finally produced a difficult bulky book *Theory of Games and Economic Behavior*, whose academic reaction was initially rather quiet, requiring many years for its full recognition by social and natural scientists. It is of utmost importance to draw a definite line between "games under risk," represented by *The Final Problem* in Doyle's detective stories, and "games under uncertainty," shown by *The Adventure of the Empty House*. We believe that the second kind of games, in which "animal spirits" à la John M. Keynes play a key role in decision-making, remain to be fairly unexplored, requiring for future research.

**Keywords** John von Neumann · Oskar Morgenstern · Game theory · The duel between Sherlock Holmes and Professor Moriarty · Sherlock Holmes versus Colonel Moran · Games under risk · Games under uncertainty · The role of animal spirits

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This chapter is an enlarged English version of Sakai (2001), with new figures. I am grateful to Oskar Morgenstern (Princeton), James Friedman (Rochester), Edward Zabel (Rochester), and Mitsuo Suzuki (Tokyo Kogyo) for helpful suggestions.

## 1.1 “Specter of Game Theory”: An Introduction

*The Manifesto of the Communist Party* (German original: *Manifest der Kommunistischen Partei*) was an 1848 pamphlet published as the joint work of two giants in philosophy and social science: Karl Marx (1818–1883) and Friedrich Engels (1820–1895). Although it was a rather small pamphlet containing only 100 pages or so, it has been very influential in the economic profession until today. Remarkably, it had a very famous introduction beginning with the following sentence:

A specter is haunting in Europe—the specter of communism. (Marx and Engels 1848, p. 1)

It seems that a similar phenomenon is taking place in modern economics today. On the analogy of *the Communist Manifesto*, we can metaphorically express such a new phenomenon as follows:

A specter is haunting in the Economics profession the specter of Game Theory.

Interestingly enough, the theory of games, or more simply game theory, was also the joint product of two superstars in science: John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977). The mathematician Neumann and the economist Morgenstern successfully combined their knowledge and skill to a revolutionary theory of games and economic behavior, based on the interactions of strategies of many players. When the final product of their collaboration appeared as a bulky book *Theory of Games and Economic Behavior* (von Neumann and Morgenstern 1944), namely the year near to the end of the long and dreadful Second World War, an authoritative mathematical journal wrote the following impressive sentence in an ecstasy of joy:

Posterity may regard this book as one of the major scientific achievements of the first half of the twentieth century. (*American Mathematical Society Bulletin* 1944)

It seems, however, that Mitsuo Suzuki (1999) had an entirely different opinion of the new theory of game. Although he himself was one of the outstanding pioneers of game theory in Japan, he once lamented over the effectiveness and applicability as follows:

When I [Suzuki] started doing research in game theory, my work was exposed to criticism from Japanese academia. To be honest, I undergo a neglect, or even attack, from so many people for my “apparently fruitless effort.” Then I had to practice my perseverance, sincerely hoping that my lonely effort would be rewarded someday in the not too distance future. (Suzuki 1999, p. 7)

In retrospect, the theory of games was once ignored by so many people and even by feared by some people. There were a number of reasons for such unjustifiable treatment in the academia. First of all, the name “game theory” per se appeared to be provocative and amusing, thus spreading the wrong impression among the general public. In everyday conversation, if we accuse some persons of “playing games,” we mean that they are not serious enough about a difficult situation, or that they are

deliberately misleading us or making us do unnecessary things. For example, people can use trump cards to enjoy many games such as poker and bridge.<sup>1</sup>

It is quite remarkable to see that in commemorating the fifth anniversary of the publication of *Theory of Games and Economic Behavior* (1944), the Princeton University Press made the following announcement:

A great book often requires so many years for its general recognition. When it is finally recognized as such by the public, its influential range will far-exceed the mere readership, thus appealing to the whole society. (*American Scientist* 1949, quoted by Poundstone 1992, p. 63)

To be honest, only 4000 volumes were sold for 5 years from 1944. The original Neumann–Morgenstern book was not only scarcely read by professional economists, but also almost ignored by many libraries. Interestingly enough, however, it was enthusiastically bought by some professional gamblers.

Second, the contents of the book were filled with the apparently strange combination of powerful mathematics and special economics, with strange equations and odd figures/tables. As a result, so many researchers who had a habit of sweeping a troublesome problem under the carpet displayed strong risk aversion to game theory.

Furthermore, the book itself was written in a sort of “German English,” definitely not in standard British English. Since von Neumann was born in Budapest, the Kingdom of Hungary, and Oskar Morgenstern in a small town of the German Empire, their communications were usually done in German rather than English.<sup>2</sup>

Because of those reasons aforementioned, game theory was born as “an unfortunate child.” We must add, however, that when grown up, such “a luckless child” became “a great figure of many talents.” The purpose of this chapter is to briefly discuss how such a great transformation has been accomplished for those long years. As the saying goes, Rome was not built in a day.

The contents of this paper are as follows. In the second section, we will discuss a short history of theory of games, from its lonely birth to popular maturity. It is really important as well as interesting to know how the collaboration between von Neumann and Morgenstern, two distinguished scholars in different fields and from different countries, became possible in the times of hardships and wars. The third

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<sup>1</sup>For the sad history of game theory in its early days, Mitsuo Suzuki once lamented it as follows: “The theory of games was born and regarded long as heretical. Its wide recognition as an important contribution to science took really long years” (Suzuki 1999, p. 7). Suzuki has been a lone pioneer in game theory in Japan and worked hard as a professor at Tohoku University and Tokyo Institute of Technology. Fortunately, his long and lonely struggle is now amply rewarded.

<sup>2</sup>The original book *Theory of Games and Economic Behavior* (1944) was a very bulky book, full of mathematical equations and strange notations. It was treated with “fear and respect.” In commemorating the fifth anniversary of its publication, the Princeton University Press voiced the following announcement in *The American Scientist*: “Recognition of first-rate books often requires so many years. Once it is rightly recognized as such, however, its influence will far-exceed the narrowed scope of readers, probably reaching the general public in the world.” To tell the truth, the volume of the first edition was only 4000. Understandably, it was hardly read by economists and purchased by a limited number of libraries. Remarkably, however, it was bought by some curious gamblers. For details, see Poundstone 1992, p. 63.

section will repeatedly confirm that Arthur Conan Doyle's novel featuring Sherlock Holmes as a famous detective constantly plays a key role for the birth and development of the revolutionary theory of strategies and games. It will be shown that the famous duel between Sherlock Holmes and Professor Moriarty gives us the original form of standard "zero-sum two-person games." As the saying goes, it is really a "mission impossible" to create something from nothing. Several remarks will be made in the final fourth section.

## 1.2 The Two Memorable Years for Game Theory: 1928 as the "Year of Birth" and 1944 as the "Year of Maturity"

This section will outline the birth and development of game theory in a historical perspective. As is seen in Table 1.1, there are the two memorable years for game theory: first, 1928 as the "Year of Birth", and second, 1944 as the "Year of Maturity."

**Table 1.1** John von Neumann and Oskar Morgenstern: their lives and collaboration on game theory

John von Neumann	Oskar Morgenstern
1903 Born in Budapest, Hungary. His father was a wealthy Jewish banker	1902 Born in Görlitz, Germany. His mother was said to be a noblewoman
1928 "Theory of social games" (the German original), this was historically the first mathematical paper on game theory	1928 "Economic forecasting" (the German original). The Holmes-Moriarty duel was first noticed as a "troublesome game"
<i>»1928 was regarded as the YEAR OF BIRTH for game theory</i>	
1933 Appointed as a lifetime professor at the Institute for Advanced Study, Princeton University, New Jersey	1935 "Perfect foresight" (the German original). The Holmes-Moriarty duel was again noted as an obstacle to perfect foresight. Then, Edward Chech pointed Morgenstern to Neumann's paper (1928) above
1937 Became a naturalized citizen of the U.S.	1938 While Morgenstern was visiting American universities, the Nazis took over in Vienna
<i>(Both Neumann and Morgenstern stayed at Princeton)</i>	
1939 After giving an after-luncheon talk, Morgenstern talked with Neumann about game theory. Their talk continued for a long time	
1940 Morgenstern wrote a greatly enlarged version of Neumann's old paper (1928). When Neumann saw it, he suggested to Morgenstern: "Why don't we write this paper together?" Their earnest collaboration began	
1944 The first edition of <i>Theory of Games and Economic Behavior</i> was published by Princeton University Press	
<i>»Thus, regarded as the YEAR OF MATURITY for game theory</i>	
1957 Neumann passed away	1977 Morgenstern passed away

To begin with, we will focus on the first memorable year of 1928. Historically speaking, this is the year in which the world economy reached the height of its prosperity. Although many European countries suffered a great deal from the aftermath of the First World War, they gradually recovered in the late 1920s, so that many people really began to dream of the everlasting capitalist economy. As the old ballade often tells us, however, all things are uncertain and must pass. In 1928, a year earlier than the outbreak of the Great Depression, ordinary people enjoyed their daily lives, being apparently confident of the continuation of their “golden days.”

It is in this 1928 that von Neumann, a native son of Budapest, the Kingdom of Hungary, wrote the following outstanding paper on “social game theory”;

von Neumann (1928), “Zur Theorie der Gesellschaftsspiele,” (English translation: The theory of social game), *Mathematische Annalen*, Vol. 100.

There are two things noticeable. First, this was an old technical paper that was written in 1928, with the language used being German, but not English. Second, although it discussed people’s social games, it was published in a mathematical journal, thus representing the unique combination of social science and mathematics.<sup>3</sup>

John von Neumann was born in Budapest, Kingdom of Hungary, which was then an integral part of the Austro-Hungarian Empire. He was the eldest son of a wealthy and nonobservant Jewish family, with his father being an influential banker. He himself was a very clever boy with bright mathematical skill and often called “child prodigy.” While by the early age of 8, he was familiar with differential and integral calculus, he found special interest in history as well. He started his lectures as a *Privatdozent* at the University of Berlin in 1928. On New Year’s Day in 1930, he married Marietta Kövesi at Budapest University. Von Neumann and Marietta had one child, a daughter, Marina, born in 1935. In the early 1970s, Yasuhiro Sakai became an assistant professor of mathematical economics at the University of Pittsburgh. Very fortunately, Sakai soon got acquainted very well with Maria, who was then highly respected as Professor Marina von Neumann Whitman, eagerly teaching international economics. In hindsight, it was really amazing to trace the existence of “Econ Connection” in mathematician von Neumann’s early career through both his first wife and his only daughter.<sup>4</sup>

<sup>3</sup>For details, see Aumann and Hart (1992), Dasgupta et al. (1992), Dixit and Nalebuff (1991), Luce and Raiffa (1957, rev. 1989), Nakayama (1997), Nakayama et al. (2000), Okada (1996, rev. 2011), Sakai (2001), Shubik (1967, 1982), and Varoufakis (2001).

<sup>4</sup>For the life and work of John von Neumann, see Poundstone (1992). Neumann has only one daughter, whose name is Marina von Neumann Whitman. When Marina was very young, she was a very clever girl, and said to George Gamow, a well-known science writer, that other than pure mathematics, she was much clever than her father, John von Neumann. And she added that even in pure mathematics, she was almost as clever as John (see Preface, Gamow 1947, rev. 1961). In the early 1970s, Sakai was an assistant professor of mathematical economics at the University of Pittsburgh. Sakai still has a very fond memory of Marina, already a noted professor of international economics, who once told Sakai that she had lost much interest in pure and applied mathematics.

In 1933, von Neumann was offered a lifetime professorship at the Institute for Advanced Study, Princeton University. In Princeton, he liked to play loud German march music on his phonograph, annoying his neighbors including Albert Einstein, the creator of the theory of relativity. It seemed that even in the United States, Neumann was fond of German culture and music except “Nazi connection,” enjoying lively conversation with his friends in German.

To put it perhaps a little strongly, Oskar Morgenstern, the hard-working economist whose destiny was to become von Neumann’s good collaborator, seemed to be influenced by German culture more strongly than von Neumann. Morgenstern was born in Görlitz, Germany, a city near the border with Poland. He grew up in Vienna, Austria, and graduated from the University of Vienna and got his Ph.D. in political science, later becoming a professor in economics at the same university in 1928. During his visit to Princeton University in 1938, something he did not expect happened all of a sudden: Adolf Hitler took over Vienna and expanded his political and military influence over the whole Europe. Since Morgenstern was unfairly dismissed as “politically unbearable” from the University of Vienna, he decided to remain in the United States. He became a member of the faculty at Princeton University and particularly gravitated toward the Institute for Advance Study in which von Neumann already stayed as a lifetime professor.<sup>5</sup>

In February 1939, Morgenstern gave an after-luncheon talk on the business cycles at the Nassau Club, von Neumann happened to be there with Niels Bohr, a distinguished particle physicist from Denmark. Both von Neumann and Bohr invited Morgenstern that afternoon for tea at Fine Hall, and all of them spent several enjoyable hours talking about games and experiments. Afterward, Morgenstern and von Neumann had many opportunities to talk about wide-ranging topics. One day, Morgenstern mentioned to von Neumann that Morgenstern was very much interested in studying von Neumann’s (1928) paper on game theory. When Morgenstern had a chance to talk to von Neumann about his pet work on incompatibility between perfect foresight and social games, he had a rather unexpected reply from von Neumann. In all honesty, von Neumann told Morgenstern that von Neumann had done no further work on game theory for those many years after 1928.<sup>6</sup>

This made Morgenstern to study von Neumann’s paper on game theory very seriously. More specifically, Morgenstern referred to the episode of the pursuit of Sherlock Holmes by Professor Moriarty in Conan Doyle’s famous detective story. Morgenstern explained to von Neumann in some detail that Holmes and Moriarty could never be resolved on the basis of one of them “out-thinking” the other. The

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<sup>5</sup>For the life of Oskar Morgenstern and his collaboration with von Neumann, see Morgenstern (1976) and Suzuki (1994). Also see Morgenstern (1958, 1972).

<sup>6</sup>By chance, the year of 1928 was the Year of the Dragon in terms of the Japanese animal calendar. In that memorial year, we saw the birth of many famous Japanese economists including Hirofumi Uzawa as a worldwide pioneer of general equilibrium theory, growth theory, and the economics of global warming, and Mitsuo Suzuki as an outstanding promoter of game theory and its applications. Twelve years later than Uzawa and Suzuki, Sakai was born in 1940, another Year of the Dragon.