Cengiz Kahraman · Irem Ucal Sari · Basar Oztaysi · Selcuk Cebi · Sezi Cevik Onar · A. Çağrı Tolga *Editors* 

# Intelligent and Fuzzy Systems

Intelligence and Sustainable Future Proceedings of the INFUS 2023 Conference, Volume 1



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# **Preface**

INFUS is an acronym for intelligent and fuzzy systems. INFUS 2019 was an on-site conference organized in Istanbul, Türkiye. INFUS 2020 and INFUS 2021 conferences were organized as online conferences because of pandemic conditions. INFUS 2022 conference was organized as both online and on-site conference in Izmir with the cooperation of Yasar University and Izmir Bakircay University. INFUS 2023 is the fifth conference of this series organized by Istanbul Technical University.

The theme of INFUS 2023 conference this year is *Intelligent and Sustainable Future*. Intelligence can be used in a wide variety of ways to manage environmental impacts and climate change such as clean sustainable supply chains, environmental monitoring and enforcement, and advanced weather and disaster forecasting. The emergence of artificial intelligence (AI) and its growing impact on many industries require research into how it can be used to achieve the Sustainable Development Goals. Applications of AI can create a sustainable and eco-friendly future. AI is a promising tool for the production of new materials that help in building a sustainable environment. The sustainability of biological diversity is another very important problem since many animal species are extinct or endangered. Therefore, intelligence can be used to study animal behavior patterns. Soil pollution is another important problem today as population growth, intensive farming, and other activities increase day by day. Since food production is the key to sustain human life, we can maintain environmental sustainability by monitoring crops and soils and maximize the crop yields, while having less impact on the environment through AI-augmented agriculture. The excessive consumption of natural resources by humans has a detrimental effect on water resources. The level of garbage accumulating in the oceans is higher than ever before. Artificial intelligence tools should be used to ensure environmental sustainability. Artificial intelligence can be used in automated garbage collection vehicles; it can help solve problems such as illegal fishing and discharge of industrial wastewater into water bodies and illegal dumping of solid wastes into the seas. The use of intelligence for a livable future has become a necessity. A program focusing on intelligence and sustainability future, which is the theme of this year's INFUS 2023 conference, is foreseen. INFUS 2023 aims to bring together the latest theoretical and practical intelligent and fuzzy studies on sustainable future in order to create a discussion environment.

Researchers from more than 30 countries such as Türkiye, Russia, China, Iran, Poland, India, Azerbaijan, Bulgaria, Spain, Ukraine, Pakistan, South Korea, UK, Indonesia, USA, Vietnam, Finland, Romania, France, Uzbekistan, Italy, and Austria contributed to INFUS 2023. Our invited speakers this year are Prof. Krassimir Atanassov, Prof. Vicenc Torra, Prof. Janusz Kacprzyk, Prof. Ahmet Fahri Özok, and Prof. Ajith Abraham, and Prof. Irina Perfilieva. It is an honor to include their invaluable speeches in our conference program. We appreciate their voluntary contributions to INFUS 2023, and we hope to see them at INFUS conferences for many years. This year, the number of submitted papers became 291. After the review process, about 40% of these papers have

been rejected. More than 50% of the accepted papers are from other countries outside Türkiye.

We again thank all the representatives of their countries for selecting INFUS 2023 as an international scientific arena to present their valuable research results. We are honored and aware of our responsibility that our participants have chosen us in a highly competitive environment with hundreds of conferences in the same field and organized in close dates to each other. INFUS conference manages high-cost international conference participation processes for the benefit of the participants, with lower registration fees but more well-known expert invitations and rich social activities.

We also thank the anonymous reviewers for their hard works in selecting high-quality papers of INFUS 2023. Each of the organizing committee members provided invaluable contributions to INFUS 2023. INFUS conferences would be impossible without their efforts. We hope meeting all of our participants next year in Türkiye one more time with a new research theme at a new city and new social activities.

We would like to thank our publisher Springer Publishing Company, Series Editor Prof. Janusz Kacprzyk, Interdisciplinary and Applied Sciences and Engineering, and Editorial Director Thomas Ditzinger, last but not least, Project Coordinator Nareshkumar Mani for their supportive, patient, and helpful roles during the preparation of this book.

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# Contextual Bipolar Database Queries: A Conjunctive and Disjunctive Perspective

Janusz Kacprzyk<br/>1,2( $\boxtimes$  ) and Sławomir Zadrożny<br/>1

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**Abstract.** We extend our approach to bipolar database queries which makes it possible to use a necessary (required, mndatory, obligatory) and optional (desired) condition connected with a non-conventional aggregation operator "and, if possible" exemplified by "find a house which is "inexpensive" and, if possible, located possibly 'close to public transportation" in which "and, if possible,..." is not the traditional "and". We extend first this formulation by context exemplified by "find houses which are inexpensive and – if possible, with respect to other houses in town – are possibly close to public transportation". We use a logical representation of the "and, if possible..." operator but also mention some other ones, notably based on the winnow operator. Moreover, we present a new formulation of a contextual bipolar query, "find a house which is inexpensive or - if impossible, with respect to other houses in own - is new". We mention an inclusive and exclusive character of these two formulations. We present a context awareness related view of these bipolar queries, and provide some remarks on intention awareness.

**Keywords:** database query  $\cdot$  bipolar query  $\cdot$  context  $\cdot$  fuzzy logic  $\cdot$  user intention  $\cdot$  user preference  $\cdot$  context awareness  $\cdot$  intention awareness

# 1 Introduction

We are concerned with database querying in which a human user intends to find information, here database records (tuples), which represent his/her intentions and preferences. Natural language, which is the only fully natural way of articulation and communication for the humans, is usually employed to pose questions, queries, requests, etc. Here, using examples from real estate, such a query may be "find all houses that are inexpensive and close to public transportation".

Such queries with imprecisely specified (via fuzzy sets) terms have a long tradition, and among more advanced versions one camn mention our query

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based on fuzzy logic with linguistic quantifiers (cf. Kacprzyk and Ziółkowski [21], Kacprzyk, Ziółkowski and Zadrożny [22]).

One of the next relevant developments in our contexts is the bipolar queries which are ment to explicitly account for the bipolarity (yes-no, pro-con, ...) in human judgments, intentions and preferences (cf. Zadrożny and Kacprzyk [32], and later papers). Then, atop of the bipolar queries we include context, in two versions, (cf. Zadrożny, Kacprzyk and Dziedzic [35]. This yields what may be termed a context-aware perspective of bipolar querying. Then, we extend it to what may be termed an intention-aware perspective.

# 2 Bipolar Fuzzy Queries

Historically, the term "bipolar query" was proposed, in our perspective, by Dubois and Prade [7,9]. The very essence is that in a query two types of query conditions are used to express negative and positive user preferences which commonly happen in human discourse and appear through a *bipolar scale* which specifies:

- some degree of being *negative*, i.e., to be rejected,
- some degree of being *positive*, i.e., to be accepted.

noindent and in practice two such bipolar scales are used:

- bipolar univariate and
- unipolar bivariate,

and the in the former there is one scale with three main levels of, respectively, negative, neutral and positive evaluation, gradually changing from one end of the scale to another, usually represented by [-1,1], and in the latter two independent scales for a positive and negative evaluation, usually represented by [0,1] are used. We will use the latter one.

In our field of fuzzy logic and possibility theory, this topic was considered by many authors, e.g. Dubois and Prade and their collaborators, e.g.: Benferhat, Dubois, Kaci, Prade [8], Dubois and Prade [7,9], cf. also Dubois and Prade [10], Dziedzic, Kacprzyk and Zadrozny [11], Hadjali, Kaci and Prade [12], Lietard and Rocacher [24], Matthé, De Tré, Zadrożny, Kacprzyk and Bronselaer [26], etc.

The key problem is to properly define the semantics of the negative and positive evaluations (gradually given). In our works we assume that the objects (tuples) with the negative evaluation are rejected and the positive evaluation contributes to the overall evaluation of an object only if it is not rejected. Therefore, even if the positive evaluations play a weaker role, they are equally important as the negative evaluations in the case when there are not rejected objects through the positive evaluations. A good representation is here via a special aggregation operator "and, if possible...".

A bipolar query may generally be written as:

$$C$$
 and possibly (i.e. and, if possible) $P$  (1)

exemplified by "find a house which is inexpensive (C) and possibly close to public transportation (P)" meant as that this query is satisfied by a tuple t only if either one of the two conditions holds:

- 1. it satisfies (of course, possibly to a high degree) both conditions C and P, or
- 2. it satisfies only C and there is no tuple which satisfies both conditions.

The key problem is a proper aggregation method to reflect this "and, if possible ..." which is clearly not the traditional "and".

The concept of such a bipolar query appeared first in Lacroix and Lavency [23] who used in the query (C, P) two conditions: C which stands for what is required (mandatory) and P which stands for what is preferred (desired), meant as: if at least one tuple satisfies both the mandatory and desired condition then the "and, if possible" operator is interpreted as the standard conjunction ("and") and otherwise only the mandatory condition is taken into account.

Such an aggregation operator has been later proposed independently by Dubois and Prade [6] in default reasoning and by Yager [27,28] in multicriteria decision making, cf. also Bordogna and Pasi [2] in information retrieval.

Moreover, the bipolar queries with the "and, if possible" operator may be also be viewed as a special case of Chomicki's [5] queries with preferences which are based on an extra relational algebra operator, the winnow. (cf. Zadrożny and Kacprzyk [32]) but this will not be considered here.

In Lacroix and Lavency [23], with the crisp (nonfuzzy) conditions C and P, a bipolar query (C, P) can be processed via the "first select using C then order using P" strategy, i.e., by finding tuples satisfying C and, second, choosing from among them those satisfying P, if any. Some fuzzifications of the original Lacroix and Lavency's approach are proposed by Zadrożny [29], and Zadrożny and Kacprzyk [31,32]; for some other approaches, see Bosc et al. [3], or Lietard. Rocacher and Bosc [25], etc.

In our general form of a bipolar query (1), C is the complement of the negative assessment (e.g., "price is inexpensive"), and P – the positive assessment (e.g., located "near public transportation"). Then, the semantics of the bipolar query (1) is:

– a tuple t belongs to the answer set of the query (1) if it satisfies (P(t)) and C(t) clearly are binary predicates):

$$C(t)$$
 and possibly  $P(t) \equiv C(t) \land \exists s (C(s) \land P(s)) \Rightarrow P(t)$  (2)

– and if there are tuples satisfying both P and C, then (2) boils down to  $C \wedge P$  while otherwise it boils down to C alone.

The fuzzification proposed by the authors (cf. Zadrożny and Kacprzyk [32] for a comprehensive account) can be done via a direct fuzzification of (2) (with fuzzy predicates):

$$C(t)$$
 and possibly  $P(t) \equiv C(t) \land \land \exists s \ (C(s) \land P(s)) \Rightarrow P(t)$  (3)

and via a direct fuzzification of the winnow operator (cf. Chomicki [5]) but it will not be used here (cf. Zadrożny and Kacprzyk [32]).

Then, in this formulation one can use a specific form of the conjunction and disjunction, i.e. a t-norm and t-conorm (s-norm), and the negation, i.e. a specific De Morgan Triple.

In addition to the "and, if possible...", which has a clear conjunctive character, in Kacprzyk, Zadrożny and Dziedzic [34] w new operator, "or, if impossible..." is proposed, which has a clear disjunctive character. Then, one can define a bipolar query with the "or, if impossible" operator to be written as:

$$P$$
 or, if impossible  $C$  (4)

and exemplified by:

to be interpreted as: if there is no house which is innexpensive, then and only then it is impossible to satisfy the first condition and the answer set comprises houses satisfying just the second condition, i.e., who are large, if any. On the other hand, if there is such a house which is inexpensive, then only such houses are retrieved, and if they are is not important. Therefore, in the former case the query (4) reduces to the condition C alone while in the latter case it reduces to the P a condition lone.

This can formally be written, for the fuzzy C and P, with the minimum and maximum operators standing for the, respectively, the conjunction and disjunction, with T(.) denoting the truth values from [0,1], as:

$$T(P(t) \text{ or, if impossible } C(t)) = \max(\mu_P(t), \min(1 - \max_{s \in R} \mu_P(s), \mu_C(t))) \quad (6)$$

Putting it in a different way, the use of "or, if impossible" is meant as to aggregate the negative and positive conditions of a bipolar query (C, P). That is, for "and, if possible…" the (complement of) negative condition C is a constraint to be satisfied, i.e. mandatory (necessary, obligatory), while the positive condition P is to be satisfied only if possible, i.e., if its satisfaction does not imply a conflict with the satisfaction of C. On the other hand, for the "or, if impossible…" as in (4) we want the positive condition P to be satisfied, and only if this is not possible, then the satisfaction of C matters.

Technically, the "and, if possible..." and "or, if impossible..." operators are related to each other (cf. Kacprzyk, Zadrożny and Dziedzic [34]).

# 3 Bipolar Queries Under a Context: An Inclusive and Exclusive Perspective

In general, when we deal with all kinds of human activities, judgements, assessments, etc. context is of utmost importance and a prerequicite for meaningful analyses. In the field of bipolar queries to databases the inclusion of context has

provided much momentum and triggered a new class of *contextual bipolar query*, proposed by Zadrożny, Kacprzyk and Dziedzic [34,35]; cf. also Kacprzyk and Zadrożny [17], and Kacprzyk and Zadrożny [19].

Briefly speaking, for the bipolar query with the required/desired semantics as considered here, i.e. due to (1), in virtually all realistic cases the "and possibly" in (1) is specified with the satisfaction of both C and P to be meant in a certain context. For instance, usually while looking for an inexpensive house it should be taken into account to which part of a city this applies which implies a contextual bipolar query:

to be meant to be satisfied (all to a possibly high degree) by a house if:

- 1. it is inexpensive and close to public transportation, or
- 2. it is inexpensive and there is no other hotel located in the same region which is both inexpensive and close to public transportation.

This new "and possibly + context" operator may be formalized as that the context is equated with a part of the database defined by an additional binary predicate W, i.e.,

$$Context(t) = \{ s \in R : W(t,s) \}$$
(8)

where R denotes the whole database (relation).

Therefore, the "and possibly + context" has three arguments:

C and possibly P with respect to W

with C and P meant as the required and desired conditions, respectively, and W standing for the context.

Then, 8 is interpreted as:

$$C(t)$$
 and possibly  $P(t)$  with respect to  $W \equiv (9)$   

$$\equiv C(t) \land (\exists s(W(t,s) \land C(s) \land P(s)) \Rightarrow P(t))$$

This form is used here though an equivalent winnow operator based form can also be employed (cf. Kacprzyk and Zadrożny [19].

Following the reasoning on the bipolar queries with the "and, if possible..." and "or, if impossible..." operators, we can apply the similar arguments now in the case of the contextual bipolar queries. We have shown above how the conjunctive case of "and, if possible..." can be dealt with, and below we will show how to deal with the disjunctive type, i.e. "or, if impossible...".

The generic form of the contextual bipolar query with the "or, if impossible..." operator is:

$$P$$
 or, if impossible  $C$  with respect to  $W$  (10)

and its semantics can be expressed as:

$$P(t)$$
 or, if impossible  $C(t)$  with respect to  $W \equiv P(t) \vee (\neg \exists_{s \in R} (P(s) \wedge W(t, s)) \wedge C(t))$  (11)

and for the minimum and maximum operators standing for, respectively, the conjunction and disjunction, we have the degree of truth T(.):

$$T(P(t) \text{ or, if impossible } C(t) \text{ with respect to } W) = \max(\mu_P(t),$$
 (12)

$$\min(1 - \max_{s \in R} \min(\mu_P(s), \mu_W(t, s)), \mu_C(t)))$$
 (13)

# 4 Remarks on a Way from Context Aware to Intention Aware Bipolar Queries

The extension of the concept of a bipolar database query, assumed here to be in a fuzzy logic and possibility theory based setting, can clearly be considered to be an example of a context-aware solution in which a new quality is obtained by the introduction of context as a crucial element of the problem formulation. As a further step, we can extend our solution to the human centric and friendly database querying, which can already provide very powerful tools and techniques to reflect human preferences and intentions. A promising option is here to extend the present solution, which could be termed context-aware, to a qualitatively new setting in which the context awareness is extended into intention awareness that is intended to reduce and facilitate the interaction between the computer system and its user that is of a crucial importance in the case of database querying, exemplified here by a synergistic and proactive interaction between a real estate agent and a customer. More specifically, the challenge is here to try to predict which action the user will take next knowing what he/she has done in the past and now, as well as his/her personal characteristics, cognitive biases, etc. These problems constitute real challenges and may need the use of, for instance, neural networks, decision trees, Bayesian analyses, etc.

# 5 Conclusions

The bipolar database queries are first presented, mainly from the perspective of the authors' works, using a fuzzy logic and possibility theory based perspective. Then, a new extension of the bipolar database queries, the contextual bipolar queries are introduced, based on the authors' settings, in which the elements (conditions) of the bipolar database queries are semantically related to context. Two forms of the introduction of context are proposed, the so called conjunctive one which corresponds to the aggregation of query conditions via "and, if possible...", to the so called disjunctive one which correspond to the aggregation of query conditions via "or, if impossible...'. finally, a possibility of an extension of the above context awareness of the bipolar queries considered to the intention awareness is mentioned.

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# Intuitionistic Fuzzy Modal Topological Structures Based on Two New Intuitionistic Fuzzy Modal Operators

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**Abstract.** In the present paper, four Intuitionistic Fuzzy Modal Topological Structures (IFMTSs) are described and some of their properties are discussed. While in the first paper over an IFMTS the modal operators were the simplest (standard) ones, here particular cases of three of the extended modal operators over intuitionistic fuzzy sets are used. Some properties of the new operators are discussed.

**Keywords:** intuitionistic fuzzy modal operator  $\cdot$  intuitionistic fuzzy set  $\cdot$  intuitionistic fuzzy topological operator  $\cdot$  intuitionistic fuzzy topological structure

**AMS:** 03E72

# 1 Introduction

The concept of an Intuitionistic Fuzzy Modal Topological Structure (IFMTS) was introduced in [2]. It was based on the definitions by Kazimierz Kuratowski (1896–1980) proposed in [6] for topological structures that satisfy the conditions:

C1 
$$cl(A\Delta B) = cl(A)\Delta cl(B)$$
,  
C2  $A \subseteq cl(A)$ ,  
C3  $cl(cl(A)) = cl(A)$ ,  
C4  $cl(O) = O$ ,

where  $A, B \in X$ , X is some fixed set of sets with a minimal element O, cl is the topological operator "closure" and  $\Delta: X \times X \to X$  is the operation that generates cl; and

```
\begin{array}{l} \text{I1 } in(A\nabla B)=in(A)\nabla in(B),\\ \text{I2 } in(A)\subseteq A,\\ \text{I3 } in(in(A))=in(A),\\ \text{I4 } in(I)=I, \end{array}
```

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where  $A, B \in X$ , X is the same set and I is its maximal element, in is the topological operator "interior" and  $\nabla : X \times X \to X$  is the operation that generates in (see also [10]).

In the Intuitionistic Fuzzy Sets (IFSs) theory there are some types of operators – modal, topological, level and others. The present paper is a modification of the ideas of [2]. Here, the standard modal operators are changed with new intuitionistic fuzzy modal operators that are particular cases of three of the extended intuitionistic fuzzy modal operators.

Later, the operators that satisfy the conditions C1–C4 will be termed as "operators of closure type" and those satisfying the conditions I1–I4 will be termed as "operators of interior type".

# 2 Short Remarks on Intuitionistic Fuzzy Sets

IFSs are one of the early extensions of fuzzy sets proposed by Lotfi Zadeh (1921–2017) [14]. When a set E, called "universum" is fixed and A is its subset, each IFS in E has the form:

$$A^* = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in E \},$$

where the functions  $\mu_A : E \to [0,1]$  and  $\nu_A : E \to [0,1]$  define, respectively, the degree of membership and the degree of non-membership of the element  $x \in E$  to the set  $A \subseteq E$ , and for each  $x \in E$ :

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

As usual, instead of  $A^*$  for brevity, the notation A is used.

Over the IFSs a lot of operations, relations and operators are defined (see [1]). The most utilized among them, which we will need below, are

```
\begin{array}{lll} A \subseteq B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \ \& \ \nu_A(x) \geq \nu_B(x)); \\ A \supseteq B & \text{iff} & B \subseteq A; \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \ \& \ \nu_A(x) = \nu_B(x)); \\ \neg A & = & \{\langle x, \nu_A(x), \mu_A(x) \rangle | x \in E\}; \\ A \cap B & = & \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \in E\}; \\ A \cup B & = & \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle | x \in E\}. \end{array}
```

The first two (simplest) analogues of the topological operators "closure" and "interior" (defined over IFSs) are introduced in [1] as follows:

$$\mathcal{C}(A) = \{ \langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y) \rangle | x \in E \},$$
  
$$\mathcal{I}(A) = \{ \langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y) \rangle | x \in E \}.$$

Now, over IFSs a lot of modal operators are defined. Three of them are the following (see [1]).

$$G_{\alpha,\beta}(A) = \{ \langle x, \alpha \mu_A(x), \beta \nu_A(x) \rangle | x \in E \},$$
  

$$H_{\alpha,\beta}(A) = \{ \langle x, \alpha \mu_A(x), \nu_A(x) + \beta \pi_A(x) \rangle | x \in E \},$$
  

$$J_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x) + \alpha \pi_A(x), \beta \nu_A(x) \rangle | x \in E \},$$

where A is an IFS and  $\alpha, \beta \in [0, 1]$ .

For the needs of the present research, we will use particular cases of operators  $H_{\alpha,\beta}$  and  $J_{\alpha,\beta}$  that we will introduce below and in the next Section we will study their behaviour.

# 3 Two New Modal Operators

Here, we define the two (particular) modal operators

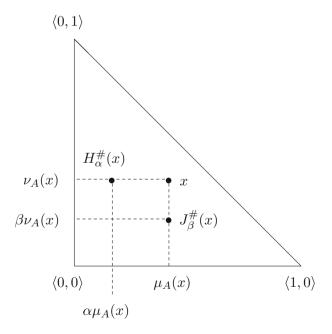
$$H_{\alpha}^{\#}(A) = \{ \langle x, \alpha.\mu_A(x), \nu_A(x) \rangle | x \in E \},$$
  
$$J_{\alpha}^{\#}(A) = \{ \langle x, \mu_A(x), \alpha.\nu_A(x) \rangle | x \in E \},$$

where A is an IFS and  $\alpha \in [0, 1]$ .

We see immediately that for each IFS A and for each  $\alpha \in [0,1]$ :

$$H_{\alpha}^{\#}(A) = H_{\alpha,0}(A),$$
  
 $J_{\alpha}^{\#}(A) = J_{0,\alpha}(A).$ 

The geometrical interpretation of both operators is shown on Fig. 1., where  $H_{\alpha}^{\#}(x)$  and  $J_{\beta}^{\#}(x)$  are the results of the applying of the two operators over element  $x \in E$ .



**Fig. 1.** The geometrical interpretation of  $H_{\alpha}^{\#}(x)$  and  $J_{\beta}^{\#}(x)$ .