

SECOND EDITION

# NOISE AND VIBRATION ANALYSIS

SIGNAL ANALYSIS AND  
EXPERIMENTAL PROCEDURES

ANDERS BRANDT



WILEY



## **Noise and Vibration Analysis**



# **Noise and Vibration Analysis**

Signal Analysis and Experimental Procedures

Second Edition

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## Contents

**About the Author** *xix*

**Preface** *xxi*

**Acknowledgments** *xxv*

**List of Abbreviations** *xxvii*

**Annotation** *xxix*

### **1** **Introduction** *1*

1.1 Noise and Vibration *1*

1.2 Noise and Vibration Analysis *2*

1.3 Application Areas *3*

1.4 Analysis of Noise and Vibrations *4*

1.4.1 Experimental Analysis *4*

1.5 Standards *5*

1.6 Becoming a Noise and Vibration Analysis Expert *5*

1.6.1 The Virtue of Simulation *6*

1.6.2 Learning Tools and the Format of this Book *6*

### **2** **Dynamic Signals and Systems** *9*

2.1 Introduction *9*

2.2 Periodic Signals *11*

2.2.1 Sine Waves *11*

2.2.2 Complex Sines *12*

2.2.3 Interacting Sines *13*

2.2.4 Orthogonality of Sines *15*

2.3 Random Signals *16*

2.4 Transient Signals *17*

2.5 RMS Value and Power *18*

2.6 Linear Systems *19*

2.6.1 The Laplace Transform *20*

2.6.2 The Transfer Function *23*

2.6.3 The Impulse Response *24*

2.6.4 Convolution *25*

2.7 The Continuous Fourier Transform *29*

2.7.1	Characteristics of the Fourier Transform	31
2.7.2	The Frequency Response	33
2.7.3	Relationship Between the Laplace and Frequency Domains	33
2.7.4	Transient Versus Steady-State Response	34
2.8	Chapter Summary	35
2.9	Problems	36
	References	38

### **3 Time Data Analysis** 39

3.1	Introduction to Discrete Signals	39
3.1.1	Discrete Convolution	40
3.2	The Sampling Theorem	40
3.2.1	Aliasing	42
3.2.2	Discrete Representation of Analog Signals	43
3.2.3	Interpolation and Resampling	45
3.3	Filters	48
3.3.1	Analog Filters	48
3.3.2	Digital Filters	50
3.3.3	Smoothing Filters	52
3.3.4	Acoustic Octave Filters	53
3.3.5	Analog RMS Integration	55
3.3.6	Frequency Weighting Filters	56
3.4	Time Series Analysis	57
3.4.1	Min- and Max-Analysis	57
3.4.2	Time Data Integration	58
3.4.3	Time Data Differentiation	62
3.4.4	FFT-Based Processing	65
3.5	Chapter Summary	66
3.6	Problems	67
	References	68

### **4 Statistics and Random Processes** 71

4.1	Introduction to the Use of Statistics	71
4.1.1	Ensemble and Time Averages	72
4.1.2	Stationarity and Ergodicity	72
4.2	Random Theory	73
4.2.1	Expected Value	73
4.2.2	Errors in Estimates	73
4.2.3	Probability Distribution	74
4.2.4	Probability Density	75
4.2.5	Histogram	75
4.2.6	Sample Probability Density Estimate	76
4.2.7	Average Value and Variance	76
4.2.8	Central Moments	78
4.2.9	Skewness	78

4.2.10	Kurtosis	79
4.2.11	Crest Factor	79
4.2.12	Correlation Functions	80
4.2.13	The Gaussian Probability Distribution	81
4.3	Statistical Methods	83
4.3.1	Hypothesis Tests	83
4.3.2	Test of Normality	85
4.3.3	Test of Stationarity	87
4.3.3.1	Frame Statistics	87
4.3.3.2	The Reverse Arrangements Test	87
4.3.3.3	The Runs Test	90
4.4	Quality Assessment of Measured Signals	91
4.5	Chapter Summary	94
4.6	Problems	95
	References	96
<b>5</b>	<b>Fundamental Mechanics</b>	97
5.1	Newton's Laws	97
5.2	The Single Degree-of-Freedom System (SDOF)	98
5.2.1	The Transfer Function	98
5.2.2	The Impulse Response	99
5.2.3	The Frequency Response	102
5.2.4	The Q-Factor	105
5.2.5	SDOF Forced Response	105
5.3	Alternative Quantities for Describing Motion	106
5.4	Frequency Response Plot Formats	108
5.4.1	Magnitude and Phase	108
5.4.2	Real and Imaginary Parts	109
5.4.3	The Nyquist Plot – Imaginary Versus Real Part	111
5.5	Determining Natural Frequency and Damping Ratio	113
5.5.1	Peak in the Magnitude of FRF	114
5.5.2	Peak in the Imaginary Part of FRF	114
5.5.3	Resonance Bandwidth (3 dB Bandwidth)	114
5.5.4	Circle in the Nyquist Plot	115
5.6	Rotating Mass	115
5.7	Some Comments on Damping	116
5.7.1	Hysteretic Damping	117
5.8	Models Based on SDOF Approximations	118
5.8.1	Vibration Isolation	118
5.8.2	Resonance Frequency and Stiffness Approximations	120
5.9	The Two Degree of Freedom System (2DOF)	121
5.10	The Tuned Damper	123
5.11	Chapter Summary	125
5.12	Problems	126
	References	127

<b>6</b>	<b>Modal Analysis Theory</b>	129
6.1	Waves on a String	129
6.2	Matrix Formulations	131
6.2.1	Degree of Freedom	132
6.3	Eigenvalues and Eigenvectors	132
6.3.1	Undamped System	132
6.3.2	Mode Shape Orthogonality	136
6.3.3	Modal Coordinates	138
6.3.4	Proportional Damping	140
6.3.5	General Damping	142
6.4	Frequency Response of MDOF Systems	146
6.4.1	Frequency Response from $[M]$ , $[C]$ , $[K]$	146
6.4.2	Frequency Response from Modal Parameters	147
6.4.3	Frequency Response from $[M]$ , $[K]$ , and $\zeta$ – Modal Damping	151
6.4.4	Mode Shape Scaling	152
6.4.5	The Effect of Node Lines on FRFs	153
6.4.6	Antiresonance	154
6.4.7	Impulse Response of MDOF Systems	155
6.5	Free Decays	155
6.6	Chapter Summary	156
6.7	Problems	157
	References	158
<b>7</b>	<b>Transducers for Noise and Vibration Analysis</b>	159
7.1	The Piezoelectric Effect	159
7.2	The Charge Amplifier	160
7.3	Transducers with Built-In Impedance Converters, “IEPE”	162
7.3.1	Low-Frequency Characteristics	163
7.3.2	High-Frequency Characteristics	164
7.3.3	Transducer Electronic Data Sheet, TEDS	165
7.4	The Piezoelectric Accelerometer	165
7.4.1	Frequency Characteristics	166
7.4.2	Mounting Accelerometers	167
7.4.3	Electrical Noise	168
7.4.4	Choosing an Accelerometer	168
7.5	The Piezoelectric Force Transducer	170
7.6	The Impedance Head	171
7.7	The Impulse Hammer	172
7.8	Accelerometer Calibration	173
7.9	Measurement Microphones	174
7.10	Microphone Calibration	175
7.11	The Geophone	175
7.12	MEMS-based Sensors	176
7.13	Shakers for Structure Excitation	177
7.14	Some Comments on Measurement Procedures	178

7.15	Problems	180
	References	181
<b>8</b>	<b>Frequency Analysis Theory</b>	183
8.1	Periodic Signals – The Fourier Series	183
8.2	Spectra of Periodic Signals	185
8.2.1	Frequency and Time	186
8.3	Random Processes	187
8.3.1	Spectra of Random Processes	187
8.4	Transient Signals	189
8.5	Interpretation of Spectra	189
8.6	Chapter Summary	191
8.7	Problems	192
	References	193
<b>9</b>	<b>Experimental Frequency Analysis</b>	195
9.1	Frequency Analysis Principles	195
9.1.1	Nonparametric Frequency Analysis	196
9.2	Octave and Third-Octave Band Spectra	197
9.2.1	Time Constants	197
9.2.2	Real-time Versus Serial Measurements	198
9.3	The Discrete Fourier Transform (DFT)	198
9.3.1	The Fast Fourier Transform, FFT	200
9.3.2	The DFT in Short	200
9.3.3	The Basis of the DFT	202
9.3.4	Periodicity of the DFT	202
9.3.5	Properties of the DFT	205
9.3.6	Relation Between DFT and Continuous Spectrum	206
9.3.7	Leakage	206
9.3.8	The Picket-Fence Effect	209
9.3.9	Time Windows for Periodic Signals	211
9.3.9.1	Amplitude Correction of Window Effects	212
9.3.9.2	Power Correction of Window Effects	212
9.3.9.3	Comparison of Common Windows	214
9.3.9.4	Frequency Resolution	218
9.3.10	Time Windows for Random Signals	219
9.3.11	Oversampling in FFT Analysis	219
9.3.12	Circular Convolution and Aliasing	219
9.3.13	Zero Padding	221
9.3.14	Frequency Domain Processing	222
9.3.15	Zoom FFT	223
9.4	Chapter Summary	224
9.5	Problems	225
	References	226

<b>10</b>	<b>Spectrum and Correlation Estimates Using the DFT</b>	229
10.1	Averaging	229
10.2	Spectrum Estimators for Periodic Signals	230
10.2.1	The Autopower Spectrum	231
10.2.2	Linear Spectrum	232
10.2.3	Phase Spectrum	233
10.3	Estimators for PSD and CSD	233
10.3.1	The Periodogram	234
10.3.2	Welch's Method	235
10.3.3	Window Correction for Welch Estimates	236
10.3.4	Bias Error in Welch Estimates	237
10.3.5	Random Error in Welch Estimates	242
10.3.6	The Smoothed Periodogram Estimator	248
10.3.7	Bias Error in Smoothed Periodogram Estimates	249
10.3.8	Random Error in Smoothed Periodogram Estimates	250
10.4	Estimators for Correlation Functions	250
10.4.1	Correlation Estimator by Long FFT	251
10.4.2	Correlation Estimator by Welch's Method	253
10.4.3	Variance of the Correlation Estimator	254
10.4.4	Effect of Measurement Noise on Correlation Function Estimates	256
10.5	Estimators for Transient Signals	258
10.5.1	Windows for Transient Signals	259
10.6	A Signal Processing Framework for Spectrum and Correlation Estimation	260
10.7	Spectrum Estimation in Practice	262
10.7.1	Linear Spectrum Versus PSD	263
10.7.2	Example of a Spectrum of a Periodic Signal	264
10.7.3	Practical PSD Estimation	266
10.7.4	Spectrum of Mixed Property Signal	269
10.7.5	Calculating RMS Values in Practice	269
10.7.6	RMS from Linear Spectrum of Periodic Signal	270
10.7.7	RMS from PSD	270
10.7.8	Weighted RMS Values	271
10.7.9	Integration and Differentiation in the Frequency Domain	272
10.8	Multichannel Spectral and Correlation Analysis	273
10.8.1	Matrix Notation for MIMO Spectral Analysis	274
10.8.2	Arranging Spectral Matrices in MATLAB/Octave	275
10.8.3	Multichannel Correlation Functions	276
10.9	Chapter Summary	276
10.10	Problems	277
	References	278
<b>11</b>	<b>Measurement and Analysis Systems</b>	281
11.1	Principal Design	282
11.2	Hardware for Noise and Vibration Analysis	283

11.2.1	Signal Conditioning	283
11.2.2	Analog-to-Digital Conversion, ADC	284
11.2.2.1	Quantization and Dynamic Range	284
11.2.2.2	Setting the Measurement Range	285
11.2.2.3	Sampling Accuracy	286
11.2.2.4	Anti-alias Filters	288
11.2.2.5	Sigma-Delta ADCs	290
11.2.3	Practical Issues	290
11.2.4	Hardware Specifications	292
11.2.4.1	Absolute Amplitude Accuracy	292
11.2.4.2	Anti-alias Protection	292
11.2.4.3	Simultaneous Sampling	293
11.2.4.4	Cross-Channel Match	293
11.2.4.5	Dynamic Range	293
11.2.4.6	Cross-Channel Talk	294
11.2.5	Transient (Shock) Recording	295
11.3	FFT Analysis Software	295
11.3.1	Block Processing	296
11.3.2	Data Scaling	297
11.3.3	Triggering	297
11.3.4	Averaging	298
11.3.5	FFT Setup Parameters	299
11.4	Chapter Summary	299
11.5	Problems	300
	Problems	300
	References	301
<b>12</b>	<b>Rotating Machinery Analysis</b>	<b>303</b>
12.1	Vibrations in Rotating Machines	303
12.2	Understanding Time-Frequency Analysis	304
12.3	Rotational Speed Signals (Tachometer Signals)	306
12.4	RPM Maps	308
12.4.1	The Waterfall Plot	309
12.4.2	The Color Map Plot	310
12.5	Smearing	310
12.6	Order Tracks	312
12.7	Synchronous Sampling	314
12.7.1	DFT Parameters after Resampling	317
12.8	Averaging Rotation-Speed-Dependent Signals	317
12.9	Adding Change in RMS with Time	318
12.10	Parametric Methods	322
12.11	Chapter Summary	323
12.12	Problems	324
	References	325

<b>13</b>	<b>Single-input Frequency Response Measurements</b>	327
13.1	Linear Systems	328
13.2	Determining Frequency Response Experimentally	328
13.2.1	Method 1 – The $H_1$ Estimator	329
13.2.2	Method 2 – The $H_2$ Estimator	330
13.2.3	Method 3 – The $H_c$ Estimator	331
13.3	Important Relationships for Linear Systems	333
13.4	The Coherence Function	333
13.5	Errors in Determining the Frequency Response	334
13.5.1	Bias Error in FRF Estimates	335
13.5.2	Random Error in FRF Estimates	337
13.5.3	Bias and Random Error Trade-offs	339
13.6	Coherent Output Power	339
13.7	The Coherence Function in Practice	340
13.7.1	Nonrandom Excitation	341
13.8	Impact Excitation	342
13.8.1	The Force Signal	343
13.8.2	The Response Signal and Exponential Window	345
13.8.3	Impact Testing Software	345
13.8.4	Compensating for the Influence of the Exponential Window	347
13.8.5	Sources of Error	349
13.8.6	Improving Impact Testing by Alternative Processing	350
13.9	Shaker Excitation	351
13.9.1	Signal-to-noise Ratio Comparison	352
13.9.2	Pure Random Noise	352
13.9.3	Burst Random Noise	355
13.9.4	Pseudo-random Noise	355
13.9.5	Periodic Chirp	356
13.9.6	Stepped-sine Excitation	356
13.10	Examples of FRF Estimation – No Extraneous Noise	357
13.10.1	Pure Random Excitation	357
13.10.2	Burst Random Excitation	358
13.10.3	Periodic Excitation	360
13.11	Example of FRF Estimation – With Output Noise	360
13.12	Examples of FRF Estimation – With Input and Output Noise	362
13.12.1	Sources of Error during Shaker Excitation	362
13.12.2	Checking the Shaker Attachment	362
13.12.3	Other Sources of Error	364
13.13	Chapter Summary	365
13.14	Problems	367
	References	368
<b>14</b>	<b>Multiple-Input Frequency Response Measurement</b>	369
14.1	Multiple-Input Systems	369
14.1.1	The 2-Input/1-Output System	370
14.1.2	The 2-Input/1-Output System – Matrix Notation	371

14.1.3	The $H_1$ Estimator for MIMO	372
14.1.4	Multiple Coherence	373
14.1.5	Computation Considerations for Multiple-Input System	375
14.1.6	The $H_v$ Estimator	376
14.1.7	Other MIMO FRF Estimators	377
14.2	Conditioned Input Signals	377
14.2.1	Conditioned Output Signals	380
14.2.2	Partial Coherence	380
14.2.3	Ordering Signals Prior to Conditioning	381
14.2.4	Partial Coherent Output Power Spectra	382
14.2.5	Backtracking the H-Systems	382
14.2.6	General Conditioned Systems	384
14.3	Bias and Random Errors for Multiple-Input Systems	384
14.4	Excitation Signals for MIMO Analysis	384
14.4.1	Pure Random Noise	385
14.4.2	Burst Random Noise	385
14.4.3	Periodic Random Noise	385
14.4.4	The Multiphase Stepped-Sine Method (MPSS)	386
14.5	Data Synthesis and Simulation Examples	387
14.5.1	Burst Random – Output Noise	387
14.5.2	Burst and Periodic Random – Input Noise	389
14.5.3	Periodic Random – Input and Output Noise	391
14.6	Real MIMO Data Case	393
14.7	Chapter Summary	396
14.8	Problems	397
	References	398
<b>15</b>	<b>Orthogonalization of Signals</b>	401
15.1	Principal Components	401
15.1.1	Principal Components Used to Find Number of Sources	403
15.1.2	Data Reduction	406
15.2	Virtual Signals	410
15.2.1	Virtual Input Coherence	411
15.2.2	Virtual Input/Output Coherence	413
15.2.3	Virtual Coherent Output Power	414
15.3	Noise Source Identification (NSI)	417
15.3.1	Multiple Source Example	417
15.3.2	Automotive Example	421
15.4	Chapter Summary	422
15.5	Problems	423
	References	424
<b>16</b>	<b>Experimental Modal Analysis</b>	425
16.1	Introduction to Experimental Modal Analysis	425
16.1.1	Main Steps in EMA	426

16.2	Experimental Setup	427
16.2.1	Points and DOFs	427
16.2.2	Selecting Measurement DOFs	428
16.2.3	Measurement System	429
16.2.4	Sensor Considerations	430
16.2.5	Data Acquisition Strategies	430
16.2.6	Suspension	431
16.2.7	Measurement Checks	432
16.2.8	Calibration	434
16.2.9	Data Acquisition	434
16.2.10	Mode Indicator Functions	434
16.2.11	Data Quality Assessment	437
16.2.12	Checklist	437
16.3	Introduction to Modal Parameter Extraction	437
16.4	SDOF Parameter Extraction	440
16.4.1	The Least Squares Local Method	440
16.4.2	The Least Squares Global Method	441
16.4.3	The Least Squares (Local) Polynomial Method	442
16.5	The Unified Matrix Polynomial Approach, UMPA	443
16.5.1	Mathematical Framework	443
16.5.2	Choosing Model Order	446
16.5.3	Matrix Coefficient Normalization	447
16.5.4	Data Compression	449
16.6	Time Versus Frequency Domain Parameter Extraction for EMA	452
16.7	Time Domain Parameter Extraction Methods	454
16.7.1	Converting Bandpass Filtered FRFs into IRFs	455
16.7.2	The Ibrahim Time Domain Method	456
16.7.3	The Multiple-Reference Ibrahim Time Domain Method (MITD)	459
16.7.4	Prony's Method	462
16.7.5	The Least Squares Complex Exponential Method	464
16.7.6	Polyreference Time Domain	464
16.7.7	The Modified Multiple-Reference Ibrahim Time Domain Method (MMITD)	468
16.8	Frequency Domain Parameter Extraction Methods	470
16.8.1	The Least Squares Complex Frequency Domain Method	471
16.8.2	The Frequency Domain Direct Parameter Identification Method (FDPI)	474
16.8.3	The Frequency Z-Domain Direct Parameter Method, FDPIz	477
16.8.4	The Complex Mode Indicator Function, CMIF Method	478
16.9	Methods for Mode Shape Estimation and Scaling	480
16.9.1	Least Squares Frequency Domain – Single Reference Case	480
16.9.2	Least Squares Frequency Domain – Multiple Reference Case	482
16.9.3	Least Squares Frequency Domain – Multiple Reference Without MPFs	484
16.9.4	Least Squares Time Domain	485
16.9.5	Scaling Modal Model When Poles and Mode Shapes Are Known	486
16.10	Evaluating the Extracted Parameters	486

16.10.1	Synthesized FRFs	486
16.10.2	The MAC Matrix	487
16.11	Chapter Summary	489
16.12	Problems	491
	References	492
<b>17</b>	<b>Operational Modal Analysis (OMA)</b>	<b>495</b>
17.1	Principles for OMA	496
17.2	Data Acquisition Principles	497
17.3	OMA Modal Parameter Extraction for OMA	498
17.3.1	Spectral Functions for OMA Parameter Extraction	499
17.3.2	Correlation Functions for OMA Parameter Extraction	502
17.3.3	Half Spectra	504
17.3.4	Time versus Frequency Domain Parameter Extraction for OMA	504
17.3.5	Modal Parameter Estimation Methods for OMA	505
17.3.6	Least Squares Frequency Domain, OMA Versions	506
17.4	Scaling OMA Modal Models	508
17.4.1	Scaling an OMA Model Using the Mass Matrix	509
17.4.2	The OMAH Method	509
17.5	Chapter Summary	512
17.6	Problems	514
	References	514
<b>18</b>	<b>Advanced Analysis Methods</b>	<b>517</b>
18.1	Shock Response Spectrum	517
18.2	The Hilbert Transform	520
18.2.1	Computation of the Hilbert Transform	521
18.2.2	Envelope Detection by the Hilbert Transform	522
18.2.3	Relating Real and Imaginary Parts of Frequency Response Functions	524
18.3	Cepstrum Analysis	527
18.3.1	Power Cepstrum	527
18.3.2	Complex Cepstrum	530
18.3.3	The Real Cepstrum	530
18.3.4	Inverse Cepstrum	530
18.4	The Envelope Spectrum	531
18.5	Creating Random Signals with Known Spectral Density	533
18.6	Identifying Harmonics in Noise	535
18.6.1	The Three-Parameter Sine Fit Method	535
18.6.2	Periodogram Ratio Detection, PRD	537
18.7	Harmonic Removal	539
18.7.1	Frequency Domain Editing, FDE	539
18.7.2	Cepstrum-Based Harmonic Removal Methods	540
18.8	Chapter Summary	542
18.9	Problems	543
	References	544

<b>19</b>	<b>Practical Vibration Measurements and Analysis</b>	<b>547</b>
19.1	Introduction to a Plexiglas Plate	547
19.2	Forced Response Simulation	550
19.2.1	Frequency Domain Forced Response for Periodic Inputs	550
19.2.2	Frequency Domain Forced Response for Random Inputs	551
19.2.3	Time Domain Computation of Forced Response for Any Inputs	551
19.2.3.1	Time Domain Response by Frequency Domain Computation	551
19.2.3.2	Time Domain Response by Digital Filters	552
19.2.4	Plexiglas Plate Forced Response Example	555
19.3	Spectra of Periodic Signals	556
19.4	Spectra of Random Signals	559
19.5	Data with Random and Periodic Content	561
19.5.1	Car Idling Sound	561
19.5.2	Container Ship Measurement	565
19.6	Operational Deflection Shapes – ODS	567
19.6.1	Plexiglas Plate ODS Example – Single Reference	568
19.6.2	Plexiglas Plate ODS Example – Multiple-Reference	570
19.7	Impact Excitation and FRF Estimation	572
19.8	Plexiglas EMA Example	578
19.8.1	FRF Quality Assessment	578
19.8.2	EMA Modal Parameter Extraction, MPE	580
19.9	Methods for EMA Modal Parameter Estimation, MPE	585
19.9.1	Time Domain Variable Settings	586
19.9.2	High-Order Methods for EMA MPE	590
19.9.3	Low-Order Methods for EMA MPE	592
19.9.4	The Complex Mode Indicator Function, CMIF	594
19.9.5	Calculating Scaled Mode Shapes	596
19.10	Conclusions of EMA MPE	599
19.11	OMA Examples	600
19.11.1	OMA Using Synthesized Data for Plexiglas Plate	600
19.11.2	OMA on Measured Data of Plexiglas Plate	607
19.11.3	OMA of a Suspension Bridge	612
19.11.4	OMA on Container Ship	617
	References	622

**Appendix A Complex Numbers** 625

**Appendix B Logarithmic Diagrams** 629

**Appendix C Decibels** 633

**Appendix D Some Elementary Matrix Algebra** 635

**Appendix E Eigenvalues and the SVD 639**

E.1 Eigenvalues and Complex Matrices 639

E.2 The Singular Value Decomposition (SVD) 640

**Appendix F Organizations and Resources 643**

**Appendix G Checklist for Experimental Modal Analysis Testing 645**

**Bibliography 647**

**Index 659**



## About the Author

Anders Brandt is currently professor and Head of the Department of Mechanical and Production Engineering at Aarhus University in Denmark. He received a MSc degree in electrical engineering from Chalmers University of Technology, Sweden, in 1986, and a Licentiate of Engineering degree in medical electronics from the same university in 1989 with a thesis on bone conduction hearing. For the next 20 years, he worked with support, education, and consultancy in industry in Sweden and abroad, in the areas of applied signal analysis. In 1996, he started Axiom EduTech, a company dedicated to serve industry and academia with his expertise in advanced signal analysis methods for vibration analysis. During this time he gave over 250 short courses on various topics such as frequency analysis, modal analysis, order tracking, and vibration testing. He was also teaching at universities on similar topics. In 2009, he left the company and joined University of Southern Denmark (SDU) as an associate professor, building up a research group focusing on research within operational modal analysis and structural health monitoring. He has supervised and cosupervised 11 PhD students and 31 master's students to completion and was promoted to full professor in 2019. He has published over 100 cited papers in the fields of vibration analysis and structural health monitoring. He left SDU to become Head of Department at Aarhus University in 2021. He is the author of the free ABRAVIBE toolbox for MATLAB and GNU Octave, and maintains the site [www.abravibe.com](http://www.abravibe.com) from which the toolbox and other educational material may be downloaded. The toolbox is used throughout universities and industry worldwide and has over 5,000 registered users. He also has a YouTube channel which contains lectures for many of the chapters of his book.



## Preface

The second edition of this book includes three new chapters: Chapter 16 on experimental modal analysis (EMA) and modal parameter estimation, Chapter 17 on operational modal analysis (OMA), and Chapter 19 in which I have included several examples of how to apply the many techniques presented in the book, both on real data and on synthesized data, i.e., data generated by some numerical model. The latter chapter includes real data from some of my recent research that are also available through my website. It is my hope that these new chapters will make the book even more comprehensive for educators, students, and practitioners alike. In addition to these new chapters, I have also rewritten a few parts, notably the part about correlation function estimators (Sections 10.4.1 and 10.4.2), for which I have found in recent years, that it is easier for my students to understand if presented for correlation as a convolution. Chapter 10 now includes a section presenting a new framework for signal processing that I developed in my recent research. In chapter 19 is also included a presentation of a new way of implementing impact testing which offers new advantages not found in traditional measurement systems. This is also available in the accompanying toolbox, ABRAVIBE, which is available from my website [www.abravibe.com](http://www.abravibe.com), along with almost everything from this book, including a solutions manual for all problems in the book. The ABRAVIBE toolbox for MATLAB (or GNU Octave) is a very comprehensive toolbox with functionality for most of the techniques presented in this book, including rotating machinery analysis, spectrum analysis, EMA, and OMA. My YouTube channel also complements this book with many lectures.

The material in the first edition of this book, published in 2011, had been developing in my mind for more than 20 years of teaching at the time of its first publication. During these years, I had been teaching over 250 shortcourses for engineers in the industry on techniques for experimental noise and vibration analysis and also on how to use commercial measurement and analysis systems. In addition, in the late 1990s, I developed and taught three master's level courses in experimental analysis of vibrations at Blekinge Institute of Technology in Sweden.

Noise and vibration analysis is an interdisciplinary field, incorporating diverse subjects such as mechanical dynamics, sensor technology, statistics, and signal processing. Whereas there are many excellent and comprehensive books in each of these disciplines, there has been a lack of introductory material for the engineering student who first starts to make noise and/or vibration measurements, or the engineer who needs a reference in their daily

life. In addition, there are few textbooks in this field presenting the techniques as they are actually used in practice. This book is an attempt to fill this void.

My aim for this book is that it may serve both as a course book and as supplementary reading in university courses, as well as providing a handbook for engineers or researchers who measure and analyze acoustic or vibration signals. The level of the book makes it appropriate both for undergraduate and graduate levels, with a proper selection of the content. In addition, the book should be a good reference for analysts who use experimental results and need to interpret such results. To satisfy these rather different purposes, for some of the topics in the book I have included more detail than would be necessary for an introductory text. To facilitate its use as a handbook, I have also included a short summary at the end of each chapter where some of the key points of the chapter are repeated.

This book contains background theory explaining the majority of analysis methods used in modern commercial software for noise and vibration measurement and analysis. It also includes a number of tools which are usually not found in commercial systems, but which are still useful for the practitioner. With modern computer-based software, it is easy to export data to, e.g., MATLAB/Octave (see below), and apply the techniques there.

Since it is an introductory text, most of the content of this book is of course available in more specialized textbooks and scientific papers. A few parts, however, include some improvements of existing techniques. I will mention these points in the descriptions of the appropriate chapters below.

Signal analysis is traditionally a field within electrical engineering, whereas most engineers and students pursuing noise and vibration measurements are mechanical or civil engineers. The aim has therefore been to make the material accessible particularly to students and engineers of these latter disciplines. For this reason, I have included introductions to the Laplace and Fourier transforms – both essential tools for understanding, analyzing, and solving problems in dynamics. Electrical engineering students and practitioners should still find many of the topics in the book interesting.

Signal analysis is a subject which is best learned by practicing the theories (perhaps that is a universal truth for all areas?). I have therefore incorporated numerous examples using MATLAB or GNU Octave throughout the book. Further examples and an accompanying toolbox which can be used with either MATLAB or GNU Octave can be downloaded from my website. More information about this is located in Section 1.6. I strongly recommend the use of these tools as a complement to reading this book, regardless of whether you are a student, a researcher, or an industry practitioner.

Chapter 2 introduces dynamic signals and systems with the aim of being an introduction particularly for mechanical and civil engineering students. In this chapter, the classification of signals into periodic, random, and transient signals is introduced. The chapter also includes linear system theory and a comprehensive introduction to the Laplace and Fourier transforms, both important tools for understanding and analyzing dynamic systems.

In Chapter 3, some fundamental concepts of sampled signals are presented. Starting with the sampling theorem and continuing with digital filter theory, this chapter presents some important applications of digital filters for fractional octave analysis and for integrating and differentiating measured signals.

Chapter 4 introduces some applied statistics and random process theory from a practical perspective. It includes an introduction to hypothesis testing as this tool is sometimes used

for testing stationarity of data. This chapter also gives an introduction to the application of statistics for data quality assessment, which is becoming more important with the large amounts of data collected in many applications of noise and vibration analysis.

Chapters 5 and 6 provide an introduction to the theory of mechanical vibrations. I anticipate that the contents of these two chapters will already be known to many readers, but I have found it important to include them because my presentation focuses on the experimental implications of the theory, unlike the presentation in most mechanical vibration textbooks, and because some later chapters in the book need a foundation with a common nomenclature.

In Chapter 7, the most important transducers used for measurements of noise and vibration signals are presented, specifically the accelerometer, the force sensor, and the microphone. Because piezoelectric sensors with built-in signal conditioning (the so-called IEPE sensors) are widely used today, this technology is presented in some depth. In this chapter, I also present some personal ideas on how to become a good experimentalist.

The analysis techniques mostly used in this field are based on the Discrete Fourier Transform (DFT), computed by the FFT. Spectrum analysis is therefore an important part of this book and Chapters 8–10 are spent on this topic. Chapter 8 introduces basic frequency analysis theory by presenting the different signal classes, and the different spectra used to describe the frequency content of these signals.

In Chapter 9, the DFT and some other techniques used to experimentally determine the frequency content of signals are presented. The properties of the DFT, which are very important to understand when interpreting experimental frequency spectra, are presented relatively comprehensively.

Chapter 10 includes a comprehensive presentation of how spectra from periodic, random, and transient signals, and mixes of these signal classes should be estimated in practice. Chapter 10 also includes a comprehensive explanation of Welch's method for PSD estimation, including overlap processing, as this is the method used in virtually all commercial software. The treatment of practical spectral analysis in this chapter should also be of use to engineers outside the field of acoustics and vibrations who want to calculate and/or interpret spectra by using the FFT.

In Chapter 11, the design of modern data acquisition and measurement systems is described from a user perspective. In this chapter, both hardware and software issues are penetrated.

Chapter 12 addresses order tracking, which is a common technique for analysis of rotating machinery equipment. The chapter describes the most common techniques used to measure such signals both with fixed sampling frequency and with synchronous sampling.

Frequency response functions are important measurement functions in experimental noise and vibration analysis and are used, for example, in EMA. Chapter 13 therefore covers techniques for measuring frequency responses for single-input/single-output (SISO) systems. Both impact excitation and shaker excitation techniques are presented in detail. In Chapter 14, the techniques are extended to multiple-input/multiple-output (MIMO) systems.

Chapter 15 presents some relatively advanced techniques used for multichannel analysis, namely principal components and virtual signals. These techniques are commonly used for noise path analysis and noise source identification in many of the sophisticated software

packages available commercially. I present these concepts in some depth, since they are not readily available in other textbooks.

Chapter 16 is a new chapter introducing EMA in terms of how to perform such tests. The chapter also includes a thorough explanation of the mathematical background of modal parameter estimation (MPE) with the Unified Matrix Polynomial Approach (UMPA) framework, and presents most algorithms currently used in commercial systems for EMA, and also for OMA, since the MPE is essentially the same for these two techniques, with some small differences.

Chapter 17 is a new chapter that presents OMA. Since most MPE algorithms are common for EMA and OMA, only those methods that are unique for OMA are presented in this chapter. The chapter also presents the fundamental basis for OMA; the decomposition of correlation functions and spectral densities into modal parameters.

In Chapter 18, which is essentially what constituted Chapter 16 in the first edition of the book, I have collected a number of more advanced techniques that engineers in this field should be acquainted with. This chapter presents, in order, the shock response spectrum, the Hilbert transform with applications, the cepstrum and envelope spectrum, and how to produce Gaussian time signals with known spectral density. The chapter has also been extended with two recently developed methods for removing harmonics, or separating signals into random and harmonic parts, cepstral editing, and the frequency domain editing method.

In the Appendices, I have included some fundamentals on complex numbers, logarithmic diagrams and the decibel unit, matrix theory, eigenvalues, and the singular value decomposition. The reader who does not feel confident with some of these concepts will hopefully find enough theory in these appendices to follow the text in this book. The last appendix contains some references to good sources for more information within the noise and vibration community. I hope the newcomer to this field can benefit from this list.

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## List of Abbreviations

2DOF	two degrees-of-freedom system
AC	alternating current
ADC	analog-to-digital converter
AFDE	automatic frequency domain editing
BT	bandwidth-time (product)
CMIF	complex mode indicator (indication) function
CSD	cross-spectral density function
DAC	digital-to-analog converter
DC	direct current
DFT	discrete Fourier transform
DOF	degree-of-freedom (point and direction)
EMA	experimental modal analysis
ESD	energy spectral density
FDD	frequency domain direct identification
FDE	frequency domain editing
FDPI	frequency domain direct parameter Identification
FE	finite element
FEM	finite element method
FFT	fast Fourier transform
FIR	finite impulse response (filter)
FRF	frequency response function
HF	high frequency
HP	highpass
IDFT	inverse discrete Fourier transform
IEPE	integrated electronics piezoelectric (sensor)
IFFT	inverse fast Fourier transform
IIR	infinite impulse response (filter)
IRF	impulse response function
ITD	Ibrahim time domain
LF	low frequency
ISO	international standardization organization
LSCE	least squares complex exponential
LSCF	least squares complex frequency domain

LSFD	least squares frequency domain
MAC	modal assurance criterion
MEMS	microelectro-mechanical systems (sensors)
MDOF	multiple degrees-of-freedom
MIF	mode indicator function
MITD	multiple-reference Ibrahim time domain
MIMO	multiple-input/multiple-output
MISO	multiple-input/single-output
MMITD	modified multiple-reference Ibrahim time domain
MPE	modal parameter estimation
MPSS	multiphase stepped sine
MrMIF	modified real mode indicator function
MvMIF	multivariate mode indicator function
NSI	noise source identification
NSR	noise-to-signal ratio
ODS	operating deflection shape
OMA	operational modal analysis
PDF	probability density function
PRD	periodogram ratio detection
PSD	power spectral density
PTD	polyreference time domain
RMS	root mean square
RPM	revolutions per minute
SDOF	single degree-of-freedom
SIMO	single-input/multiple-output
SISO	single-input/single-output
SNR	signal-to-noise ratio
SRS	shock response spectrum
SVD	singular value decomposition
TEDS	transducer electronic data sheet