Mechanisms and Machine Science

Rajiv Tiwari · Y. S. Ram Mohan · Ashish K. Darpe · V. Arun Kumar · Mayank Tiwari *Editors*

Vibration Engineering and Technology of Machinery, Volume I

Select Proceedings of VETOMAC XVI 2021

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Mechanisms and Machine Science

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The book is dedicated to Late Prof. J. S. Rao

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Preface

The International Conference on Vibration Engineering and Technology of Machinery (VETOMAC) is annually held series of conferences essentially to promote Vibration Engineering and Technology of Machinery. The first VETOMAC was organized at IISc Bengaluru, India, in the year 2000 with the intention of encouraging scientific and technical cooperation and exchange across the globe. The 16th VETOMAC was held at B.M.S. College of Engineering, Bengaluru, India, during 16 to 18 December 2021. This edition of VETOMAC was dedicated to the memory of its founder, Prof. J. S. Rao, who passed away on July 04, 2020 and in pursuance of his vision, the theme of this conference is kept as 'Integrated Vehicle Health Management (IVHM)'. This conference is the 16th in the series of annual symposia that started in Bengaluru, India (2000), and were subsequently held in Mumbai, India (2002), Kanpur, India (2004), Hyderabad, India (2007), Wuhan, China (2009), New Delhi, India (2010), Hong Kong, China (2011), Vaddeswaram, India (2012), Nanjing, China (2013), Manchester, UK (2014), Taiwan (2015), Warsaw, Poland (2016), Queensland, Australia (2017), Lisbon, Portugal (2018), and Curitiba, Brazil (2019).

Late Prof. Jammi Srinivasa Rao (1939–2020) popularly known as JS in the field of Vibration Engineering was first person to acquire doctorate in Design stream from IIT Kharagpur in independent India. He was a Distinguished Professor of rotor dynamics and vibration engineering at IIT-Delhi, Counsellor-Science and Technology at the Indian Embassy of USA, Washington DC, Visiting Professor at various Universities abroad and Consultant to over 30+ industries. He is one of the 13 founding fathers of the International Federation for the Promotion of Mechanism and Machine Science (IFToMM) Poland in 1969. He established 'The Vibration Institute of India (TVII)'. An international scientific journal launched by him in 2002 has groomed itself into yearly six volume *Journal of Vibration Engineering and Technologies* (JVET) co-published with Springer. His authorship includes 200+ reputed journal and 300+ conference papers and twenty-two machinery dynamics books. Professor Rao was instrumental in initiating, collaborating and promoting the Vibration Engineering and Technology of Machinery (VETOMAC) conference since its inception in 2000 until 2019. B.M.S. College of Engineering has organized this prestigious conference in the memory of Prof. J. S. Rao (who was also an INAE Distinguish Professor at BMSCE).

VETOMAC XVI covered four main broad categories in Vibration and Technology of Machinery fields: Vibration Analysis, Condition Monitoring Based on Vibrations, Rotor Dynamics and Tribology and allied areas. Plenary lectures were given on the theme of conference, 'Integrated Vehicle Health Management (IVHM)' by Dr. V. K. Saraswat, National Institution for Transforming India (NITI) Aayog, India and Dr. Kota Harinarayana, Chairman (Board of Governor) Indian Institute of Technology Varanasi. The technical keynote and contributed papers were presented in hybrid mode due to prevailing COVID-19 situation all over the world during conference period. Presenters were from across the globe, including countries like USA, UK, France, Australia, Ireland, Italy, Poland, Portugal, Brazil, Sweden, Mexico, China, Taiwan, India, and Nepal. The present book series have two volumes. Volume I (present volume) contains total 38 contributed technical papers and Volume II contains remaining 37 contributed technical papers after rigorous peer review and revision.

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Bengaluru, India — Y. S. Ram Mohan
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Dr. V. Arun Kumar currently working as Chief Technology Menor, at Dheya Technologies Ltd., Bengaluru, and as Adjunct Professor in B.M.S. College of Engineering, Bengaluru, has obtained his B.E. Mechanical Engineering, from Bengaluru University, M.Sc. Engg. from Madras University, and Ph.D. from IIT, Madras. He worked in Propulsion Division, National Aerospace Laboratories for 34 years and retired as Director Grade Scientist/Head, Propulsion Division. His major areas of research interests include vibration reduction in rotating machineries, advanced bearing/dampers systems, etc. He has to his credits ten patents granted (another five under processing) and over 100 publications. He has received National Design award from NDRF in Mechanical Engineering and Biren Roy award from Aeronautical Society of India, apart from outstanding performance award from NAL.

Dr. Mayank Tiwari is currently Professor in the Department of Mechanical Engineering, Indian Institute of Technology Patna, since 2013. He worked in General Electric from 2001 to 2013 in the Global Research and Development Division and the Aviation Division in the John F. Welch Technology Centre Bengaluru. He completed his Ph.D. from IIT Delhi in the area of nonlinear Rotor Dynamics in 1998 after which he worked as Postdoctoral Fellow in the Acoustic and Dynamics Laboratory of the Ohio State University USA. Dr. Mayank Tiwari works in the area of rotor dynamics and tribology. He has more than 30 papers in international journals and patents in USA, France, and India in the area of vacuum tribology, X-ray tube rotors, wind turbine gear box. Dr. Mayank Tiwari also has two design registrations in the India Patent Office.

Identification in a Magnetically Levitated Rigid Rotor System Integrated with Misaligned Sensors and Active Magnetic Bearings

Prabhat Kumar and Rajiv Tiwari

Abstract Misalignment is amongst the serious faults occurring in rotating machinery and the normal operations of machine will be crucially influenced under this state. Therefore, there is a need for exploring the motion of a faulty rotor system and identifying the faults for untroubled performance of machines. In this paper, a numerical investigation has been presented on the dynamic action of an unbalanced as well as misaligned stiff rotor system having two discs at the offset positions and mounted on active magnetic bearings at both the shaft ends. The rotor is assumed to be in combined misalignment with AMBs and non-contact displacement sensors. The force arising from misaligned AMB is derived along with description of the mathematical modelling of misaligned sensors based on the virtual trial misalignment strategy. In this approach, the trial misalignment is additionally and virtually given to the rotor by providing an additional bias current to AMBs. This helps in creating additional misalignments in both AMBs and sensors relative to the rotor operating axis. Dynamic equations of the rotor-sensor-AMB model is derived based on moment equilibrium method with the consideration of inertia force, unbalance force, gyroscopic effects and misaligned AMB forces. Further, the equations are solved by developing and arranging SIMULINK™ blocks to obtain the time dependent displacement response and AMB current signal. The foremost aim of the article is to explain the vibrational effects on the rotor due to unbalance, eddy current proximity sensors misalignment and AMBs residual misalignment. Apart from this, the paper would also identify the initial offset of sensors.

Keywords Active magnetic bearing · Virtual trial misalignment · Unbalance · Rigid rotor · Gyroscopic moment · Identification

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Nomenclature

a_1, a_2	Distances of the rotor center of gravity (C.G.) from AMB1 as well as AMB2
e_1, e_2	Disc 1 and Disc 2 eccentricities
(i_{x1}, i_{y1}) and (i_{x2}, i_{y2})	Control currents at AMB 1 as well as AMB 2 in the vertical (x) and horizontal (y) directions
(i_{Tx1}, i_{Ty1}) and (i_{Tx2}, i_{Ty2})	Trial bias currents at AMB 1 as well as AMB 2 in the vertical (x) and horizontal (y) directions
I_d, I_{P1}, I_{P2}	Rotational inertia of the rotor, disc 1 as well as disc 2
(k_{sx1}, k_{sy1}) and (k_{sx2}, k_{sy2})	Stiffness (displacement) values of AMB 1 as well as AMB 2 in the vertical (x) and horizontal (y) directions
(k_{ix1}, k_{iy1}) and (k_{ix2}, k_{iy2})	Stiffness (current) constants of AMB 1 as well as AMB 2 in the vertical (x) and horizontal (y) directions
k_P, k_D, k_I	Proportional, Derivative as well as Integral factors of PID controller
l, l_1 and l_2	Rotor length, disc 1 and disc 2 distances with respect to the rotor C.G. point
m, m_{d1} and m_{d2}	Mass of the rotor, disc 1 as well as disc 2
i_0, s_0	Bias current, Nominal AMB air gap
t	Time
u_x, u_y	Rotor displacements in the vertical (x) and horizontal (y) directions

Greek Letter

β_1, β_2	Disc 1 and Disc 2 unbalance phases
$\left(\delta^a_{x1}, \delta^a_{y1}\right)$ and $\left(\delta^a_{x2}, \delta^a_{y2}\right)$	Residual (without trial) misalignments of AMB 1 as well as AMB 2 in the vertical (x) and horizontal (y) directions
$\left(\delta^s_{x1}, \delta^s_{y1}\right)$ and $\left(\delta^s_{x2}, \delta^s_{y2}\right)$	Residual (without trial) misalignments of sensors at AMB 1 and AMB 2 in the vertical (x) and horizontal (y) directions
$\left(\Delta^r_{x1}, \Delta^r_{y1}\right)$ and $\left(\Delta^r_{x2}, \Delta^r_{y2}\right)$	Rotor virtual trial misalignments at AMB 1 as well as AMB 2 locations in the vertical (x) and horizontal (y) directions

$(d\Delta_{x1}, d\Delta_{y1})$ and $(d\Delta_{x2}, d\Delta_{y2})$	Air gap between rotor and sensors (created manually) at AMB 1 and AMB 2 in the vertical (x) and horizontal (y) directions
ω	Rotor spin speed

1 Introduction

Modern industries need a properly designed high-speed rotating machinery (i.e., pumps, turbines, compressors, aircraft gas turbine engines, etc.) for multiple applications along with effective as well as efficient performance. Rotating element, i.e. rotors in the machines, in general have the support with conventional bearings, such as the rolling element bearings, hydrodynamic journal bearings. However, due to various operational constraints in these bearings, as such they are suitable for lower or moderate speeds, require lubrication and chances of wear and tear [1], the researchers in the present day are concentrating on active magnetic bearings (AMBs) [2]. AMB is a non-contact kind of bearing device committed for overcoming the performance restrictions of conventional bearings. The capabilities of AMB technology (consisting of electric magnets, proximity displacement sensors, controller and current amplifiers [3]) include active control of rotor system dynamics through adaptable damping and stiffness factors of the controller, lubrication free operation, which leads to provide high speed rotation in the air, low power loss, etc. [4]. Moreover, the rotor in the AMB structure can be stably operated at multiple higher spin speeds and for various system parameters. The AMB technology is also utilized for identification and diagnosis of multiple faults in high-speed turbomachinery, such as the centrifugal pumps and gas turbine engines [5, 6].

Malfunctions such as unbalance in rotor and misalignment in the coupled shaft as well as supported bearings are the predominant causes for excessive vibrations in a rotor system. Unbalance in a rotor may be caused from non-coinciding nature of its inertia and geometric axes [7]. This occurs due to manufacturing defects, heterogeneity of raw materials, maintenance issues, etc. [8]. On the other hand, the misalignment fault may occur as a result of inappropriate assembly of several machine components, system installation errors, etc. Moreover, this fault also arises from uninterrupted functioning of the rotor setup as well as uneven temperature based expansion and contraction of the rotor support structure [9]. There may be misalignment in the coupled shaft (i.e., between two rotating shafts at coupling location) as well as among the rotor and supported bearings, i.e., traditional bearings [9] or active magnetic bearings [10] under different types of misalignment i.e., parallel, angular and combined form. Under the influence of these rotor faults, there may be drastic troubles in the operation of entire machine, which may minimize the efficiency and productivity of a factory. Higher level of vibrations arising from faults and sudden breaking of some components of the machine may also cause hazardous accidents to the workers. To resolve these concerns and make risk-free life, it is essentially required to analyze the fault signatures and their effect on the system's dynamics. The suppression of the rotor vibrations based on balancing and aligning the system or using AMB and identifying the faults quantitatively using appropriate online monitoring techniques are also the crucial tasks.

In view of this, various literature surveys have been done through research papers. Morton [11] presented a modal balancing technique in order to balance an elastic shaft, in which he was not needed trial weights and values of bearing parameters. It was observed through numerical simulation that the technique was very useful for balancing purpose up to different critical speeds. A method was proposed by Krodkiewski [12] for identification of magnitude and phase of residual unbalance in a multi-bearing rotor system. He also included white noise in the displacement signals and tested the method for checking its accuracy. Identification results were found to be quite accurate and the method was highly robust. To evaluate the imbalance state of a rotor system, a technique based on measuring pedestal vibration was proposed by researchers [13]. The developed technique was noticed to be insensitive in determining the unbalance characteristics against noise signals but the bearing and mass parameters were slightly sensitive. A review on rotor dynamic nature, distinct balancing and vibration controlling methods was presented by Zhou and Shi [14]. Later, Menshikov [15] considered a rotor system with two elastic bearing supports and identified parameters associated with unbalance fault using the inverse method. Researchers [16] utilized the equivalent load concept described in [17] to identify single as well as double unbalances in a rotating machine. However, the considered machine was very simple in terms of the number of rotating components. Thereafter, Yao et al. [18] presented two distinct techniques to obtain optimized values of the rotor unbalance parameters. The first technique was a combination of the modal expansion method as well as optimization algorithm, whereas the second technique was the incorporation of the inverse method into the first technique. In the recent publication, a joint-input state estimation algorithm was utilized in identifying force due to unbalance in a stiff rotor model having traditional bearing supports [19]. An estimation methodology was developed for identifying unbalance characteristics in a simple stiff rotor-AMBs test rig set up [20]. Afterwards, AMB was used as a vibration controlling and fault identification device in an elastic shaft-discs-bearings system [21]. Later, they have applied a virtual trial unbalance method for the purpose of rotor balancing using AMB technology [22].

A long year back, the mathematical equations of coupled rotor misalignment force as well as moment were developed by authors in [23]. In the paper, the distinct kinds of coupling were also described and the effect of misalignment fault on these coupling were studied in detail. Hori and Uematsu [9] investigated stability analysis of a system consisting of two rotors (coupled together) with four number of journal bearings. The misalignment (parallel as well as angular) was considered in the supported bearings. The rotating system model was examined numerically by employing the transverse matrix technique and Newton–Raphson method. Researchers [24, 25] investigated the effects of coupling misalignment in the combined form (i.e., both lateral and angular misalignments) on vibrations of rotating machinery. They observed that the misalignment fault can be identified by looking into the vibration peak at twice the rotational speed. Afterwards, the consequence of lateral misalignment on the system's dynamics (two rigidly coupled Jeffcott rotors) was studied by Al-Hussain and Redmond [26]. Later, Al-Hussain [26] incorporated the pure angular misalignment between the two rigid rotors and generated the vibrational signature of the system using both the Newmark and Newton Raphson techniques. [27]. Three theoretical AMB models based on the four, six and eight number of electromagnets were proposed by Bouaziz et al. [28] to analyze the dynamic behavior of two degrees-of-freedom misaligned rotor-AMBs support system. In the same line, Messaoud et al. [29] presented an AMB integrated model for investigating rotor system dynamics considering angular misalignment. The consequence of distinct misalignments was also explored on vibrational response and stator current signature, in a misaligned coupled rotor having fluid bearing supports [30]. The orbital response and Fourier transform of both vibration and current were also effectively utilized along with the vibration and current waveforms, in identifying the unique characteristics of misalignment fault. Jang and Khonsari [31] described about misalignment in journal bearings in his review paper. Further, the dimensionless vibrational behavior of a fully floated misaligned rigid rotor-AMB system was analyzed numerically and found that there was rapid increment in magnitudes of non-dimensional system responses with a little enhancement in the disc's eccentricity and AMB's misalignment amount [32, 33]. Later, AMB technology was utilized by Srinivas et al. [34] in order to control vibrations arising due to unbalance in the rotor as well as misalignment at coupling position in a rotor-train model. They have also identified various system and fault parameters using a suitable steering function in the rotor system modelling. Kuppa and Lal [35] have also incorporated AMB as an active vibration mitigation in a misaligned turbo-generator model, which was comprised of two coupled rigid rotors with elastic bearing supports. With the help of a developed identification algorithm, the disc unbalance parameters, conventional bearings and AMBs parameters, as well as coupling's damping and stiffness constants were also accurately estimated.

Through the ongoing research studies it has been observed that the publications are quite available in analyzing the unbalance and misalignment faults for the system diagnosis and their detection in a rotor model hold mostly by conventional bearings and very less on AMBs support. Researchers have also been involved in reviewing AMB technology and misalignment in journal bearings as well as couplings and active balancing techniques. However, the dynamic investigation on the rotor system levitated by misaligned AMBs and connected with misaligned sensors for measuring displacement responses have not been addressed till now. Hence, the present article explores the development of mathematical model of several system components, and also the dynamic interaction among unbalance, AMBs misalignment as well as sensors misalignment faults. Identification of sensor residual misalignments is also accomplished employing a virtual trial misalignment (VTM) strategy.

In this paper, a novel VTM concept is developed for investigating the vibrational effects of the unbalance as well as misalignment faults on a magnetically levitated rotating system. A stiff and misaligned rotor mounted on two AMBs is mathematically modelled with innovative derivation of AMB force considering the misalignment in AMBs and proximity sensors. The system's dynamic equations are derived and resolved by developing a Simulink™ model to acquire the time series displacement as well as controlling current data. Dynamic effect of AMBs as well as sensors residual misalignments on the rotor performance has been presented in the paper. Further, the sensors' misalignment (residual amount) placed at the AMBs location are also estimated utilizing the innovative concept of sensor air gaps for without trial and additional (with trial) misalignments along with the amounts of rotor virtual trial misalignments.

2 System Description

To examine the dynamic behavior and understand the effect of unbalance as well as residual misalignment in a magnetically levitated system, a stiff rotor fastened with two rigid discs at offset positions and mounted on two misaligned radial AMBs is accounted as depicted in Fig. 1a. Due to the rigid behavior of the rotor throughout the operation, the displacements are to be measured at the left and right positioned AMBs (i.e., AMB 1 and AMB 2). For measuring the displacement response, the eddy current proximity sensors are included in the rotor model at AMB places. The major chance for AMB misalignment may be due to offsetting in sensors, which arise from manufacturing, system's assembly and test set up errors in the initial time. Therefore, the misalignment of sensors placed at AMB positions is also taken into account (refer Fig. 1b, c). Misalignment of the rotor with radial AMBs and sensors are supposed to be combined (parallel and angular) misalignment. This is on account of different amounts of x- and y-directional AMBs residual misalignment (δ^a_{x1}, δ^a_{y1}, δ^a_{x2} and δ^a_{y2}) and sensors residual misalignment (δ^s_{x1}, δ^s_{y1}, δ^s_{x2} and δ^s_{y2}) located at AMB 1 and AMB 2 places. The center position of eddy current proximity sensors, actuators of AMB and the rotor are, respectively, shown by O, A, and C. Both AMBs are assumed to be anisotropic and dissimilar stiffness parameters. Modelling of the system also incorporates the gyroscopic effect due to offset discs. Moreover, the effect of leakage in the magnetic flux as well as loss of eddy current are also considered in the AMB system to align with the practical situation. A proportional-integral-derivative (PID) controller is utilized in AMBs for the rotor stability and as an agent for reducing its undesirable vibration.

3 Mathematical Modelling of the System

This section explains the mathematical model involved in completely describing the unbalanced as well as misaligned stiff rotor system presented in Fig. 1a. The whole modelling includes the force model due to disc unbalance, force coming from misaligned AMB in the presence of offsets in sensors and inertia force model. Based on the rigid nature of the rotor during operation, its translational and angular deflections at C.G. point can be expressed as a function of translational deflection at

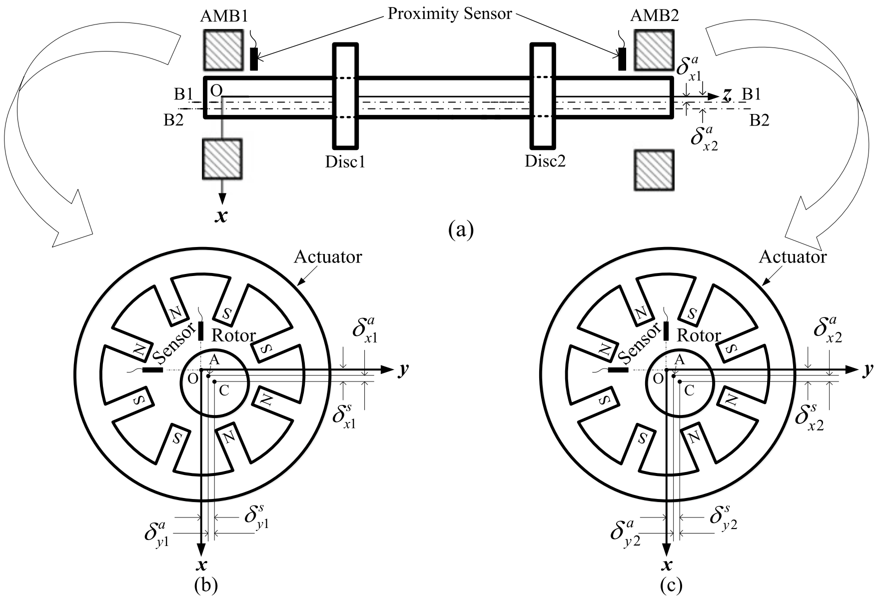

Fig. 1 **a** A rigid rotor-misaligned AMB model fixed with sensors in the x–z plane **b**, **c** residual misalignment of AMBs (δ_{x1}^a, δ_{y1}^a, δ_{x2}^a, δ_{y2}^a) and sensors (δ_{x1}^s, δ_{y1}^s, δ_{x2}^s, δ_{y2}^s) with the rotor at AMB 1 as well as AMB 2 locations in the x–y plane

AMB 1 as well as AMB 2 sites following Fig. 2. In Fig. 2, A1 and A2 denote for AMB 1 as well as AMB 2 locations, D1 and D2 for Disc 1 and Disc 2 positions and G for the location of rotor center of gravity.

The relation for displacements can be expressed as

$$\begin{aligned} u_x =& \bar{a}_2 u_{x1} + \bar{a}_1 u_{x2}; u_y = \bar{a}_2 u_{y1} + \bar{a}_1 u_{y2};\ \varphi_y \\ &= (-u_{x1} + u_{x2})/l; \varphi_x = \left(u_{y1} - u_{y2}\right)/l \end{aligned} \tag{1}$$

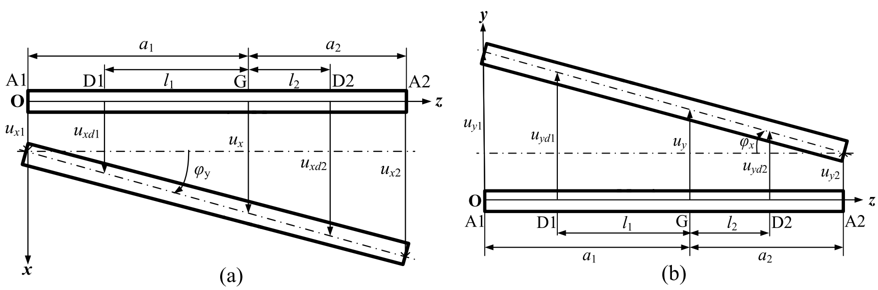

Fig. 2 Translational as well as angular deflections of rigid shaft at various locations in **a** x–z plane **b** y–z plane

with

$$\overline{a}_1 = \frac{a_1}{l};\ \overline{a}_2 = \frac{a_2}{l}$$

where the symbols φ_y and φ_x represent for the rotor angular deflections in the *x*–*z* and *y*–*z* planes. The symbols a_1 and a_2 are the distances of C.G. from left AMB and right AMB, respectively.

3.1 Modelling of Unbalance Force

The *x*- and *y*-directional unbalance force due to Disc 1 and Disc 2 can be written as

$$\begin{aligned} f_{unbx1} &= m_{d1}e_1\omega^2\cos(\omega t + \beta_1);\ f_{unby1} = m_{d1}e_1\omega^2 sin(\omega t + \beta_1) \\ f_{unbx2} &= m_{d2}e_2\omega^2\cos(\omega t + \beta_2);\ f_{unby2} = m_{d2}e_2\omega^2 sin(\omega t + \beta_2) \end{aligned} \tag{2}$$

where m_{d1} and m_{d2} are, respectively, the masses of Disc 1 as well as Disc 2. The unbalance eccentricities and phases of Disc 1 and Disc 2 are represented by (e_1, β_1) and (e_2, β_2). The symbol ω is the rotor spin speed.

3.2 Modelling of Misaligned AMB Force Considering Offsets in Sensors

For perfect alignment case (in which the rotor, sensors and AMB axes are coinciding, i.e. the points O, A and C are at the same point in Fig. 1b, c and an equal gap of air is available in the lower pole as well as upper pole of AMB as well as the back and front side poles of AMBs), Schweitzer and Maslen [36] have given the below expression for AMB force (assuming AMB 1 and *x*-direction force) as

$$f_{x1} = k_1\left\{\frac{(i_0 + i_{x1})^2}{(s_0 - u_{x1})^2} - \frac{(i_0 - i_{x1})^2}{(s_0 + u_{x1})^2}\right\} \tag{3}$$

The linearized form of Eq. (3) is expressed as

$$f_{x1} = k_{sx1}u_{x1} + k_{ix1}i_{x1} \tag{4}$$

where the stiffnesses k_{s1} and k_{i1} are indicated as

$$k_{s1} = \frac{4k_1 i_0^2}{s_0^3}; k_{i1} = \frac{4k_1 i_0}{s_0^2};\ k_1 = \frac{1}{4}\mu_0\eta N_1^2 A_{a1}\cos\frac{\alpha}{2} \tag{5}$$

Here, η is the constant term indicating for consideration of magnetic flux leakage, eddy current loss, etc. [37]. The magnetic pole area, number of turning coils and angle between two consecutive poles in AMB are denoted by A_{a1}, N_1 and α, respectively. The symbol μ_0 is the vacuum permeability of free space having $4\pi \times 10^{-7}$ H/m value. With different pole area and number of turning coils (i.e., A_{a2} and N_2), the stiffness coefficients of AMB 2 will also change. Moreover, the AMB 1 force in the vertical (y) direction (i.e., f_{y1}) will follow Eq. (4) with replacement of k_{sx1} by k_{sy1} and k_{ix1} by k_{iy1} due to its anisotropic nature, u_{x1} by u_{y1} and i_{x1} by i_{y1}.

However, in a real practice, it is very challenging to acquire a perfectly balanced and aligned rotor system. Even if precise alignment is assured, it cannot be sustained for an extensive period. Misalignment fault in the system can exist while machine's operation owing to various external aspects, such as disruption in the base support structure, thermal expansion and contraction of machine components, etc. [38]. In a fully floated rotor-AMB apparatus, the sensor measurement errors (due to its offset placed position presented in Fig. 1b, c) while locating the centre position of AMB may also cause the rotor misalignment with the supported AMB [39]. Following this, Fig. 3 presents the cross-sectional view of rotor and misaligned sensors placed at qth AMB site in the x–y plane, for the case of residual misalignment and additional trial misalignment (in which the virtual trial misalignment is provided to the rotor in addition to sensor residual misalignments). In Fig. 3b, the amounts Δ^r_{xq} and Δ^r_{yq} are the user-provided virtual trial misalignments to the rotor using AMB bias current in the vertical (x) and horizontal (y) directions, respectively (this will be discussed more briefly in Sect. 3.3.1). The points (1, 2) in Fig. 3a and points (1', 2') in Fig. 3a situated at the shaft represent for the points from where the transverse translational displacements can be measured by residually and additionally misaligned proximity sensors.

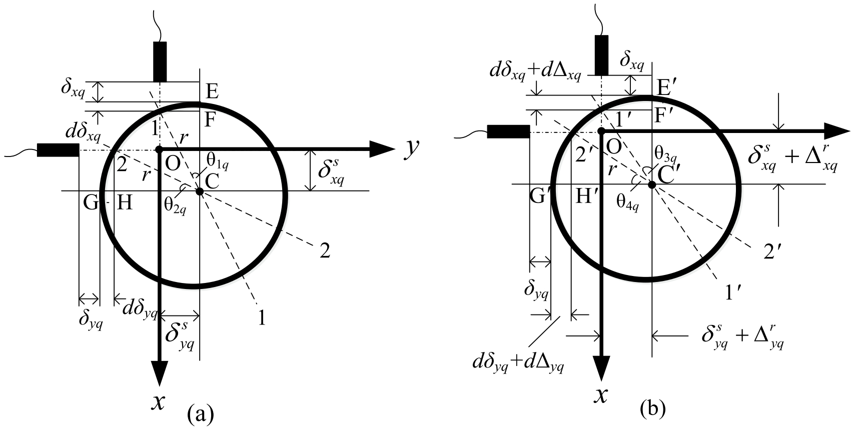

Fig. 3 Sectional view of the shaft and misaligned sensors available at qth AMB site (where $q = 1$ for AMB 1, and $q = 2$ for AMB 2) **a** residual misalignment (δ^s_{xq}, δ^s_{yq}) **b** additional trial misalignment ($\delta^s_{xq} + \Delta^r_{xq}$, $\delta^s_{yq} + \Delta^r_{yq}$)

Owing to misalignment of AMB in the nth direction (where $n = 1, 2$ as indicated in Fig. 3a by dashed line), the gap between the rotor and qth AMB is modified, which gives rise to the amount $(s_0 - \delta_{nq}^a)$ as the lower gap and $(s_0 + \delta_{nq}^a)$ as the upper gap, respectively. By putting these updated gaps into Eq. (3), the force in the nth direction from residually misaligned AMB can be formulated as

$$f_{nq}^m = k_q \left\{ \frac{\left(i_0 + i_{nq}^m\right)^2}{\left(s_0 - \delta_{nq}^a - u_{nq}^m\right)^2} - \frac{\left(i_0 - i_{nq}^m\right)^2}{\left(s_0 + \delta_{nq}^a + u_{nq}^m\right)^2} \right\};\ n = 1,\ 2 \tag{6}$$

where 'm' superscript represents the condition for residually misaligned AMB. The displacement of the rotor detected from qth misaligned (residually) sensors at the positions 'n' (i.e., $n = 1, 2$ in Fig. 3a) is shown by u_{nq}^m. Current output of the PID controller due to displacement u_{nq}^m as input is indicated with i_{nq}^m. Further, consideration of less vibration as compared to the air gap in the lower and upper poles, i.e. $u_{nq}^m \ll (s_0 - \delta_{nq}^a)$ and $u_{nq}^m \ll (s_0 + \delta_{nq}^a)$ and negligence of the higher-order terms, such as $(u_{nq}^m)^2$, $(i_{nq}^m)^2$, $u_{nq}^m (i_{nq}^m)^2$ and $i_{nq}^m u_{nq}^m$, the linearized form of Eq. (6) is given as

$$f_{nq}^m = k_{snq}^m u_{nq}^m + k_{inq}^m i_{nq}^m + f_{cnq}^m \tag{7}$$

with

$$k_{snq}^m = \frac{k_{snq}}{\left(1 - \delta_{nq}^2\right)^2};\quad k_{inq}^m = \frac{k_{inq}\left(1 + \delta_{nq}^2\right)}{\left(1 - \delta_{nq}^2\right)^2};\quad f_{cnq}^m = \frac{f_q \delta_{nq}}{\left(1 - \delta_{nq}^2\right)^2};$$
$$f_q = \frac{4 k_q i_0^2}{s_0^2};\ \delta_{nq} = \frac{\delta_{nq}^a}{s_0} \tag{8}$$

Using Eq. (7) for $(n = 1, 2)$ and Fig. 3a, the misaligned AMB forces (f_{xq}^m, f_{yq}^m) can be expressed in the vector form as

$$\begin{Bmatrix} f_{xq}^m \\ f_{yq}^m \end{Bmatrix} = [S_q] \begin{bmatrix} k_{s1q}^m & 0 \\ 0 & k_{s2q}^m \end{bmatrix} \begin{Bmatrix} u_{1q}^m \\ u_{2q}^m \end{Bmatrix} + [S_q] \begin{bmatrix} k_{i1q}^m & 0 \\ 0 & k_{i2q}^m \end{bmatrix} \begin{Bmatrix} i_{1q}^m \\ i_{2q}^m \end{Bmatrix} + \begin{Bmatrix} f_{cxq}^m \\ f_{cyq}^m \end{Bmatrix} \tag{9}$$

with

$$[S_q] = \begin{bmatrix} \cos\theta_{1q} & \sin\theta_{2q} \\ \sin\theta_{1q} & \cos\theta_{2q} \end{bmatrix};\ \begin{Bmatrix} f_{cxq}^{m1} \\ f_{cyq}^{m1} \end{Bmatrix} = [S_q] \begin{Bmatrix} f_{c1q}^{m1} \\ f_{c2q}^{m1} \end{Bmatrix}$$

where $[S_q]$ denotes for the transformation matrix at qth AMB site [40]. Further, with the help of Fig. 3a, the AMBs' stiffness constants $(k_{s1q}^m, k_{s2q}^m, k_{i1q}^m, k_{i2q}^m)$ and the rotor displacements and currents $(u_{1q}^m, u_{2q}^m, i_{1q}^m, i_{2q}^m)$ can be computed with reference to x- and y-coordinate axes as below

$$\begin{Bmatrix} k_{s1q}^m \\ k_{s2q}^m \end{Bmatrix} = [R_q] \begin{Bmatrix} k_{sxq}^m \\ k_{syq}^m \end{Bmatrix};\ \begin{Bmatrix} k_{i1q}^m \\ k_{i2q}^m \end{Bmatrix} = [R_q] \begin{Bmatrix} k_{ixq}^m \\ k_{iyq}^m \end{Bmatrix};\ \begin{Bmatrix} u_{1q}^m \\ u_{2q}^m \end{Bmatrix}$$
$$= [R_q] \begin{Bmatrix} u_{xq}^m \\ u_{yq}^m \end{Bmatrix};\ \begin{Bmatrix} i_{1q}^m \\ i_{2q}^m \end{Bmatrix} = [R_q] \begin{Bmatrix} i_{xq}^m \\ i_{yq}^m \end{Bmatrix}, [R_q] = \begin{bmatrix} \cos\theta_{1q} & \sin\theta_{1q} \\ \sin\theta_{2q} & \cos\theta_{2q} \end{bmatrix} \tag{10}$$

On substituting the relations of Eq. (10) into Eq. (9) and afterwards on simplification, the compact form of misaligned force vector due to qth AMB (i.e., $q = 1$ for AMB 1 and $q = 2$ for AMB 2) is written as

$$\mathbf{f}_{AMBq}^m(t) = \mathbf{K}_{s\theta q}^m \boldsymbol{\eta}_{AMBq}^m(t) + \mathbf{K}_{i\theta q}^m \mathbf{i}_{cq}^m(t) + \mathbf{f}_{cq}^m \tag{11}$$

with

$$\mathbf{f}_{AMBq}^m(t) = \begin{Bmatrix} f_{xq}^m \\ f_{yq}^m \end{Bmatrix};\ \boldsymbol{\eta}_{AMBq}^m(t) = \begin{Bmatrix} u_{xq}^m \\ u_{yq}^m \end{Bmatrix};\ \mathbf{f}_{cq}^m = \begin{Bmatrix} f_{cxq}^m \\ f_{cyq}^m \end{Bmatrix};\ \mathbf{i}_{cq}^m(t) = \begin{Bmatrix} i_{xq}^m \\ i_{yq}^m \end{Bmatrix};$$
$$\mathbf{K}_{s\theta q}^m = \begin{bmatrix} \left(a_{1q}k_{sxq}^m + b_{1q}k_{syq}^m\right) & \left(b_{1q}k_{sxq}^m + c_{1q}k_{syq}^m\right) \\ \left(b_{1q}k_{sxq}^m + c_{1q}k_{syq}^m\right) & \left(c_{1q}k_{sxq}^m + d_{1q}k_{syq}^m\right) \end{bmatrix};\ f_{cxq}^m = \frac{f_q\delta_{xq}}{\left(1-\delta_{xq}^2\right)^2};$$
$$\mathbf{K}_{i\theta q}^m = \begin{bmatrix} \left(a_{1q}k_{ixq}^m + b_{1q}k_{iyq}^m\right) & \left(b_{1q}k_{ixq}^m + c_{1q}k_{iyq}^m\right) \\ \left(b_{1q}k_{ixq}^m + c_{1q}k_{iyq}^m\right) & \left(c_{1q}k_{ixq}^m + d_{1q}k_{iyq}^m\right) \end{bmatrix};\ f_{cyq}^m = \frac{f_q\delta_{yq}}{\left(1-\delta_{yq}^2\right)^2};$$
$$a_{1q} = \cos^3\theta_{1q} + sin^3\theta_{2q};\ b_{1q} = \sin\theta_{1q}\cos^2\theta_{1q} + \sin^2\theta_{2q}\cos\theta_{2q};$$
$$c_{1q} = \sin^2\theta_{1q}\cos\theta_{1q} + \cos^2\theta_{2q}\sin\theta_{2q};\ d_{1q} = sin^3\theta_{1q} + \cos^3\theta_{2q} \tag{12}$$

$$k_{sxq}^m = \frac{k_{sxq}}{\left(1-\delta_{xq}^2\right)^2};\ k_{ixq}^m = \frac{k_{ixq}\left(1+\delta_{xq}^2\right)}{\left(1-\delta_{xq}^2\right)^2};\ k_{syq}^m = \frac{k_{syq}}{\left(1-\delta_{yq}^2\right)^2};$$
$$k_{iyq}^m = \frac{k_{iyq}\left(1+\delta_{yq}^2\right)}{\left(1-\delta_{yq}^2\right)^2};\ \delta_{xq} = \frac{\delta_{xq}^a}{s_0};\ \delta_{yq} = \frac{\delta_{yq}^a}{s_0} \tag{13}$$

In Eq. (11), the rotor displacement and AMB's control current vectors at the qth AMB site are represented by $\boldsymbol{\eta}_{AMBq}^m$ and $\mathbf{i}_{cq}^m$. Further, the mathematical expression of the control current vector $\mathbf{i}_{cq}^m$, for the condition of misalignment in AMB and sensors can be written as

$$\mathbf{i}_{cq}^m(t) = -\begin{bmatrix} k_P & 0 & k_I & 0 & k_D & 0 \\ 0 & k_P & 0 & k_I & 0 & k_D \end{bmatrix} \left\{ u_{xq}^m \ u_{yq}^m \ \int u_{xq}^m \mathrm{d}t \ \int u_{yq}^m \mathrm{d}t \ \dot{u}_{xq}^m \ \dot{u}_{yq}^m \right\}^T \tag{14}$$

3.3 Modelling of Misaligned Sensors

This section explores the mathematical strategies for evaluating the angles (θ_{1q}, θ_{2q}) for sensors residual misalignment and (θ_{3q} and θ_{4q}) for their additional (trial) misalignments as well as identifying their residual misalignments (δ^s_{xq}, δ^s_{yq}). Using Fig. 3a, the following relationships i.e., Eqs. (15) and (16) can be established.

$$\left.\begin{array}{l}\overline{\mathrm{EC}} = \overline{\mathrm{EF}} + \overline{\mathrm{FC}} \Rightarrow r = d\delta_{xq} + r\cos\theta_{1q} \Rightarrow d\delta_{xq} = r\left(1 - \cos\theta_{1q}\right) \\ \overline{\mathrm{GC}} = \overline{\mathrm{GH}} + \overline{\mathrm{HC}} \Rightarrow r = d\delta_{yq} + r\cos\theta_{2q} \Rightarrow d\delta_{yq} = r\left(1 - \cos\theta_{2q}\right)\end{array}\right\} \tag{15}$$

$$\sin\theta_{1q} = \frac{\delta^s_{yq}}{r};\ \sin\theta_{2q} = \frac{\delta^s_{xq}}{r} \tag{16}$$

Similarly, assisted by Fig. 3b, the exhibiting relations are

$$\left.\begin{array}{l}\overline{E'C'} = \overline{E'F'} + \overline{F'C'} \Rightarrow r = \left(d\delta_{xq} + d\Delta_{xq}\right) + r\cos\theta_{3q} \Rightarrow \left(d\delta_{xq} + d\Delta_{xq}\right) = r\left(1 - \cos\theta_{3q}\right) \\ \overline{G'C'} = \overline{G'H'} + \overline{H'C'} \Rightarrow r = \left(d\delta_{yq} + d\Delta_{yq}\right) + r\cos\theta_{4q} \Rightarrow \left(d\delta_{yq} + d\Delta_{yq}\right) = r\left(1 - \cos\theta_{4q}\right)\end{array}\right\} \tag{17}$$

$$\sin\theta_{3q} = \frac{\delta^s_{yq} + \Delta^r_{yq}}{r};\ \sin\theta_{4q} = \frac{\delta^s_{xq} + \Delta^r_{xq}}{r} \tag{18}$$

Here, 'r' is the shaft radius. Subsequently, the subtraction of Eq. (17) from Eqs. (15) and (18) from Eq. (16), respectively, give the next equations as Eqs. (19) and (20)

$$\cos\theta_{1q} - \cos\theta_{3q} = \frac{d\Delta_{xq}}{r} \text{ and } \cos\theta_{2q} - \cos\theta_{4q} = \frac{d\Delta_{yq}}{r} \tag{19}$$

$$sin\theta_{3q} - \sin\theta_{1q} = \frac{\Delta^r_{yq}}{r} \text{ and } sin\theta_{4q} - \sin\theta_{2q} = \frac{\Delta^r_{xq}}{r} \tag{20}$$

The distances ($\delta_{xq} + d\delta_{xq}$) and ($\delta_{yq} + d\delta_{yq}$) in Fig. 3a are, respectively, the vertical and horizontal directions air gap of the sensors with the rotor without trial misalignment, whilst ($\delta_{xq} + d\delta_{xq} + d\Delta_{xq}$) and ($\delta_{yq} + d\delta_{yq} + d\Delta_{yq}$) in Fig. 3b are the gaps with consideration of trial misalignment together with sensors residual misalignment. These air gaps are well known parameters set by a user for detecting the vibrational displacement at qth AMB location. Subtraction of air gaps for with and without trial misalignments would provide, respectively, the quantities $d\Delta_{xq}$ and $d\Delta_{yq}$, that are required in Eq. (19). Furthermore, the values of trial misalignments Δ^r_{xq} and Δ^r_{yq} needed in Eq. (20) are also known quantities. The method for calculating these values will be elaborated in Sect. 3.3.1. Afterwards, the θ_{1q} and θ_{2q} magnitudes can be acquired by solving Eqs. (19) and (20) as below

$$\theta_{1q} = \tan^{-1}\left(\frac{d\Delta_{xq}}{\Delta^r_{yq}}\right) - \sin^{-1}\left\{\frac{\left(d\Delta_{xq}\right)^2 + \left(\Delta^r_{yq}\right)^2}{2r^2}\right\} \text{ and}$$

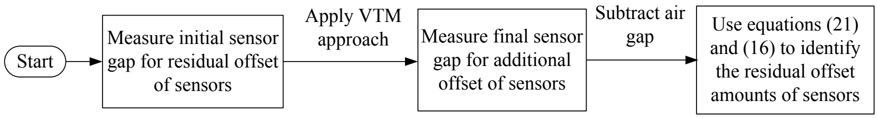

Fig. 4 Flow diagram describing the procedure to identify sensors residual misalignments

$$\theta_{2q} = \tan^{-1}\left(\frac{d\Delta_{yq}}{\Delta^{r}_{xq}}\right) - \sin^{-1}\left\{\frac{\left(d\Delta_{yq}\right)^{2} + \left(\Delta^{r}_{xq}\right)^{2}}{2r^{2}}\right\} \tag{21}$$

Moreover, the angles θ_{3q} and θ_{4q} in Fig. 3b can be obtained by using Eq. (19). Further, the residual misalignments of sensors (δ^{s}_{xq}, δ^{s}_{yq}) can be identified by substituting the values of θ_{1q} and θ_{2q} into Eq. (16). For more clarification in the procedures to identify residual misalignments of sensors located at qth AMB location, the flow chart diagram is given in Fig. 4.

3.3.1 Virtual Trial Misalignment (VTM) Concept

In this section, a novel VTM technique is explored to provide known misalignments (i.e., Δ^{r}_{xq} and Δ^{r}_{yq}) to the rotor, so that the center position of rotor gets shifted from the point (C) in Fig. 3a to the point (C') in Fig. 3b. These trial misalignments give rise to additional trial misalignments of sensors (located at qth AMB position) with respect to the rotor center, i.e. the distances $\delta^{s}_{xq} + \Delta^{r}_{xq}$, $\delta^{s}_{yq} + \Delta^{r}_{yq}$ in Fig. 3b. The word used here as *virtual* is due to shifting of rotor relative to AMB current position using trial bias current. Moreover, the trial misalignments among the rotor and AMBs as well as the rotor and sensors can also be created by *physical* methods as elaborated in Kumar and Tiwari [39]. However, this physical technique is less effective, time-consuming and more laborious in comparison to the virtual method. Therefore, it is good to focus on the VTM concept. In this process, the vertical and horizontal directions trial magnetic forces (i.e., f_{Txq} and f_{Tyq}) derived from bias currents are initiated at the qth AMB site to acquire the amounts of the vertical and horizontal trial misalignments. These additional forces serving as virtual trial misalignment (VTM) excitation forces, are developed besides controlling forces of AMBs, which are expressed as below

$$f_{Txq} = k^{T}_{ixq} i_{Txq}; f_{Tyq} = k^{T}_{iyq} i_{Tyq} \tag{22}$$

with

$$k^{T}_{ixq} = \frac{4k_{q}\left(i_{0} + i_{Txq}\right)}{s_{0}^{2}};\ k^{T}_{iyq} = \frac{4k_{q}\left(i_{0} + i_{Tyq}\right)}{s_{0}^{2}} \tag{23}$$

Here, the terms k_{ixq}^{T} and k_{iyq}^{T} appeared after supplying extra bias currents (i.e., i_{Txq} and i_{Tyq}) to AMB for developing trial misalignments, virtually. Moreover, the values of trial misalignments are to be known by equating Eq. (22) with the trial constant forces of misaligned AMB [41], as

$$f_{Txq} = k_{ixq}^{T} i_{Txq} = \frac{f_q \Delta_{xq}}{\left(1 - \Delta_{xq}^2\right)^2};\ f_{Tyq} = k_{iyq}^{T} i_{Tyq} = \frac{f_q \Delta_{yq}}{\left(1 - \Delta_{yq}^2\right)^2};$$
$$f_q = \frac{4 k_q i_0^2}{s_0^2};\ \Delta_{xq} = \frac{\Delta_{xq}^{r}}{s_0}; \Delta_{yq} = \frac{\Delta_{yq}^{r}}{s_0} \tag{24}$$

Further, the next equation (Eq. (25)) can be obtained by substituting the terms (k_{ixq}^{T} and k_{iyq}^{T}) of Eq. (23) into Eq. (24) as

$$i_{Txq}^2 + i_0 i_{Txq} + \frac{i_0^2 \Delta_{xq}}{\left(1 - \Delta_{xq}^2\right)^2} = 0;\ i_{Tyq}^2 + i_0 i_{Tyq} + \frac{i_0^2 \Delta_{yq}}{\left(1 - \Delta_{yq}^2\right)^2} = 0 \tag{25}$$

With the known quantities of these four parameters, such as trial bias currents (i_{Txq}, i_{Tyq}), nominal bias current and AMB air gap (i_0 and s_0), Eq. (25) helps in evaluating the values of rotor vertical and horizontal trial misalignments (i.e., Δ_{xq}^{r} and Δ_{yq}^{r}). These amounts will be useful in identifying the sensors' residual misalignments as elaborated in Sect. 3.3.

3.4 Dynamic Equations of the Considered Rotating System

Dynamic equations of the considered magnetically levitated rotating machine as shown in Fig. 1a have been derived by using the moment equilibrium method [42]. For this, the moment due to unbalance forces, misaligned AMB forces considering offsets in sensors, inertia force and the disc's gyroscopic effect in the x–z plane and y–z plane have taken about AMB 1 as well as AMB 2 positions. The equations of motion (EOM) of the rotor model in the matrix form can be expressed as

$$\mathbf{M}\Delta\ddot{q}^{m}(t) - \omega \mathbf{G}\Delta\dot{\mathbf{q}}^{m}(t) = \mathbf{f}_{unb} + \mathbf{f}_{AMB}^{m} \tag{26}$$

where the mass matrix **M** and gyroscopic matrix **G** are

$$\mathbf{M} = \begin{bmatrix} \left(m\bar{a}_2^2 + i_d\right) & 0 & \left(m\bar{a}_1\bar{a}_2 - i_d\right) & 0 \\ 0 & \left(m\bar{a}_2^2 + i_d\right) & 0 & \left(m\bar{a}_1\bar{a}_2 - i_d\right) \\ \left(m\bar{a}_1\bar{a}_2 - i_d\right) & 0 & \left(m\bar{a}_1^2 + i_d\right) & 0 \\ 0 & \left(m\bar{a}_1\bar{a}_2 - i_d\right) & 0 & \left(m\bar{a}_1^2 + i_d\right) \end{bmatrix};\ \mathbf{G} = \begin{bmatrix} 0 & i_p & 0 & -i_p \\ i_p & 0 & -i_p & 0 \\ 0 & -i_p & 0 & i_p \\ -i_p & 0 & i_p & 0 \end{bmatrix} \tag{27}$$

with