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Sujaul Chowdhury · Abdullah Al Sakib

Numerical Exploration of Fourier Transform and Fourier Series

The Power Spectrum of Driven Damped Oscillators

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Preface

This book contains practical demonstrations of numerically obtaining the Fourier transform of given numerical data. In particular, we demonstrate how to obtain the frequencies that are present in data numerically, using what is called discrete Fourier transform. We have used programs written in Mathematica in this regard.

To obtain the Fourier series, we first need to know the frequencies that we need to use. Here lies a need for the Fourier transform. We find that if we have the Fourier series for a function $y(t)$, we can plot the Fourier series in an extended interval of time t. We find that a function need not be periodic to be expressed analytically as a Fourier series. But after expressing the function as a Fourier series, we can plot it in an extended interval of time and get repeated or periodic plots of the original non-periodic function.

This book also contains numerical solutions of differential equations of driven damped oscillators using the 4th-order Runge-Kutta method using programs written in Mathematica. Data of the numerical solution are compared with analytical solutions and are fed to a discrete Fourier transform program to obtain frequency content of the oscillator using programs written in Mathematica.

The behavior of mechanical systems such as driven damped oscillators can be depicted by plotting velocity versus displacement, rather than displaying displacement as a function of time. This velocity versus displacement coordinate system is known as phase space. The trajectory in phase space provides another perspective of a system, and often it is more valuable than the displacement versus time plot. We have depicted the motion of a simple harmonic oscillator, a damped harmonic oscillator and a driven damped oscillator in phase space.

This book contains the first and first-hand practical demonstrations of obtaining discrete Fourier transform of data numerically using Mathematica. We have explored the use of discrete Fourier transform using Mathematica. Besides graduate students for the course titled Computational Physics, this book will prove useful to all of Physical Science and Engineering who often need to know the frequencies that are present in numerical data.

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Exploring Fourier Transform and Fourier Series Approximation Numerically

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Abstract

This chapter contains practical demonstrations on numerically obtaining Fourier transform of given numerical data. In particular, we demonstrate how to obtain the frequencies that are present in the data numerically using what is called discrete Fourier transform. We also demonstrate how to numerically obtain Fourier series approximation to any function. Programs were written in Mathematica in this regard.

1.1 Frequency Content in Oscillatory Motion

Suppose, we have a given set of numerical data, for an oscillatory motion. For example, suppose, we have a set of values of displacement y of a particle for a given set of values of time t. We can numerically get the frequencies that are present in the data by obtaining Fourier transform. This is outlined in the following.

Fourier transform pair is given by

$$y(t) = \frac{1}{\sqrt{2\pi}} \int_{-\propto}^{+\propto} Ae^{i\omega t}\, d\omega \tag{1.1}$$

and

$$A = \frac{1}{\sqrt{2\pi}} \int_{-\propto}^{+\propto} y(t)\, e^{-i\omega t}\, dt \tag{1.2}$$

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Equation (1.2) tells us about frequency content in Eq. (1.1). In other words, Eq. (1.2) tells us which frequencies are present in the data for y and relative importance or dominance of these frequencies.

To find A numerically, we replace Eq. (1.2) by

$$A = \frac{1}{\sqrt{2\pi}} \int_0^T y(t)\, e^{-i\omega t}\, dt \tag{1.3}$$

as an approximation. Here 0 to T is a time interval in which we have a set of numerical data for $y(t)$. T need not be true period in the oscillatory motion. We take $y(t) = y(t + T)$.

We know from *trapezoidal rule* for numerical integration that

$$\int_0^T v\, dt = \frac{h}{2}\left[v_0 + 2(v_1 + v_2 + v_3 + \cdots + v_{n-1}) + v_n\right] \tag{1.4}$$

where v_i's are equally spaced values of v, in the interval 0 to T; h is the spacing. If $v_0 = v_n$, we get

$$\int_0^T v\, dt = h(v_1 + v_2 + v_3 + \cdots + v_n) \tag{1.5}$$

As such, Eq. (1.3) can now be written as

$$A(n) = h \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{N} y_k\, e^{-i\omega_{n_1} t_k} \tag{1.6}$$

where y_k is value of y for $t = t_k = k\,h$, $\omega_{n_1} = n_1\, \omega_1 = n_1\, (2\pi/T)$ with $T = N\,h$. $n_1 = 0, 1, 2, 3, \ldots, N$. $\omega_0 = 0$ for $n_1 = 0$ corresponds to the zero frequency or dc component of the signal $y(t)$ that does not oscillate. $n = n_1/T$ Eq. (1.6) gives

$$A(n) = \frac{h}{\sqrt{2\pi}} \sum_{k=1}^{N} y_k\, e^{-i\left(n_1 \frac{2\pi}{Nh}\right)k\,h}$$

or,

$$A(n) = \frac{h}{\sqrt{2\pi}} \sum_{k=1}^{N} y_k\, e^{-i\left(\frac{2\pi}{N}\right)k n_1} \tag{1.7}$$

or,