

Helge Holden · Ragni Piene *Editors*



# The Abel Prize 2018 – 2022



# **The Abel Prize**



Niels Henrik Abel 1802–1829  
The only contemporary portrait of Abel, painted by Johan Gørbitz in 1826  
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Helge Holden • Ragni Piene  
Editors

# The Abel Prize 2018-2022



THE  
ABEL  
PRIZE



Det Norske  
Videnskaps-Akademi  
The Norwegian Academy  
of Science and Letters



Springer

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Photo of R. Langlands on front page and Curriculum Vitae by Peter Badge/Typos1 and on part title page by Heiko Junge/NTB Scanpix

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Photo of L. Lovász and A. Wigderson on front page and Curriculum Vitae by Peter Badge/Typos1 and on part title page by Tamas Szigeti (Lovász) and Abel Prize (Wigderson)

Photo of D. Sullivan on front page and Curriculum Vitae by Peter Badge/Typos1 and on part title page by Naina Helén Jåma

ISSN 2661-829X

ISSN 2661-8303 (electronic)

The Abel Prize

ISBN 978-3-031-33972-1

ISBN 978-3-031-33973-8 (eBook)

<https://doi.org/10.1007/978-3-031-33973-8>

This work was supported by Norwegian Academy of Science and Letters

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*Au reste il me paraît que si l'on veut faire des progrès dans les mathématiques il faut étudier les maîtres et non pas les écoliers.*

Niels Henrik Abel<sup>†</sup>

*En reste il me paraît que si l'on veut faire des progrès dans les mathématiques il faut étudier les maîtres et non pas les écoliers. —*

---

<sup>†</sup> “Finally, it appears to me that if one wants to make progress in mathematics, one should study the masters, not their students.” In: “Memoires de Mathématiques par N. H. Abel”, Paris, August 9, 1826, in the margin of p. 79. Original (Ms.fol. 351 A) in The National Library of Norway. Reprinted with permission.

# Preface

This book constitutes the fourth volume<sup>1</sup> in a series on the Abel Laureates, covering the period 2018–2022.

As in previous volumes there is one part per year. Each part starts with the full citation from the Abel Committee, followed by an autobiographical piece by the laureate(s). The autobiographical pieces are enhanced by photos – old and new. Then comes a description of the scientific accomplishments of the laureate(s). The parts end with a curriculum vitae and a complete bibliography of each laureate.

In the first part, James Arthur writes on the work of Robert Langlands, while in the second part, Simon Donaldson presents the work of Karen K. Uhlenbeck. The third part contains Vitaly Bergelson, Eli Glasner, and Benjamin Weiss’s article on the work of Hillel Furstenberg, as well as Alex Eskin, David Fisher, and Dmitry Kleinbock’s article on the work of Gregory Margulis. In the fourth part, the work of Lázló Lovász is presented by Martin Grötschel and Jaroslav Nešetřil, and that of Avi Wigderson by Boaz Barak, Yael Kalai, Ran Raz, Salil Vadhan, and Nisheeth Vishnoi. In the fifth part Edson de Faria, Sebastian von Strien, and Shmuel Weinberger write on the work of Dennis Sullivan.

The traditional award ceremonies in Oslo could not take place in 2020 and 2021 due to the COVID-19 pandemic. They were replaced by award ceremonies in the Norwegian embassies in the countries of the laureates. Lectures were given by the laureates, but there were no traditional Abel lectures. Fortunately, in 2022, we could return to the normal award ceremony in Oslo, where Dennis Sullivan received the Abel Prize from His Majesty King Harald. We were happy that also the laureates from the two previous years, Hillel Furstenberg, Gregory Margulis, Lázló Lovász, and Avi Wigderson could be present in Oslo on that occasion.

The last part is meant to give, through a collection of photos, an idea of all the activities that took place in connection with the Abel Prize during the last five years.

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<sup>1</sup> H. Holden, R. Piene (eds.): *The Abel Prize 2003–2007. The First Five Years*, Springer, Heidelberg, 2010, *The Abel Prize 2008–2012*, Springer, Heidelberg, 2014, and *The Abel Prize 2013–2017*, Springer, Heidelberg, 2019.

The back matter contains updates regarding publications and curriculum vitae for all laureates. Finally, we list the members of the Abel Committee and the Abel Board for the period 2018–2022.

The annual interview of the Abel Laureate(s) – The Abel Prize Interviews – can be watched on the video channel of the Norwegian Academy of Science and Letters. Transcripts of the interviews have been published, and publication details can be found in the back matter.

We would like to express our gratitude to the laureates for collaborating with us on this project, especially for providing the autobiographical pieces and the photos. We would like to thank the mathematicians who agreed to write about the scientific work of the laureates, and thus are helping us in making the laureates' work known to a broader audience.

Thanks go Marius Thaulé and Erlend Due Børve for their  $\LaTeX$  expertise and the preparation of the bibliographies as well as copyediting the manuscripts.

The technical preparation of the manuscript was financed by the Niels Henrik Abel Board.

Trondheim, Norway

Oslo, Norway

June 6, 2023

Helge Holden

Ragni Piene



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**Part I**  
**2018 Robert P. Langlands**



*“for his visionary program connecting  
representation theory to number theory”*



THE  
ABEL  
PRIZE



# Citation

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2018 to **Robert P. Langlands**, Institute for Advanced Study, Princeton, New Jersey, USA,

*“for his visionary program connecting representation theory to number theory.”*

The Langlands program predicts the existence of a tight web of connections between automorphic forms and Galois groups. The great achievement of algebraic number theory in the first third of the 20th century was class field theory. This theory is a vast generalisation of Gauss’s law of quadratic reciprocity. It provides an array of powerful tools for studying problems governed by abelian Galois groups. The non-abelian case turns out to be substantially deeper. Langlands, in a famous letter to André Weil in 1967, outlined a far-reaching program that revolutionised the understanding of this problem.

Langlands’s recognition that one should relate representations of Galois groups to automorphic forms involves an unexpected and fundamental insight, now called Langlands functoriality. The key tenet of Langlands functoriality is that automorphic representations of a reductive group should be related, via  $L$ -functions, to Galois representations in a dual group.

Jacquet and Langlands were able to establish a first case of functoriality for  $GL(2)$ , using the Selberg trace formula. Langlands’s work on base change for  $GL(2)$  proved further cases of functoriality, which played a role in Wiles’s proof of important cases of the Shimura–Taniyama–Weil conjecture.

The group  $GL(2)$  is the simplest example of a non-abelian reductive group. To proceed to the general case, Langland’s saw the need for a stable trace formula, now established by Arthur. Together with Ngô’s proof of the so-called Fundamental Lemma, conjectured by Langlands, this has led to the endoscopic classification of automorphic representations of classical groups, in terms of those of general linear groups.

Functoriality dramatically unifies a number of important results, including the modularity of elliptic curves and the proof of the Sato–Tate conjecture. It also lends weight to many outstanding conjectures, such as the Ramanujan–Peterson and Selberg conjectures, and the Hasse–Weil conjecture for zeta functions.

Functoriality for reductive groups over number fields remains out of reach, but great progress has been achieved by the work of many experts, including the Fields medallists Drinfeld, Lafforgue and Ngô, all inspired by the guiding light of the Langlands program. New facets of the theory have evolved, such as the Langlands conjectures over local fields and function fields, and the geometric Langlands program. Langlands’s ideas have elevated automorphic representations to a profound role in other areas of mathematics, far beyond the wildest dreams of early pioneers such as Weyl and Harish-Chandra.



# Autobiography\*

Robert P. Langlands

I was born and passed the first two decades of my life in the vicinity of Vancouver, British Columbia, an area now overwhelmed by immigration from the Orient, above all, China and India. More precisely, I was born in New Westminster in 1936, spent the first few years of my childhood on the shore about seventy miles to the north, in Lang Bay close to Powell River, returned to New Westminster to begin school, then moved to White Rock, where I passed my adolescence, and then went to Vancouver, of which New Westminster is now a suburb, to the University for five years, leaving in 1958 for graduate school, never to return except for short visits. I recall first the geography of the area, then the circumstances there in the nineteenth century and the beginning of the twentieth as well as the circumstances of my family and me. It is an area that has seen changes that, although peaceful have been definitive: a semi-rural, even partly rural environment, with a population that, apart from a visible, but small, indigenous component, still had close ties to the Old Country, generally meaning Great Britain and Ireland, and to Eastern Canada, has become more urban, much more cosmopolitan, and undoubtedly much more sophisticated, and apparently, much wealthier.

Canada, like its neighbour, the United States, like most countries, was built on conquest and oppression. In the area where the two countries now meet, this conquest is often referred to as a discovery, and this discovery, in so far as overland voyages are concerned is recent: Alexander Mackenzie reached the mouth of the Bella Coola river in 1793; Lewis and Clark reached the mouth of the Columbia River in 1805; Simon Fraser reached the mouth of the Fraser river in 1808. The first and last of these rivers lie in Canada, the second reaches the ocean in the USA.

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\* Based on a manuscript published in *Langlands' Program and His Mathematical World*. Some paragraphs have been removed by the editors. Reprinted with permission.

R. Langlands

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My childhood and youth were passed in a very small compass, first New Westminster and elementary school, then White Rock and high school, then Vancouver and university, I recall the geography. Vancouver lies with its back to the northern bank of the Fraser River at its mouth; New Westminster lies just a little upstream. The mouth of the river itself is formed from alluvial islands and the region south of it, Surrey and Delta, as far as the border with the US, a matter of 15–20 miles is also alluvial. When I was child, it was almost largely farmland, although some of the farmers had employment elsewhere. Three of my uncles were longshoremen (stevedores) on the docks at New Westminster and two of these also had small farms in Surrey. As I recall the only livestock consisted of fowl. White Rock lay at the time in Surrey but on the coast, almost at the point where the border with the USA first reaches the sea, not far from the mouth of a small river, the Campbell. I had at its source a more distant relative, married to a cousin, perhaps, of my grandfather, with a genuine farm: cows to be milked, something I attempted there but never since, and fruit to be harvested. He was also a water diviner and the owner of the last horse and buggy in Langley, the municipality adjacent to Surrey.

Lang Bay, about half-way between Vancouver and Bella Coola, was isolated and on the shore. My first recollections are from there, where we lived in a rented summer house, with two neighbours, an elderly woman and her grand-daughter. My memory is largely of sea, shore, the woods, which were boggy, the neighbour's fields, and a grazing goat. Occasionally but seldom, there were visitors from the south. When I came of school age, my mother, a Catholic of Irish descent, was eager to return to New Westminster where there was a parochial school and, I suppose, to her large family, three living sisters and six brothers. I flourished in the school, appreciated the nuns, who were often young, often pretty and gentle. The traditional costume, now largely, perhaps completely, abandoned, I regarded as normal. I learned to read quickly, had no trouble with the arithmetic, and skipped a grade. I liked to read, even, under the influence of the Books of Knowledge, popular at the time and pedalled door-to-door, tried, for reasons otherwise forgotten, to learn French on my own, accompanying in the various volumes a little British family, complete with dog, on its voyage to Paris and France. Like today's more desperate voyagers, I never got beyond Calais, although I was going in the opposite direction. It was many years before I returned to the language, but then with somewhat more success. My faith was also fervent for a brief period—I even toyed at the age of seven or eight with the notion of becoming a priest, something that would have corresponded to my mother's ambitions for my recently discovered academic ability, as it would have to those of many Catholic mothers of the period, but already before leaving New Westminster my faith was failing and my desire for a greater freedom growing. In particular, I wanted to leave the parochial school for the public school.

We moved to White Rock very shortly after the war, where I spent my adolescence, arriving in 1946 and leaving for university in 1953. These were not academic years. There were very few Catholics in White Rock, indeed the children with whom I consorted never saw the inside of a church. My mother was, unfortunately attached to the Church and eager, even desperate that I remain in it. The marriage was a mixed marriage and my father, to compensate for his own sins—principally, perhaps only,

gambling—would join in her efforts to bring me to church every Sunday morning, where I was a reluctant altar boy to a disagreeable priest of Irish descent. He himself stayed at home. Church meant also confession, a vicious practice, to which, for me, the only response after the age of twelve was prevarication or invention. There are, of course, many other practices that are very much worse and not far to seek. I was ashamed, as an adolescent, of my inability to resist my parents' pressure and still do not like to recall my youthful weakness. Luckily, once I left home to attend University and I was then not yet seventeen, I could abandon the Church and churches completely except for one midnight mass, the occasional funeral, and some touristic visits. Also as a concession to my wife, who was pleasing her mother, who herself appears to have been negligent about such niceties, we were married in a church, not however a Catholic church.

The Catholic Church aside, I would say that my childhood—in a society that had not yet ceased to be a frontier society, thus a society of people who had acquired independence at more of a cost to others, in this instance largely to the indigenous inhabitants, than to themselves, among people who, by and large, were indifferent to any but an extremely modest success, financial or otherwise, major success being beyond their imagination, and only a few of whom were unlucky enough to have occasion to be confronted with authority,—encouraged a natural, even if not necessarily bold, independence, a very useful characteristic for a mathematician.

I do not entirely understand my mother's relation to the Church. A good part of her large family did not take the Church nearly so seriously as she did. Her childhood had some difficult elements. Her father was, I believe, a fireman on the Canadian National Railroad, who barely survived a head-on clash with another train and did not recover from the incident, suffering in following years from epilepsy, severe psychic disorders, as well as, I understand, alcoholism. He spent a good deal of his time in an institution, although he could, apparently, return home on weekends, from Essondale to New Westminster on foot accompanied by his dog. That was a substantial trek. So my grandmother, who had been married at sixteen, was responsible for the family. She worked as a charwoman, in the houses of women who were better off. As a result, my mother, who was apparently a lively, popular young woman, on her school's basketball team, found herself wearing the cast-off clothes of her classmates. She never forgot it. My grandmother, Emily, whose maiden name was Dickson, was, so far as I know, above such feelings. She was a tremendously warm woman, beloved of her children and grandchildren. I knew her only for a few years. Dickson is not an Irish name, but my mother's ancestors seem otherwise to have been almost entirely Irish and they seem largely, perhaps entirely, to have left Ireland, usually south-east Ireland, for example, Kilkenny Co., before the famine. I believe that the South-East was not strongly affected by it. My grandfather's family name, Phelan, is distinctly Irish and has, I believe, its origins in a region farther to the west, in the town of Cork, but as I recall reading once in a history of Ireland, the tribe of the Phelans was displaced by the Norman invaders, whom it was attempting to resist, to south-eastern Ireland in the 12th Century, whence some of them came much later to Canada.

One exception to the Irish descent of my mother is an ancestor, a young German named Schildknecht who left Wittenberg or Wittenberge in Germany just before the American Revolution, in which he fought as a corporal in the South Carolina Loyalist Regiment. As compensation after Great Britain's loss, he was granted land in Ship Harbour, Nova Scotia where he settled with his wife, born in the American colonies but clearly the daughter of German emigrants. The name Schildknecht became Shellnutt and their descendants mixed with the Irish immigrants. A daughter Mary Catherine Shellnutt married an O'Bryan. My grandmother's mother was her granddaughter. They must have had a number of male children as well because the surname Shellnutt seems to be fairly common in Nova Scotia. As I observed, three of my mother's brothers were longshoremen. So was her paternal grandfather. He was killed in the famous Halifax explosion of 1917 when a French cargo ship that was carrying munitions exploded in the harbour leaving 2000 residents of the city dead and 9000 injured. He was not working at the time, rather he was, with his wife, on the way home from mass.

In these peripheral ways my mother's family was affected by the fortuitous of the world's affairs. They themselves were not much concerned with these. Even the genealogical information on both sides is not traditional, but has been, by and large, the result of efforts of a later generation. My father's family were more recent immigrants. My mother's parents moved across Canada from Halifax to New Westminster, stopping for an unsuccessful attempt at farming in the province of Saskatchewan where my mother was born. My father seems to have been conceived in England and his mother, who was apparently not prepared for life in a tent in British Columbia, returned with her children to England for a couple of years not long after his birth. That he had two sisters, one his twin, the other born a very short time before him, did not make her life any easier.

She was the sixth child in a family of seven. Her paternal grandfather—I know nothing of her mother—had been a private in the British Army, who stationed for a while in Cork, Ireland met and married, either then or later, a woman called Mary. Nothing more is known about her antecedents, nor about her surname. She may have been Irish or, perhaps, the daughter of an English soldier. I cannot say. On the other hand, like my mother's mother, perhaps even more so, she seems to have been a very resourceful and courageous woman. The marriage was apparently first recorded in Hobart, Tasmania, at the time their first child was born. Her husband died in Tasmania in 1845 at the age of 36, not in the course of his military duties but of an illness. His grave and gravestone are still to be found in a famous cemetery, the Isle of the Dead, in Tasmania. His wife managed not only to find her way back to England, with at least two sons, of which my great-grandfather was the youngest and two daughters, and to enroll the sons in the Duke of York's Military School, a school near London for the orphans of soldiers. She found employment for herself in the same institution as a laundress and for her eldest daughter as well. The whole family is listed in the 1851 Census as residing in the School. The individual indications with place of birth are as follows: (i) Flowers, Elizabeth, 13, servant of Quartermaster, Enniskillin, Ireland; Flowers, Mary, F., 39, Laundress, Cork, Ireland; Richard Flowers, 10, soldier's son, Manchester, England; Robert Flowers, 10, sol-



Fig. 1: Five months old (Photo: private)

dier's son, Hobart, Van Diemen's Land, Australia. The sons were not twins, but the birth of the second was only eight months after that of the first. There was also a second sister, Mary Ann, eight years old at the time. A last son seems to have died in infancy. All in all, five children were born in about five years. Apparently the older son remained in the school and then joined the British army as a private, a rank at which he remained all his life. The younger son, Robert Flowers, my grandmother's father, asked to be released into his mother's custody, became first a draper and then an auctioneer, and with time, he became, in Newcastle upon Tyne, first a councillor and then an alderman. He seems to have been a responsible son. Although, I have no information as to his mother's fate, it is clear that his two sisters came to Newcastle upon Tyne, presumably with him, where they married and, much later, died.

My paternal grandmother was married relatively late, as she was approaching thirty, perhaps past it. My reading of the circumstances is that she expected to remain at home, in her father's house, which I believe was a substantial house in Westgate, so far as I know a well-to-do quarter in Newcastle. However at some point her father, who was a widower, decided to marry again. His new wife was considerably younger, almost thirty years, also well-to-do, and apparently took up enough space in his home that there was no longer room for my grandmother. So she herself married, a slightly younger man, my grandfather. They appear to have been members of the same Methodist congregation, but I am not certain. So far as I know, her father's marriage, however unwelcome it may have been for her, was a blessing for him in his later years.

My grandmother was the only member of her family to emigrate. From her and from my grandfather as well, not from my mother's family, I acquired the notion of the old country, a notion often invoked in their house. It was represented in their home by a bust of Kitchener, labelled Kitchener of Khartoum, invoking his famous colonial exploits in Africa. I was, myself, disabused of any notion of a special relation to the old country when I later met, as a mathematician, a number of Englishmen. I may simply have been unfortunate in my first encounters. I came to know more agreeable specimens later.

Some time after my grandparents' marriage, two or three years, the business of my paternal grandfather's father, who was a cabinet maker, seems to have collapsed, whether for general economic reasons or for illness, mental or physical, I cannot say. His whole family moved to Canada: two daughters, both of whom were married to clergymen, one apparently a missionary to the Indians of Kispiox, in northern British Columbia, where my great-grandfather is buried, and two sons, both carpenters, presumably trained in their father's shop, one of whom was later killed in an accident during the construction of the Hudson Bay building in Vancouver. My grandmother appears to have been an unhappy woman, although she was, in comparison with her husband, her children, and my mother's family, cultivated. She could play the piano and, when I began university and acquired some intellectual interests, it was from her that I borrowed the *The Imitation of Christ* by Thomas Kempis, a famous medieval work of devotional literature. I confess that I neglected to return it. So far as I know, she passed a good many hours with the Bible and other devotional matters, but by the time I became curious about the world, I had little occasion to talk to her and, she, in any case, was growing senile. Whatever cultivation she had, she had acquired, I should think, in her parents' home. I had many more maternal cousins than paternal, but there was a greater awareness of the value, at least commercial, of a university education among the paternal cousins, two had business degrees and became accountants, another had a degree in engineering from a prominent American university and worked for International Business Machines. His mother, my father's eldest sister, had been trained first as an elementary school teacher and, then, as a nurse, while her sister and her two brothers all left high-school early, presumably, at least in the case of the boys, to become apprentice carpenters. A friend of my grandfather with whom he had emigrated from Newcastle and to whom he remained close until my grandfather's death told me, at my grandfather's

funeral, that he had reproached my grandfather for favouring the eldest daughter, but was told that the others would not profit so well as she from any more extensive education. What role my grandmother played in these decisions, I do not know.

I knew her as a frail, rather withdrawn woman, who had little support from her children as she aged. My father perhaps assumed more of the charge than any other, but my mother did not cooperate. New Westminster was not a large town, and my mother's brothers and sisters would have formed a large and boisterous group, not all so pious as my mother. My grandmother did not approve of the marriage and did not, I believe, disguise her feelings. My father made, however, a better marriage than he deserved. I doubt that my grandmother was aware of his more serious failings, although she expressed in the Bible she gave him on his fifteenth birthday only a feeble hope that he would read it.

I had far more cousins on my mother's side and found them more congenial, at least as an adolescent. They were easier with each other and with their mother, although my father was certainly close to his twin sister. So far as I know, apart from me, only one or two of the very youngest of my maternal cousins attended university. This did not, necessarily, prevent them from prospering. I myself, as a high-school student, had no notion whatsoever of attending university. My dream was to quit school, as one said, as soon as the law permitted, namely at the age of fifteen, and to take to the road, hitch-hiking to Toronto. Certainly, a large number of students in White Rock left at this age and found work, often seasonal, as loggers or as unskilled labourers of one kind or another. My mother, by temperament or as a consequence of her childhood experience and of my response to reading, writing, and arithmetic, will have had some ambitions for me, but more likely as a priest or a medical doctor. It was certainly noticed at the high school I attended, by observation or from the IQ tests to which we were all subject, that I had more than the usual aptitude for academic topics, but I myself was not impressed.

The few years spent in New Westminster were an occasion to meet a good many of my numerous cousins, a good proportion of whom were of about my age. New Westminster was a pleasant city in which to pass the first half of the 1940s. It was founded in 1858 as the capital of the Colony of British Columbia and remained larger than Vancouver, itself founded considerably later, until into the twentieth century. It was well and carefully planned, not large but with broad chestnut-lined streets, spacious boulevards and parks. It was a joy to be in. Thanks to the Second World War, the streets were during my early childhood almost entirely free of motorized vehicles. The port itself, on the river, did not intrude and the town, hardly more than a mile or two square was everywhere accessible to a child between the ages of six and nine. So as an introduction to urban life it could not have been gentler. I was told later by my mother that I had some trouble at first protecting myself from larger bellicose schoolmates, but I have no memory of that. The only childhood fisticuffs I remember were in White Rock. There were only two incidents. In the first, not provoked by me, I, in a burst of fury, pummelled my larger opponent and had to be pulled off him by the spectators; in the second, provoked by me, I had my nose broken.

Very few events from New Westminster are fixed in my memory: an older brother of some playmates ran away from home, and so far as I know, never returned. A classmate at the Catholic school, a girl my age, Maruka, Ukrainian I believe, whose parents were gardeners who later opened a prosperous nursery, was taken ill by one of the childhood diseases feared at the time and died within a few days. She was also a neighbour and we walked to school together. Although I was not particularly troubled by her death at the time, her image has stayed with me. I also remember her mother weeping as my own mother tried to console her, as well as a second attempt at consolation a couple of years later, just before the war's end, when a second neighbour received the notice of her son's death. My own family, uncles in particular, were not affected. Perhaps they all had children. One uncle, the youngest, served in the Air Force although he never went abroad, and a cousin, who had been born in the American state of Montana, went off to serve in the US Navy, returning to New Westminster some time later. His own father had spent some time in the USA, having run off, I believe, to join the American Navy towards the end of the First World War. My grandfather had brought him home from a similar earlier attempt, but yielded to his obstinacy. Both the father and the son married Americans.

In White Rock there was a small area of land, an Indian reserve, reserved for the members of a local tribe, of which there were only a few members remaining. The tribe, like many others, had been decimated in the nineteenth century by a disease that arrived with the colonists. The few children went to the local school but did not remain long and were pretty much ignored by the other children. This seemed to me at the time a normal state of affairs. There were also a few Métis in the town, but they were not distinguished from the rest of the population. My parent's store and our home above it were across the street from the reserve, but not from its residences. The chief came occasionally to buy lumber or other building material and would chat with my mother, who was usually at the cash register. Those boys in the town who were fond of fishing and of solitude would spend time in the reserve because the river ran through it, as did the trail to the border and the adjacent American town of Blaine.

I was nine, almost ten, years old when we arrived in White Rock, sixteen when I left it for university, returning only in the summer, and nineteen when I left it for good. It was a pleasant, but a strange, place for an adolescent. There was the ocean and the shore, although there were few boats. Except for the crab fisherman, the owners of the few boats, namely small rowboats, were usually Americans with a summer cottage. There was little exchange between the children on our side of the border and those on the other. The Americans were considered richer and were recognizable, from the front or from the back, by a slight plumpness.

Before the war the town's principal function was as a resort village in proximity to New Westminster and, to a lesser extent because of the lack of bridges and tunnels, to Vancouver. After the war, it served a different function, or so it seems to me on reflection. The summer cottages afforded inexpensive lodging. So there were a good number of families with no father, with a father who appeared only infrequently, or a feckless father. The proprietor of the local hotel or of the local dance hall were in my eyes rich. There were other families and their children as

well, but those children whose families allowed them, for one reason or another, more freedom appealed to me.

My wife has a copy of the year book of the high school with photos of the classes, thus for grades 7 to 12. They suggest that there were about four hundred children in the school. Their faces and names are by and large familiar, but many of them came from the surrounding rural areas and many kept pretty much to themselves returning home directly after school, so that I knew much less about them and their circumstances than about those in the town itself, where there were also those youths who had left early to find work full or part time. Schooling was only compulsory until age 15. A hint of restiveness, a desire for independence, drew me to those who had left school or were free, for one reason or another, of parental constraints, but I was very young, not very bold, and could not entirely free myself from the interdictions imposed on me by my mother and, in her wake, my father. As a Catholic child, I believed, without any question that any sins would be observed not only by God above but by my recently deceased grandmother, whom I cherished, at his side.

It is not that I was up to the company of the children or youths whom I admired or envied. I had started school rather early and had skipped a grade, while a good number of my classmates had failed a grade, thus been kept back, and not just once but several times, so that they were substantially older than I was. Moreover they had substantially more freedom than I. They may not have been able to read or write with any ease, but I envied them, both the boys and the girls. My mother, curiously enough, because it was not shared by a good number of her brothers and sisters, had a fear of sin, in particular of books, not of course childrens' books, as a source of sin, that made life with her difficult. My father, whose Methodist/Wesleyan (in the diluted Canadian form of the United Church) background had not left him immune to sin, but had left him with a strong sense of propriety and of possible disapproval of the neighbours, provided no relief. So I had to struggle for whatever freedom I had. I did quickly take to foul language, although I could not use it at home. From the age of twelve to the age of fourteen, I could probably compete with the most imaginative or coarse of Indian taxi-drivers in Vancouver or of current American politicians, but between the ages of fourteen and sixteen my passion for this form of expression slowly dissipated. In those years I met, at one of the school dances, called mixers and introduced to encourage civilized social intercourse between the boys and the girls in the school, someone almost as timid as I but, in contrast to me, with plans for the future. This was a decisive, perhaps the decisive, event in my life.

A determining feature of those years may have been labour. The period after the war was an economically favourable period. My parents had moved to White Rock, away from New Westminster, probably at the urging of my mother, where they had founded a business—lumber and building supplies. My father provided the technical competence—he had training as a carpenter, although he never acquired his journeyman's papers—while my mother took care of the books. As would be normal at the time, perhaps today as well, my father was responsible for the collection of overdue accounts, which were frequent enough. This was, for him, not an agreeable task. For me, the fortunate aspect of the business was to provide me with an occupation to fill time that would otherwise have been idled away. Although in no





Fig. 2: Wedding 1956 (Photo: private)

way fragile, I was not a particularly strong youth, nor was I particularly athletic. I tried but I was younger than my classmates, so that I was never chosen for school athletic teams. On the other hand, in those days, kegs of nails, sacks of cement, agricultural tiles, plywood, plasterboard, and lumber of all kinds were loaded onto trucks by hand and unloaded in the same way. From the age of twelve or thirteen that was how I spent my time after school and on Saturdays. During the university years it was how I spent the summers, earning the funds to pay for the winter's food and lodging. It meant, above all, that I arrived at twenty reasonably robust, with a body that has not failed me, at least not seriously, over the next sixty years. It also meant that, without being particularly adroit physically, I could manage, although not with great skill, those household tasks associated with the building trades. However, as

time went on I was ever more disinclined to undertake anything outside mathematics that demanded patience. With age, mathematics demands that quality more and more.

On reflection and I have, oddly enough, never indulged myself in reflection about these matters, after my early childhood, even during, I had little to do with my father outside these common labours. There was little disagreeable about them. Although he was occasionally impatient with my lack of dexterity, it was pleasant to work with my father. He was generous with my pay, adequate to allow me to indulge my juvenile sartorial extravagance, to go to the movies and so on, and, now and again, as a diversion, he suggested that I work as a swamper, a term used only in Canada, thus as a helper on the local light-delivery truck, which was not only more leisurely, with a good deal of time spent beside the driver watching the world go by, but entailed occasionally a trip to Vancouver or New Westminster for a load of cement, drainage tiles, or sashes and doors. This was hardly work!

The postwar years were prosperous; the business thrived, ostensibly under the hand of my father, but the determination was, I believe, my mother's, although this was not apparent to me at the time. My father had a modest taste for luxury. As the business prospered, they were soon able to construct a building to house it, with an apartment upstairs for the family. With a stone facing and large plate glass windows, it was at that time and place an imposing edifice. It now houses a restaurant. With time, my father was able to buy himself a Buick, at that time a luxury automobile, and to construct a house in the best part of town, on a cliff on the shore with a splendid view over the Strait of Juan de Fuca. Unfortunately, as my mother grew older and became ill, she was no longer able to save him from himself, and a vice—gambling—that had been present from the beginning, although hidden from the children, and a constant source of anxiety to her, took over. He, and thus they, slowly lost everything. By then, I was far away. It fell to my two sisters to do what they could, and it was considerable, to mitigate the disaster.

To return to my own development, how did it happen that rather than hitchhiking to Toronto, I went to university? It may have been the effect of my new acquaintance, but that would have been an unconscious effect, although I doubt that she would have encouraged my hitchhiking plans. She would certainly not have been willing to be a part of them. It would undoubtedly have meant that we parted ways.

There were two things. First of all, in the last year of high school, we had a new teacher, Crawford Vogler, and a new textbook, a textbook that introduced us to English literature. He was a very enthusiastic, very sympathetic teacher. I recall that he gave two or three students special assignments. I was asked to report on the novel *The Ordeal of Richard Feverel* by the well-known Victorian novelist George Meredith. It was all a little puzzling to me and any expectation on his part must have been disappointed. However, as I remarked above, I will have been given an IQ test and he will have been aware of any unusual talent. That will have been the source of his disappointed confidence in me. Nonetheless, he took, either then or on some other occasion, a full class period to explain to me in front of the other students that I must go to university. Going to university meant going to the University of British Columbia. There were then no other universities in the province and the possibility

of going elsewhere would never have crossed my mind. I was impressed then and there by the suggestion and decided not only to take the entrance examinations but to study for them. I was successful; I even received a bursary to pay the academic fees.

Secondly, although my father had left school after nine years to become an apprentice, my wife, not at that time of course, was the child of a man with a more meagre educational background—born on Prince Edward Island, into a mixed, Franco-Irish marriage his mother died when he was two years old and he was given into the care of a Scottish family. So his initial language was Gaelic, but apparently somewhat frail and uncomfortable with the robust sons of the family, he left home at an early age, working in the logging camps of Quebec, where he learned French, as spoken by the loggers. He must have been quite agile, since his task was sometimes to clear log-jams. This was done by inserting an explosive in the jam and then running away, from one log to another, in order to be clear of the jam before the explosion freed the logs—a slip would be fatal.

What he had not learned neither with the Scots nor in the logging camps was to read and write. His chance came during the Great Depression, when various unions or political parties undertook not only to feed the unemployed but also to educate them. He had kept some books, by Frederick Engels, Karl Kautsky, August Bebel and other socialist authors, that he undertook to read at that time, most of which we still have. He liked to cite at length various passages from these books. He had a good memory but reading was always difficult for him. I think his wife, my mother-in-law, gave him further lessons after their marriage. My own father could read well enough, although writing was another matter. I do not think I ever received more than one brief note from him, in an emergency. One book in particular, I took away from my future father-in-law's library to read, *The Story of the World's Great Thinkers* by Ernest R. Trattner with biographies of many of the world's renowned thinkers, for example, Copernicus, Hutton, . . . , Marx, Pasteur, Freud, . . . , Einstein. I remember being particularly impressed by the story of Hutton and the age of the Earth. The book itself had been very popular, deservedly so, in the late thirties and early forties of the last century. So second-hand copies are still easily found.

I recalled at length various genealogical facts related to my family. I recall one or two related to my wife. They are striking. Her mother's family, like my mother's large with ten children, had emigrated not from England but from Scotland at the time of the First World War. My grandfather and his brother had returned to the Old Country as soldiers but they were sufficiently old that they never, so far as I know, saw battle. Lord Kitchener, who as I recalled was a familiar figure to me from my grandparents' dining room, was the Secretary of State for War for Great Britain at the beginning and had introduced the policy of sending brothers together to the battlefield on the principle that side-by-side they would fight better. The result was that many families lost all their sons at one stroke. My wife's grandfather's family seem to have been so affected, not her grandfather and one of his brother's who had also emigrated, who survived, although injured, as members of the Canadian army, but the four brothers who remained in Scotland and fought with the British army all perished, at least three apparently as a result of Kitchener's policy, which

I believe was finally abandoned. Part at least of her father's family had been in Canada much longer, descendants of a Basque fisherman and his Micmac wife, who are to be found in the census undertaken by the British after the conquest in 1763. My children are aware of their descent, although it is hardly apparent. Nevertheless, one of my daughters, the blonde among the four children, learned recently from her dentist that there is an aspect of her dental structure that is a sure sign of an indigenous ancestor.

Almost the first event marking the change from childhood years to university years were aptitude tests. They were followed by consultation with some member of the university's teaching staff. I was offered, given my arithmetic talent, such possibilities as accountancy although academic possibilities were also mentioned. I suppose I expressed an intention to study mathematics and physics and it was observed that in that case I might even want to take a Ph.D. degree. I listened and, my studies not having yet begun, returned to White Rock, where for some reason or other I visited the house of my future in-laws, who were in bed. I took the opportunity of asking my future father-in-law what a Ph.D. was. Somewhat surprisingly, now but not then, he knew. Sometime later, I consulted with a mathematician, Dr. Jennings, with the title for professors customary in Canadian universities, who suggested to me that as a mathematician there would be several foreign languages to be learned. I took his remark seriously, although, initially, seriously may not have entailed effectively. One year of French or some other language was a normal requirement. At the end of the first year I acquired a basic text for German grammar and reading it over the summer felt that I was adequately equipped in that direction, and in the second year moved on to a course in Russian.

In retrospect, these efforts were a little ridiculous, but I had occasion later to make more serious and more effective attempts to master these and other languages. I pity the mathematicians of today, not only the native speakers of English who have no occasion to learn the classical European languages, which offered until recently, outside mathematics and within mathematics as well, a great deal but also the European mathematicians and mathematicians from elsewhere, but especially the Europeans—the French who have all but destroyed Breton, the Germans, who have destroyed above all Yiddish but also less important languages as well, like Wendish/Sorbian—who now are assiduously rendering their own more and more difficult of access. They, with a notion that acquiring English is today still a cultural achievement, merit perhaps more contempt than pity. Some, of course, like the editors of Springer Verlag, are in it for the money. It may be that the response as to whether mathematics should be a matter of several languages or of one will ultimately be given by the Chinese.

In my first year of university, my principal preoccupation was some mastery of English. As I have implied I was in high-school negligent and had ignored the basics not of English orthography but of English grammar. I was assiduous—to the amusement of the other students, some of them adults, and the teacher, Dr. Morrison—consulting the dictionary for every unfamiliar word in every poem and learning, in particular, to avoid comma splices.

The mathematics was new to me but by today's standards elementary, largely, as I recall, trigonometry. The second year was again relatively slow, mathematics and a course of logic, a subject about which, as a mathematician, I have tried to inform myself but always unsuccessfully. The physics course, the Russian course, and the course on English literature, from the beginning to the nineteenth century, Chaucer, Fielding, and a good number of other authors, all appealed to me. We had been introduced to Shakespeare in high-school.

The third year was more interesting. For various reasons the multi-variable calculus courses were not successful, a bad and indifferent teacher undermined the efforts of a conscientious and potentially excellent teacher, Dr. Leimanis. However Marvin Marcus, now very old but still, I believe, with us, recommended Courant's classic book on differential and integral calculus, which I studied, but occasionally, as with the inverse function theorem, in too superficial a manner. Either in the second but more likely third year, I had two other courses, each, as I recall, half-year courses. Dr. Christian gave an excellent course on algebra from a well-known book of Dickson. Once again, I did not always grasp adequately the interest of various important points, for example, the theory of the cubic equation. The textbook in the other course, on linear algebra and geometry was also a widely used American text, the names of whose authors I forget, but during the summer, at the suggestion of Christian, I read Halmos's book on vector spaces, widely used at the time. I confess that I fell in love with the abstraction of his presentation, a passion good in its way for a mathematician, but it is best not to be overwhelmed by it. A book that, in some sense, was a surer sign of my fate as a specialist of automorphic forms and the Hecke theory was a book that I found on my own, a translation *Modern Algebra and Matrix Theory* of a once familiar German text by Schreier and Sperner where the theory of elementary divisors is treated at length.

By the fourth year, I could give myself up almost completely to mathematics. Not entirely through a fault of my own, I had abandoned any intention I may have had to become a physicist. In retrospect, I did not and do not have the right kind of imagination, but the decisive event was a course in thermodynamics in the third year. This is a difficult subject, in particular the topics of heat and entropy and in response to a homework question I wrote an extravagantly long essay, which unfortunately I did not keep. Given my age—I had just turned nineteen—and my lack of a solid pre-university education, it probably was not so bad. The teacher, an English experimentalist, chose to mock it in class. That, I think, was the turning point. Certainly, given the nature of whatever talent I had, it was for the best. I did, that year, have a physics course that I much enjoyed, the optics course, above all the experiments. I was fortunate to have a partner, Alan Goodacre, in the laboratory experiments that were an important part of the course and that in his hands always yielded the expected, thus the correct, results. There was a division of labour, in which I took the easy half, the theory, and he the difficult half, the experiments. I still have occasion to meet him occasionally in Ottawa, where he was an experimentalist with the dominion laboratories for many years.

So, in the fourth year, I focussed except for a second course in Russian on mathematics. I learned or began to learn a great deal: function theory, in particular from the third volume, which I studied on my own, of the prescribed text, a translation of a German text by Konrad Knopf, the Weierstrass theory of elliptic functions; ordinary differential equations, including something about special functions and the beginnings of spectral theory, which I supplemented later in graduate school and the years immediately following, with the book of Coddington and Levinson and with M. H. Stone's book on the spectral theory of operators in Hilbert space, both excellent preparation for the general theory of Eisenstein series, which has been a major concern of my career and which led to its major achievement, what is often referred to as the Langlands programme. Galois theory was the principal topic in another course but it went by, I am afraid, more or less unremarked. I also participated in my fourth year at university, or more likely in the following year, in a seminar on commutative algebra, based on the book *Ideal Theory* of D. G. Northcott. In any case, I managed during the following year to write, on my own initiative, a master's thesis on some idea or problem that I found in it. None of the professor's were familiar with the topic so that they were uncertain what to do. Out of the goodness of their hearts and, I suppose, because my performance was otherwise satisfactory they accepted it—even though I had had to confess at some point during the proceedings that I had found an important error in it—and let me move on to graduate school. That year 1957–58, between my four undergraduate years and my two graduate years at Yale, was one of the most demanding of my life. I had been married a year before, at a very young age, was teaching one undergraduate course, my first experience of lecturing, was taking enough courses to acquire the necessary credits for a master's degree, without which I could not move on to the important stage of a doctoral degree, and finally I was writing the master's thesis. I remember almost nothing from that year: life in a trailer with my wife; a charming girl in the freshman class I was teaching who seemed to be taken with me, an infatuation to which unmarried and otherwise uncommitted I would have been happy to respond; as well as an incident with a second professor of physics. This I remember clearly.

The occasion was the final examination of a course on mathematical methods in physics, a course for graduate students offered by this professor, an immigrant from Europe. The focus was the representation theory of finite groups, characters, orthogonality and so on. The single problem assigned was to analyze the representation of the tetrahedral group on the sum of the four tangent spaces at the vertices, each a three-dimensional space, which he thought of as provided with the natural metric. He expected the students to decompose the representation using the orthogonality relations. The best, most direct solution is of course to use a non-orthogonal basis directed away from the vertices along the edges. Then the problem is solved by inspection. He looked at all the zeros and ones in the calculation and was persuaded that I, as a thick-headed student, had inappropriately introduced the regular representation and was about to fail me, which would have meant no master's degree and no move to Yale. He seemed to be obdurate, but for unexplained reasons ultimately gave me a passing grade. Perhaps a colleague explained the solution to him.