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Jianying Zheng, Liang Xu, Qinglei Hu, Lihua Xie

Control over Communication Networks

Modeling, Analysis, and Design of
Networked Control Systems and
Multi-Agent Systems over Imperfect
Communication Channels


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To my parents, my husband Pengfei, and my daughter Coco (Jianying Zheng)
To my parents, my wife Xiaoxue, and my son Ze (Liang Xu)

Contents

About the Authors	<i>xiii</i>
Preface	<i>xv</i>
Acknowledgments	<i>xvii</i>
Acronyms	<i>xix</i>
List of Symbols	<i>xxi</i>

1	Introduction	<i>1</i>
1.1	Introduction and Motivation	<i>1</i>
1.1.1	Networked Control Systems	<i>1</i>
1.1.2	Multi-Agent Systems	<i>2</i>
1.2	Literature Review	<i>4</i>
1.2.1	Basics of Communication Theory	<i>4</i>
1.2.2	Stabilization of NCSs	<i>6</i>
1.2.2.1	Control over Noiseless Digital Channels	<i>6</i>
1.2.2.2	Control over Stochastic Digital Channels	<i>7</i>
1.2.2.3	Control over Analog Channels	<i>8</i>
1.2.3	LQ Optimal Control of NCSs over Fading Channels	<i>9</i>
1.2.4	Estimation of NCSs with Intermittent Communication	<i>11</i>
1.2.4.1	Stability of Kalman Filtering with Intermittent Observations	<i>11</i>
1.2.4.2	Remote State Estimation with Sensor Scheduling	<i>12</i>
1.2.5	Distributed Consensus of MASSs	<i>13</i>
1.3	Preview of the Book	<i>15</i>
1.4	Preliminaries	<i>18</i>
1.4.1	Graph Theory	<i>18</i>
1.4.2	Hadamard Product and Kronecker Product	<i>19</i>
	Bibliography	<i>20</i>

2	Stabilization over Power Constrained Fading Channels	29
2.1	Introduction	29
2.2	Problem Formulation	29
2.3	Fundamental Limitations	31
2.4	Mean-Square Stabilizability	35
2.4.1	Scalar Systems	36
2.4.2	Two-Dimensional Systems	37
2.4.2.1	Communication Structure	38
2.4.2.2	Encoder/Decoder Design	38
2.4.2.3	Scheduler Design	39
2.4.2.4	Scheduler Parameter Selection	40
2.4.2.5	Proof of Theorem 2.3	41
2.4.3	High-Dimensional Systems: TDMA Scheduler	44
2.4.4	High-Dimensional Systems: Adaptive TDMA Scheduler	45
2.4.4.1	Scheduling Algorithm	46
2.4.4.2	Scheduler Parameter Selection	46
2.4.4.3	Proof of Theorem 2.5	46
2.5	Numerical Illustrations	51
2.5.1	Scalar Systems	51
2.5.2	Vector Systems	52
2.6	Conclusions	53
	Bibliography	53
3	Stabilization over Gaussian Finite-State Markov Channels	57
3.1	Introduction	57
3.2	Problem Formulation	58
3.2.1	Stability of Markov Jump Linear Systems	59
3.2.2	Sojourn Times for Markov Lossy Process	60
3.3	Fundamental Limitation	61
3.4	Stabilization over Finite-State Markov Channels	64
3.4.1	Communication Structure	65
3.4.2	Observer/Estimator/Controller Design	65
3.4.3	Encoder/Decoder/Scheduler Design	67
3.4.4	Sufficient Stabilizability Conditions	68
3.5	Stabilization over Markov Lossy Channels	71
3.5.1	Two-Dimensional Systems	71
3.5.1.1	Optimal Scheduler Design	72
3.5.1.2	Scheduler Parameter Selection	74
3.5.1.3	Sufficiency Proof of Theorem 3.4	75
3.5.2	High-Dimensional Systems	77

3.5.3	Numerical Illustrations	81
3.6	Conclusions	82
	Bibliography	83
4	Linear-Quadratic Optimal Control of NCSs with Random Input Gains	85
4.1	Introduction	85
4.2	Problem Formulation	86
4.3	Finite-Horizon LQ Optimal Control	88
4.4	Solvability of Modified Algebraic Riccati Equation	91
4.4.1	Cone-Invariant Operators	91
4.4.2	Solvability	97
4.5	LQ Optimal Control	108
4.6	Conclusion	114
	Bibliography	115
5	Multisensor Kalman Filtering with Intermittent Measurements	117
5.1	Introduction	117
5.2	Problem Formulation	118
5.3	Stability Analysis	120
5.3.1	Transmission Capacity	120
5.3.2	Preliminaries	120
5.3.3	Lower Bound	121
5.3.4	Upper Bound	124
5.3.5	Special Cases	130
5.4	Examples	131
5.5	Conclusions	132
	Bibliography	133
6	Remote State Estimation with Stochastic Event-Triggered Sensor Schedule and Packet Drops	135
6.1	Introduction	135
6.2	Problem Formulation	135
6.3	Optimal Estimator	137
6.4	Suboptimal Estimators	143
6.4.1	Fixed Memory Estimator	143
6.4.2	Particle Filter	145
6.5	Simulations	149
6.6	Conclusions	151
	Bibliography	152

7	Distributed Consensus over Undirected Fading Networks	153
7.1	Introduction	153
7.2	Problem Formulation	154
7.3	Identical Fading Networks	155
7.4	Nonidentical Fading Networks	163
7.4.1	Definition of Edge Laplacian	163
7.4.2	Sufficient Consensus Conditions	164
7.5	Simulations	168
7.6	Conclusions	170
	Bibliography	170
8	Distributed Consensus over Directed Fading Networks	173
8.1	Introduction	173
8.2	Problem Formulation	174
8.3	Identical Fading Networks	174
8.3.1	Consensus Error Dynamics	175
8.3.2	Consensusability Results	177
8.3.3	Balanced Directed Graph Cases	179
8.4	Definitions and Properties of CIIM, CIM, and CEL	181
8.4.1	Definitions of CIIM, CIM, and CEL	181
8.4.2	Properties of CIIM, CIM, and CEL	182
8.5	Nonidentical Fading Networks	185
8.5.1	$\Lambda = \mu I$	189
8.5.1.1	Star Graphs	190
8.5.1.2	Directed Path Graphs	191
8.5.2	$\Lambda \neq \mu I$	192
8.6	Simulations	192
8.7	Conclusions	194
	Bibliography	195
9	Distributed Consensus over Networks with Communication Delay and Packet Dropouts	197
9.1	Introduction	197
9.2	Problem Formulation	198
9.3	Consensusability with Delay and Identical Packet Dropouts	199
9.3.1	Stability Criterion of NCSs with Delay and Multiplicative Noise	199
9.3.2	Consensusability Conditions	204
9.4	Consensusability with Delay and Nonidentical Packet Dropouts	209
9.5	Illustrative Examples	214
9.6	Conclusions	216
	Bibliography	216

10	Distributed Consensus over Markovian Packet Loss Channels	219
10.1	Introduction	219
10.2	Problem Formulation	219
10.3	Identical Markovian Packet Loss	220
10.3.1	Analytic Consensus Conditions	224
10.3.2	Critical Consensus Condition for Scalar Agent Dynamics	226
10.4	Nonidentical Markovian Packet Loss	228
10.5	Numerical Simulations	232
10.6	Conclusions	234
	Bibliography	235
11	Synchronization of the Delayed Vicsek Model	237
11.1	Introduction	237
11.2	Directed Graphs	238
11.3	Problem Formulation	239
11.4	Synchronization of Delayed Linear Vicsek Model	240
11.5	Synchronization of Delayed Nonlinear Vicsek Model	246
11.6	Simulations	249
11.7	Conclusions	253
	Bibliography	253
	Index	255

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Preface

In networked control systems (NCSs), wired or wireless communication channels are used to link components among plants, sensors, and controllers to achieve control objectives. While there are many advantages, NCSs also introduce a series of challenging problems that arise from the limited resources and unreliability of the communication networks used for information transmission. For example, due to congestion, data losses and transmission delays may occur in digital communication channels. Besides, in wireless communication networks, which are widely used in sensor networks and multi-agent systems (MASs), communication channels naturally suffer from inference, fading, and transmission noises. Since control is often used in safety or mission-critical applications, we must take the uncertainties in communication networks into consideration and investigate how they affect the stability and performance of NCSs and MASs.

The book gives a systematical and self-contained description for the analysis and design of NCSs and MASs over imperfect communication networks. Specifically, the book considers fading channels and delayed channels and includes two main parts. In the first part, the stabilization, optimal control, and remote state estimation of linear systems over channels with fading, signal-to-noise constraints, or intermittent measurements are considered. The channel requirements for the mean-square stabilization and optimal control are characterized and the optimal estimator designs and performance analysis are conducted. In the second part, the joint impact of communication channels and network topology on the consensusability of MASs is analyzed. By integrating communication and control theory, we present several fundamental results on the stabilization, optimal control, and estimation of NCSs and the consensus of MASs over imperfect channels. The book

intends to provide a unified platform for introducing the analysis and design of NCSs and MASs for researchers working in related areas.

January 2023

Jianying Zheng

Liang Xu

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The contents included in this book are the outgrowth and summary of the authors' academic research achievements on the analysis and design of networked control systems and multi-agent systems over imperfect communication channels in the past several years. Some of the materials contained herein arose from the joint work with our collaborators, and the book would not have been possible without their efforts and support. In particular, the first author is indebted to Prof. Li Qiu from the Hong Kong University of Science and Technology, who led her into the gate of scientific research. We would like to thank our collaborators, including Prof. Jiu-Gang Dong, Prof. Yilin Mo, Prof. Keyou You, Prof. Chao Yang, and Dr. Nan Xiao, for their helpful discussions, and the students Roudan Zhou and Xiao Wang for their assistance in preparing the manuscript.

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Acronyms

a.e.	almost everywhere
ARE	algebraic Riccati equation
AWGN	additive white Gaussian noise
BMI	bilinear matrix inequality
CEL	compressed edge Laplacian
CIIM	compressed in-incidence matrix
CIM	compressed incidence matrix
DEL	directed edge Laplacian
FSMC	finite-state Markov channel(s)
i.i.d.	independent and identically distributed
IIM	in-incidence matrix
IM	incidence matrix
LMI	linear matrix inequality(-ies)
LTI	linear time-invariant
LQ	linear quadratic
LQG	linear quadratic Gaussian
MARE	modified algebraic Riccati equation(s)
MAS	multi-agent system(s)
MIMO	multi-input-multi-output
MJLS	Markov jump linear system
MSE	mean-square error
MMSE	minimum mean-square error
NCS	networked control system(s)
SNR	signal-to-noise ratio
TCP	transport control protocol
TDMA	time division multiple access

List of Symbols

\mathbb{N}	the set of natural numbers
\mathbb{N}^+	the set of positive natural numbers
$\mathbb{R}(\mathbb{C})$	the set of real (complex) numbers
$\mathbb{R}^n(\mathbb{C}^n)$	the set of n -dimensional real (complex) column vectors
$\mathbb{R}^{m \times n}(\mathbb{C}^{m \times n})$	the set of $m \times n$ -dimensional real (complex) matrices
S_n	the set of $n \times n$ symmetric matrices
\mathcal{P}_n	the set of $n \times n$ positive semidefinite matrices
$\operatorname{Re}(c)$	the real part of $c \in \mathbb{C}$
$ c $	the magnitude of $c \in \mathbb{C}$
$ S $	the cardinality of set S
$\mathbf{1}$	a column vector of ones
$\mathbf{0}_{m \times n}$	an $m \times n$ matrix with all entries being zero
I_N	the N -by- N identity matrix
A'	the transpose of matrix A
A^*	the conjugate transpose of matrix A
$[A]_{ij}$	the ij th element of matrix A
$[A]_{\text{row } i}$	the i th row of matrix A
$[A]_{\text{column } j}$	the j th column of matrix A
A^{-1}	the inverse of matrix A
A^\dagger	the Moore–Penrose pseudoinverse of matrix A
$\rho(A)$	the spectral radius of matrix A
$\operatorname{tr}(A)$	the trace of matrix A
$\det(A)$	the determinant of matrix A
$\operatorname{null}(A)$	the null space of matrix A
$\operatorname{diag}(A, B)$	a diagonal matrix with diagonal entries A and B
$\begin{bmatrix} A & * \\ C & B \end{bmatrix}$	the abbreviation of the symmetric matrix $\begin{bmatrix} A & C' \\ C & B \end{bmatrix}$
$\lambda_{\min}(S)$	the minimal eigenvalue of a real symmetric matrix S
$S > 0$ ($S \geq 0$)	positive definite (semidefinite) matrix

χ^t	the sequence $\{\chi_i\}_{i=0}^t$
\otimes	the Kronecker product
\odot	the Hadamard product
$\mathbb{E}\{\cdot\}$	the expectation operator
$\mathbb{E}_x\{\cdot\}$	the expectation conditioned on the event $X = x$
$\mathcal{N}_x(\bar{x}, \Sigma)$	a random variable x with Gaussian distribution of mean \bar{x} and covariance matrix Σ
$f(x) (\Pr(x))$	the probability density function (probability) of the random variable x
$f(x y) (\Pr(x y))$	the probability density function (probability) of the random variable x conditioned on the event that $Y = y$
e	the Euler's number
$\ln(\cdot)$	the natural logarithm
$\log(\cdot)$	the logarithm to base 2
$\log_x(\cdot)$	the logarithm to base x

1

Introduction

1.1 Introduction and Motivation

1.1.1 Networked Control Systems

Due to the flexible architecture and ease of installation and maintenance, communication networks are widely used in control systems, which result in networked control systems (NCSs), where the plants, actuators, sensors, and controllers are spatially distributed and interconnected by communication channels [Schenato et al., 2007, Hespanha et al.]. NCSs are ubiquitous in industry and daily life, such as teleoperation [Arcara and Melchiorri, 2002], power systems [Wang et al., 2012], and transportation systems [Seiler and Sengupta, 2001].

Even though NCSs have the advantages of low cost, easy implementation, and expansion to large-scale applications, they also introduce new challenging problems arising from the limited resources and unreliability of the communication networks used for information transmission (see Figure 1.1). For example, the time delay may occur in digital communication channels due to data processing and transmission [Tse and Viswanath, 2005, Goldsmith, 2005]. Notably, in wireless communication networks, communication channels naturally suffer from interference, fading, and transmission noises [Tse and Viswanath, 2005, Goldsmith, 2005]. There into, fading is the time variation of channel strengths and is usually caused by two factors: one is the shadowing from obstacles; the other one is the multipath propagation [Tse and Viswanath, 2005, Goldsmith, 2005]. Packet drops can also be modeled as a special case of channel fading. Take Figure 1.2 as an illustration. The wireless signal may transmit through the car and undergo several paths before arriving at the receiver. If the phases of the received signals from different paths are the same, the signal strength is enhanced. Otherwise, the signal strength is reduced as a result of the cancellation of radio waves. Besides, the signal strength at the receiver side might be reduced due to the shadowing from

the car. Since control is often used in safety- or mission-critical applications, we must take the uncertainties in communication networks into consideration and investigate how they affect the stability and performance of control systems.

The classical control theory mainly deals with the systems with nearly perfect point-to-point connections and focuses on the design of control laws to achieve the given control performance. It can't be applied directly to the NCSs when the uncertainties in the communication network must be considered. A new control paradigm is required to deal with the interplay between control and communication. In this book, one of the main objectives is to study the stabilization, estimation, and optimal control of NCSs over channels with fading, packet drops, or delay.

1.1.2 Multi-Agent Systems

Motivated by the collective behavior in nature, such as schooling fish, flocking birds, and marching locusts, multi-agent systems (MASs) have attracted considerable research interest from the control community [Jadbabaie et al., 2003, Olfati-Saber and Murray, 2004, Olfati-Saber et al., 2007, Bliman and Ferrari-Trecate, 2008, Cao et al., 2008, Ren and Beard, 2008, You and Xie, 2010, Cao et al., 2012, Trentelman et al., 2013, Qi et al., 2016, Qiu et al., 2017, Xu et al., 2018, Zheng et al., 2018]. With the rapid development of wireless communication networks, MASs have been applied in many industrial and military applications. Such systems usually involve large numbers of autonomous agents (e.g. robots, unmanned aerial vehicles, satellites), which share information via local interactions and work together to achieve collective objectives.

For MASs, each agent can have the same or different system dynamics, resulting in different types of MASs, e.g. first- and second-order MASs, linear and nonlinear MASs, homogeneous and heterogeneous MASs. The interactions among the agents form the interaction topology, which can be fixed or time-varying. Then the cooperative control of MASs is based on the system dynamics and the interaction topology to design the control laws, which can be centralized or distributed, to fulfill a task. Typical cooperative control tasks include consensus, formation, swarming/flocking, rendezvous, etc. There into, the consensus problem, which requires all agents to agree on a certain quantity of common interests, builds the foundation of other cooperative tasks.

Existing research on consensus assumes that the communication networks among agents are perfect. However, as mentioned earlier, in practical applications, communication channels naturally suffer from fading, signal-to-noise ratio (SNR) constraints, time delay, etc. Hence, it is of great significance to study how

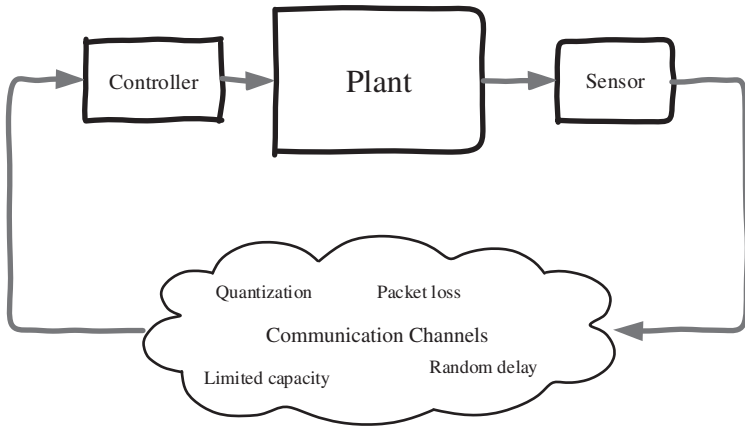


Figure 1.1 Networked control systems.

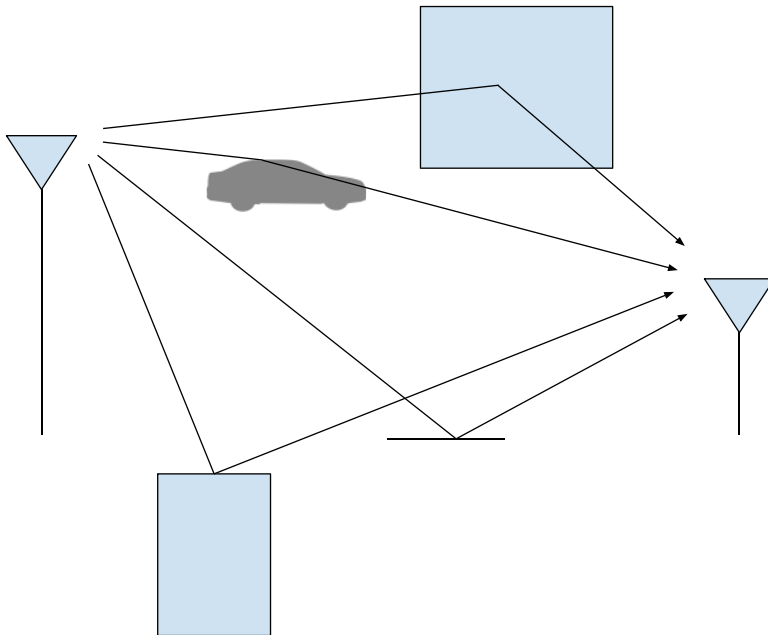


Figure 1.2 Fading phenomenon in wireless communications.

the uncertainties in communication networks influence the consensus of MASs. The other main objective of this book is to analyze the consensus problem of MASs over channels with fading, packet drops, and delay.

1.2 Literature Review

Control over communication channels/networks has been a hot research topic in the past decades [Matveev and Savkin, 2009, Como et al., 2014, You et al., 2015], motivated by the rapid developments of wireless communication technologies that enable the wide connection of geographically distributed devices and systems. However, the inclusion of wireless communication channels/networks also introduces challenges in the analysis and design of control systems due to constraints and uncertainties in wireless communications. We must take the communication channels/networks into consideration and study their impact on the stability and performance of control systems. This section briefly reviews existing results on the analysis and design of NCSs and MASs over imperfect communication channels.

1.2.1 Basics of Communication Theory

One of the main focuses of this book is to characterize the critical channel requirement such that the NCS can be mean-square stabilized. Since the communication channel is used to transmit information about the system state, as illustrated in Figure 1.1, it is expected that if the channel capacity is large enough, the feedback connected system can be mean-square stable. From this perspective, the communication channel capacity might be critical for the mean-square stabilization of control systems.

The channel capacity problem is fundamental in communication theory since it dictates the maximum data rates that can be transmitted over channels with asymptotically small error probability [Tse and Viswanath, 2005, Goldsmith, 2005]. In this subsection, we briefly review the communication channel capacity definitions and discuss why the communication theoretic channel capacity is not the critical characterization of the capacity required for controls. We only discuss discrete memoryless channels, and most of the definitions are borrowed from Cover and Thomas [2006].

A discrete memoryless channel consists of three parts: an input alphabet \mathcal{X} , an output alphabet \mathcal{Y} , and a probability transition matrix $p(y|x)$ that describes the probability of observing the output symbol y given the input symbol x . The channel is memoryless if the probability distribution of the current channel output conditioned on the current channel input is independent of previous channel inputs or outputs. The configuration of the point-to-point communication system

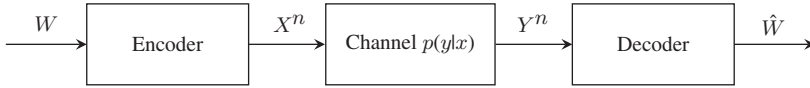


Figure 1.3 Point-to-point communication system.

is depicted in Figure 1.3. We want to transmit a message W reliably through the communication channel with appropriately designed channel encoders and decoders. The (M, n) code in a communication system is defined as follows.

Definition 1.1 (**(M, n) code**) An (M, n) code for the channel $(\mathcal{X}, p(y|x), \mathcal{Y})$ consists of three parts:

1. A message index set $\{1, 2, \dots, M\}$.
2. An encoding function $X^n : \{1, 2, \dots, M\} \rightarrow \mathcal{X}^n$, generating codewords $x^n(1), x^n(2), \dots, x^n(M)$.
3. A decoding function $g : \mathcal{Y}^n \rightarrow \{1, 2, \dots, M\}$, generating an estimate for the transmitted message index.

The performance of the code is measured by the decoding error.

Definition 1.2 (**Decoding error**) The maximal probability of error for an (M, n) code is defined as $\lambda^{(n)} = \max_{i \in \{1, 2, \dots, M\}} \Pr(g(Y^n) \neq i | X^n = x^n(i))$.

The communication channel capacity which measures the maximal capacity for reliably transmitting the information is defined below.

Definition 1.3 (**Channel capacity**) The rate R of the (M, n) code is defined as

$$R = \frac{\log M}{n} \text{ bits per transmission.}$$

A rate R is achievable if there exists a sequence of $(\lceil 2^{nR} \rceil, n)$ codes such that $\lambda^{(n)}$ tends to 0 as $n \rightarrow \infty$. The channel capacity C is then defined as the supremum of all achievable rates.

The channel capacity in Definition 1.3 is called the Shannon channel capacity since C. E. Shannon proved in the channel coding theorem that this channel capacity equals the mutual information of the channel maximized over all possible input distributions [Shannon, 2001, Cover and Thomas, 2006]:

$$C = \max_{p(x)} \mathcal{I}(X; Y),$$