

**IEEE Press Series on Electromagnetic Wave Theory**

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# Foundations of Antenna Radiation Theory

Eigenmode Analysis



*Wen Geyi*

  
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# **Foundations of Antenna Radiation Theory**

## **Eigenmode Analysis**

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Wen Geyi (Fellow, IEEE) was born in Pingjiang, Hunan, China, in 1963. He received the B.Eng., M.Eng., and Ph.D. degrees in electrical engineering from Xidian University, Xi'an, China, in 1982, 1984, and 1987, respectively. From 1988 to 1990, he was a lecturer at the Radio Engineering Department, Southeast University, Nanjing, China. From 1990 to 1992, he was an associate professor at the Institute of Applied Physics, University of Electronic Science and Technology of China (UESTC), Chengdu, China. From 1992 to 1993, he was a visiting researcher at the Department of Electrical and Computer Engineering, University of California at Berkeley, Berkeley, CA, United States. From 1993 to 1998, he was a full professor at the Institute of Applied Physics, UESTC. He was a visiting professor at the Electrical Engineering Department, University of Waterloo, Waterloo, ON, Canada, from February 1998 to May 1998. From 1996 to 1997, he was the vice chairman of the Institute of Applied Physics, UESTC, where he was the chairman of the institute from 1997 to 1998. From 1998 to 2007, he was with Blackberry Ltd., Waterloo, ON, Canada, first as a senior scientist with the Radio Frequency Department and then the director of the Advanced Technology Department. Since 2010, he has been a National Distinguished Professor with Fudan University, Shanghai, China, and the Nanjing University of Information Science and Technology (NUIST), Nanjing, where he is currently the director of the Research Center of Applied Electromagnetics. He has authored over 100 journal publications and *Foundations for Radio Frequency Engineering* (World Scientific, 2015), *Foundations of Applied Electrodynamics* (Wiley, 2010), *Advanced Electromagnetic Field Theory* (China: National Defense

Publishing House, 1999), and *Modern Methods for Electromagnetic Computations* (China: Henan Science and Technology Press, 1994). He holds more than 40 patents.

# Preface

Wireless technologies have revolutionized many different fields in industry as well as in our daily lives. As a vital device in wireless systems, antennas play an important role in boosting overall system performance. The demand on various types of antennas for different wireless applications is growing rapidly, which raises many challenges for antenna designers. For example, wireless terminals have become smaller, and antennas must be squeezed into an even smaller space. At the same time, multiple antenna systems and antennas covering multiple frequency bands are being deployed to wireless terminals to meet the increasing demand for new services and to improve the communication quality. To overcome these challenges, antenna designers need a better understanding of antenna theory.

Antenna theory usually contains three different but related subjects: generic properties of antenna, antenna analysis, and antenna synthesis. The generic properties of antenna are meant to be valid for all antennas, and they are the fundamentals of antenna design. For historical or technical reasons, many of the generic properties of antenna discovered in the last few decades have not yet been reflected in most antenna books. To include these new results in a book, one has to introduce a number of concepts that are barely touched in many antenna books, such as the stored field energy around antenna, the radiation quality factor, and the spherical vector wave functions. Antenna analysis examines the radiation properties of antenna with a specified current distribution, of which the radiated field is conventionally expressed as an integration. Such a process is, however, not always the

most efficient since the integration must be carried out for each observation point in order to find the field distribution outside the source region. The antenna synthesis, also called pattern synthesis, is the opposite process of analysis, in which the current distribution or type of antenna, including the geometry and feeding mechanism, is determined in an optimal way so that a prescribed field distribution in the far- or near-field region can be achieved. Since a continuous current distribution is not easy to realize in practice, it must be discretized and then realized by an antenna array. For this reason, various antenna synthesis methods are primarily developed for antenna array. The conventional array synthesis methods are dependent on the array factor, which is no longer effective when the array elements are not identical, the surrounding environment is too complicated, or the inter-element spacing becomes very small. New array synthesis methods based on eigenmode analysis have been developed in recent years and can overcome the existing problems associated with the array factor, but have not yet been incorporated into textbooks therefore limiting accessibility to students and researchers.

The main theme of this book is eigenmode analysis and its applications in antenna theory and design. The free space can be considered as a spherical waveguide. An antenna may therefore be viewed as a waveguide junction that connects the feeding line and the spherical waveguide, transforming the guided modes into spherical modes in transmitting mode or converting the spherical modes into guided modes in receiving mode. For this reason, it is possible to build a theory for antennas that parallels the theory for waveguides. The eigenmode analysis is the foundation of waveguide theory, and its importance in physics and engineering cannot be overstressed. An eigenmode is a possible state of a system when it is free of

excitation, and the corresponding eigenvalue often represents an important quantity of the system, for example the total energy of the system (such as in quantum mechanics) or the natural oscillation frequency (such as in a metal cavity resonator). An arbitrary state of the system can be expressed as a linear combination of the eigenmodes. If only one or a few eigenmodes dominate in the linear combination, this will significantly simplify the analysis of the problem. In the eigenmode expansion of a field, the expansion coefficients are expressed as the integrals over the source region and the integrations are only performed once. After the expansion coefficients are determined, the evaluation of the field distribution outside the source region only involves the sum of series, which decreases the computational burden and simplifies the numerical treatment most of the time as compared to the conventional integral representation.

There have been several modal theories for studying electromagnetic (EM) radiation and scattering problems. The singularity expansion method (SEM) is based on the analysis in complex frequency domain and formulated by electric field integral equation. The resonant frequencies and the modes in SEM are complex, which significantly increases the computational time and the difficulty in numerical implementations. The eigenmode expansion method (EEM) uses the eigenfunctions of an integral operator. Same with the SEM, the eigenvalues and the eigenmodes in EEM are complex numbers. In addition, the EEM lacks a solid mathematical foundation. The characteristic mode (CM) analysis is another interesting modal theory and is carried out in the real frequency domain, of which the characteristic values (eigenvalues) and CMs are all real. It is noted that all the modes involved in the CM, SEM, and EEM formulations depend not only on

the properties of the scatterer but also on the working frequency.

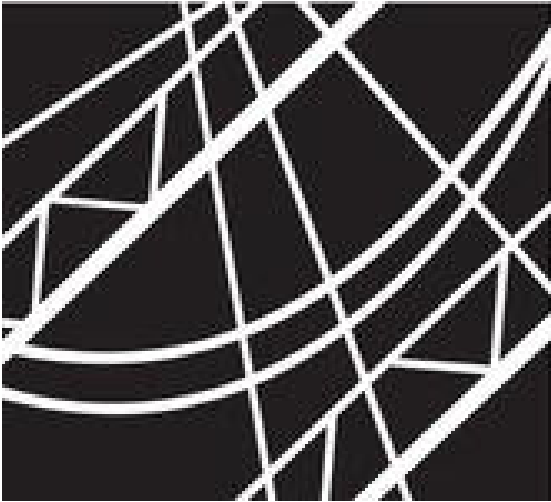
This book contains the new developments in antenna theory, with the goal to address the aforementioned problems and challenges in the best possible way and is hoped to be a useful alternative to the traditional approaches. The antenna radiation problems in both closed and open region are treated in a unified manner in terms of the eigenmodes available from the systems. The eigenmodes are derived from waveguides, cavity resonators, and spherical waveguide and are independent of frequency, and can therefore be used to expand the fields in either frequency or time domain. The organization and treatment of the proposed book is quite different from the previous books on similar topics. The method of eigenmodes, similar to the Fourier series expansion in signal analysis, is used throughout the book. The antenna analysis problems are treated by combining the method of separation of variables, Green's function, and variational method. The variational method establishes the complete set of eigenmodes and their properties, and the method of separation of variable is used to find the eigenmodes for simple geometries. The radiated field is then expanded by using the eigenmodes, from which dyadic Green's functions can be determined, avoiding the problem caused by the inappropriate selection of the eigenmodes for the expansion of a point source. When the dyadic Green's functions are applied to the integral equation formulation for an antenna, a significant computational burden can be reduced and the numerical treatment can be simplified. The array synthesis problems are also treated as an eigenvalue problem with the method of maximum power transmission efficiency (MMPTE). The variational expression is established for the power transmission efficiency (PTE) between the antenna array under design

and a testing array. An algebraic eigenvalue problem resulting from the variational principle is then solved, and the eigenvector corresponding to the maximum eigenvalue is selected as the distribution of excitations for the array under design.

The contents of the book are selected for their fundamentality and importance, and many of them are formulated in terms of eigenmode theory and appear in book form for the first time. The book not only discusses the antenna radiation problems in open space but also those in waveguide and cavity resonator, and it consists of six chapters. [Chapter 1](#) describes the basics of EM field equations and their solution methods and provides the necessary background information for later chapters. It begins with the introduction of Maxwell equations, the wave equations, and the theorems for EM fields. Three analytical tools for the solution of boundary value problems are introduced, and they are the separation of variables, Green's function, and the variational method. The main focus of this chapter is the treatment of eigenvalue problems arising in matrix theory, scalar and vector fields, and they are fundamental to our later discussions. By means of the Rayleigh quotient (a variational expression for the eigenvalue problem), the eigenmodes of the Laplacian operator acting on a scalar or a vector field are treated in a similar manner, and a complete set of eigenmodes is constructed by the variational analysis of the Rayleigh quotient. In order to understand how a vector field is decomposed into longitudinal, transverse, and harmonic components, the Helmholtz theorems for the vector fields defined in finite or infinite region are presented. As a generalization of the Helmholtz theorem, the eigenfunctions of the curl operator are also explored, in terms of which the plane-wave expansions for the fields and the dyadic Green's functions are obtained.

[Chapter 2](#) investigates the radiation problems in waveguide. The eigenvalue problems in waveguide are approached in transverse field for its generality. Various dyadic Green's functions for waveguide are derived directly from the field expansions in terms of the vector modal functions, which avoids a problem caused by the incompleteness of the eigenfunctions selected to expand a dyadic point source in the conventional study of dyadic Green's functions in waveguides. By the equivalence principle, three common waveguide discontinuity problems, the excitation of waveguide, obstacles in waveguide, and the coupling between waveguides, are analyzed and treated as a radiation problem and compactly reformulated by using the dyadic Green's functions. The radiated field in time domain is approached by the vector modal functions, and the transient processes in the waveguide are studied. For reference, the vector modal functions in typical waveguides are summarized.

[Chapter 3](#) deals with the radiation problems in metal cavity resonators. In particular, the vector modal functions in the waveguide cavity resonator are derived from the waveguide modes. The dyadic Green's functions of electric and magnetic type for a cavity resonator are established from the modal expansions of the fields. Like the waveguide theory, all the cavity-related problems are treated as a radiation problem through the use of equivalence principle. The circuit parameters for the cavity with multiple waveguide ports are evaluated by the modal analysis. The vector modal functions for typical waveguide cavity resonators are derived. It is demonstrated that the dyadic Green's functions for the waveguide cavity reduce to those for the waveguide if the two ends of the waveguide cavity are extended to infinity. The time-domain fields generated by the sources in the waveguide cavity are expanded in terms of the vector modal functions in



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