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Foundations of Antenna Radiation Theory

Eigenmode Analysis

Wen Geyi

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About the Author

Wen Geyi (Fellow, IEEE) was born in Pingjiang, Hunan, China, in 1963. He received the B.Eng., M.Eng., and Ph.D. degrees in electrical engineering from Xidian University, Xi'an, China, in 1982, 1984, and 1987, respectively. From 1988 to 1990, he was a lecturer at the Radio Engineering Department, Southeast University, Nanjing, China. From 1990 to 1992, he was an associate professor at the Institute of Applied Physics, University of Electronic Science and Technology of China (UESTC), Chengdu, China. From 1992 to 1993, he was a visiting researcher at the Department of Electrical and Computer Engineering, University of California at Berkeley, Berkeley, CA, United States. From 1993 to 1998, he was a full professor at the Institute of Applied Physics, UESTC. He was a visiting professor at the Electrical Engineering Department, University of Waterloo, Waterloo, ON, Canada, from February 1998 to May 1998. From 1996 to 1997, he was the vice chairman of the Institute of Applied Physics, UESTC, where he was the chairman of the institute from 1997 to 1998. From 1998 to 2007, he was with Blackberry Ltd., Waterloo, ON, Canada, first as a senior scientist with the Radio Frequency Department and then the director of the Advanced Technology Department. Since 2010, he has been a National Distinguished Professor with Fudan University, Shanghai, China, and the Nanjing University of Information Science and Technology (NUIST), Nanjing, where he is currently the director of the Research Center of Applied Electromagnetics. He has authored over 100 journal publications and *Foundations for Radio Frequency Engineering* (World Scientific, 2015), *Foundations of Applied Electrodynamics* (Wiley, 2010), *Advanced Electromagnetic Field Theory* (China: National Defense Publishing House, 1999), and *Modern Methods for Electromagnetic Computations* (China: Henan Science and Technology Press, 1994). He holds more than 40 patents.

Preface

Wireless technologies have revolutionized many different fields in industry as well as in our daily lives. As a vital device in wireless systems, antennas play an important role in boosting overall system performance. The demand on various types of antennas for different wireless applications is growing rapidly, which raises many challenges for antenna designers. For example, wireless terminals have become smaller, and antennas must be squeezed into an even smaller space. At the same time, multiple antenna systems and antennas covering multiple frequency bands are being deployed to wireless terminals to meet the increasing demand for new services and to improve the communication quality. To overcome these challenges, antenna designers need a better understanding of antenna theory.

Antenna theory usually contains three different but related subjects: generic properties of antenna, antenna analysis, and antenna synthesis. The generic properties of antenna are meant to be valid for all antennas, and they are the fundamentals of antenna design. For historical or technical reasons, many of the generic properties of antenna discovered in the last few decades have not yet been reflected in most antenna books. To include these new results in a book, one has to introduce a number of concepts that are barely touched in many antenna books, such as the stored field energy around antenna, the radiation quality factor, and the spherical vector wave functions. Antenna analysis examines the radiation properties of antenna with a specified current distribution, of which the radiated field is conventionally expressed as an integration. Such a process is, however, not always the most efficient since the integration must be carried out for each observation point in order to find the field distribution outside the source region. The antenna synthesis, also called pattern synthesis, is the opposite process of analysis, in which the current distribution or type of antenna, including the geometry and feeding mechanism, is determined in an optimal way so that a prescribed field distribution in the far- or near-field region can be achieved. Since a continuous current distribution is not easy to realize in practice, it must be discretized and then realized by an antenna array. For this reason, various antenna synthesis methods are primarily developed for antenna array. The conventional array synthesis methods are dependent on the array factor, which is no longer effective when the array elements are not identical, the surrounding environment is too complicated, or the inter-element spacing becomes very small. New array synthesis methods based on eigenmode analysis have been developed in recent years and can overcome the existing problems associated with the array factor, but have not yet been incorporated into textbooks therefore limiting accessibility to students and researchers.

The main theme of this book is eigenmode analysis and its applications in antenna theory and design. The free space can be considered as a spherical waveguide. An antenna may therefore be viewed as a waveguide junction that connects the feeding line and the spherical waveguide, transforming the guided modes into spherical modes in transmitting mode or converting the spherical modes into guided modes in receiving mode. For this reason, it is possible to build a theory for antennas that parallels the theory for waveguides. The eigenmode analysis is the foundation of waveguide theory, and its importance in physics and engineering cannot be overstressed. An eigenmode is a possible state of a system when it is free of excitation, and the corresponding eigenvalue often represents an important quantity of the system, for example the total energy of the system (such as in quantum mechanics) or the natural oscillation frequency (such as in a metal cavity resonator). An arbitrary state of the system can be expressed as a linear combination of the eigenmodes. If only one or a few eigenmodes dominate in the linear combination, this will significantly simplify the analysis of the problem. In the eigenmode expansion of a field, the expansion coefficients are expressed as the integrals over the source region and the integrations are only performed once. After the expansion coefficients are determined, the evaluation of the field distribution outside the source region only involves the sum of series, which decreases the computational burden and simplifies the numerical treatment most of the time as compared to the conventional integral representation.

There have been several modal theories for studying electromagnetic (EM) radiation and scattering problems. The singularity expansion method (SEM) is based on the analysis in complex frequency domain and formulated by electric field integral equation. The resonant frequencies and the modes in SEM are complex, which significantly increases the computational time and the difficulty in numerical implementations. The eigenmode expansion method (EEM) uses the eigenfunctions of an integral operator. Same with the SEM, the eigenvalues and the eigenmodes in EEM are complex numbers. In addition, the EEM lacks a solid mathematical foundation. The characteristic mode (CM) analysis is another interesting modal theory and is carried out in the real frequency domain, of which the characteristic values (eigenvalues) and CMs are all real. It is noted that all the modes involved in the CM, SEM, and EEM formulations depend not only on the properties of the scatterer but also on the working frequency.

This book contains the new developments in antenna theory, with the goal to address the aforementioned problems and challenges in the best possible way and is hoped to be a useful alternative to the traditional approaches. The antenna radiation problems in both closed and open region are treated in a unified manner in terms of the eigenmodes available from the systems. The eigenmodes are derived from waveguides, cavity resonators, and spherical waveguide and are independent of frequency, and can therefore be used to expand the fields in either frequency or time domain. The organization and treatment of the proposed book is quite different from the previous books on similar topics. The method of eigenmodes, similar to the Fourier series expansion in signal analysis, is used throughout the book. The antenna analysis problems are treated by combining the method of separation of variables, Green's function, and variational method. The variational method establishes the complete set of eigenmodes and their properties, and the method of separation of variable is used to find the eigenmodes for simple geometries. The radiated field is then expanded by using the eigenmodes, from which dyadic Green's functions can be determined, avoiding the problem caused by the inappropriate selection of the eigenmodes for the expansion of a point source. When the dyadic Green's functions are applied to the integral equation formulation for an antenna, a significant computational burden can be reduced and the numerical treatment can be simplified. The array synthesis problems are also treated as an eigenvalue problem with the method of maximum power transmission efficiency (MMPTE). The variational expression is established for the power transmission efficiency (PTE) between the antenna array under design and a testing array. An algebraic eigenvalue problem resulting from the variational principle is then solved, and the eigenvector corresponding to the maximum eigenvalue is selected as the distribution of excitations for the array under design.

The contents of the book are selected for their fundamentality and importance, and many of them are formulated in terms of eigenmode theory and appear in book form for the first time. The book not only discusses the antenna radiation problems in open space but also those in waveguide and cavity resonator, and it consists of six chapters. Chapter 1 describes the basics of EM field equations and their solution methods and provides the necessary background information for later chapters. It begins with the introduction of Maxwell equations, the wave equations, and the theorems for EM fields. Three analytical tools for the solution of boundary value problems are introduced, and they are the separation of variables, Green's function, and the variational method. The main focus of this chapter is the treatment of eigenvalue problems arising in matrix theory, scalar and vector fields, and they are fundamental to our later discussions. By means of the Rayleigh quotient (a variational expression for the eigenvalue problem), the eigenmodes of the

Laplacian operator acting on a scalar or a vector field are treated in a similar manner, and a complete set of eigenmodes is constructed by the variational analysis of the Rayleigh quotient. In order to understand how a vector field is decomposed into longitudinal, transverse, and harmonic components, the Helmholtz theorems for the vector fields defined in finite or infinite region are presented. As a generalization of the Helmholtz theorem, the eigenfunctions of the curl operator are also explored, in terms of which the plane-wave expansions for the fields and the dyadic Green's functions are obtained.

Chapter 2 investigates the radiation problems in waveguide. The eigenvalue problems in waveguide are approached in transverse field for its generality. Various dyadic Green's functions for waveguide are derived directly from the field expansions in terms of the vector modal functions, which avoids a problem caused by the incompleteness of the eigenfunctions selected to expand a dyadic point source in the conventional study of dyadic Green's functions in waveguides. By the equivalence principle, three common waveguide discontinuity problems, the excitation of waveguide, obstacles in waveguide, and the coupling between waveguides, are analyzed and treated as a radiation problem and compactly reformulated by using the dyadic Green's functions. The radiated field in time domain is approached by the vector modal functions, and the transient processes in the waveguide are studied. For reference, the vector modal functions in typical waveguides are summarized.

Chapter 3 deals with the radiation problems in metal cavity resonators. In particular, the vector modal functions in the waveguide cavity resonator are derived from the waveguide modes. The dyadic Green's functions of electric and magnetic type for a cavity resonator are established from the modal expansions of the fields. Like the waveguide theory, all the cavity-related problems are treated as a radiation problem through the use of equivalence principle. The circuit parameters for the cavity with multiple waveguide ports are evaluated by the modal analysis. The vector modal functions for typical waveguide cavity resonators are derived. It is demonstrated that the dyadic Green's functions for the waveguide cavity reduce to those for the waveguide if the two ends of the waveguide cavity are extended to infinity. The time-domain fields generated by the sources in the waveguide cavity are expanded in terms of the vector modal functions in waveguide, and the transient responses in the cavity resonator are examined.

Chapter 4 discusses the generic properties of antenna. Typical antenna parameters are summarized. Complete set of vector modal functions for the spherical waveguide is rigorously constructed from spherical harmonics, and the modal expansions of the dyadic Green's functions are derived from the field expansions. A general definition of the stored field energy of antenna is proposed by means of a conservation law for the stored field energies in an arbitrary medium. Two methods for evaluating the radiation quality factor are elucidated. One is from the input impedance of antenna and the other is via the current distribution. The modal quality factors are thoroughly examined, and their finite power series expansions are obtained. The upper bounds on the product of gain and bandwidth for both directional and omnidirectional antenna are presented, and their applications in small antenna design are demonstrated. The upper bounds answer a common question of how much space should be requested to accommodate an antenna to realize a specified performance. The radiated fields from a transient source are studied through the field expansions in terms of the vector modal functions for the spherical waveguide.

In antenna analysis, the induced current distribution on antenna is either given or to be determined from an impressed source. In many cases, one must resort to numerical techniques to solve an integral equation or a set of differential equations derivable from Maxwell equations with boundary conditions to find the induced current distribution. Chapter 5 is devoted to the modal analysis of typical antennas. The free space is considered as a spherical waveguide, and the radiated field is expressed as a linear combination of spherical vector wave functions. The integral equations for an antenna consisting of composite materials are derived by the modal expansions of dyadic Green's functions. Instead of using the integral representation of the fields in conventional antenna analysis, typical antennas, including dipole, loop, aperture, and patch antenna, are all analyzed by the spherical wave functions or the eigenmodes in both near- and far-field regions. An arbitrary scatterer is said to be resonant if its stored electric field energy is equal to the stored magnetic field energy. Based on this definition, a method for computing the resonant modes is proposed and applied to the antenna design.

Antenna synthesis involves using well-organized optimization methods to find the current distribution so as to achieve a specified field distribution in the near- or far-field region. A single antenna is often around one wavelength in size, and its radiation pattern covers a wide angle and thus exhibits poor directivity. In order to enhance the directivity and increase the flexibility of shaping the radiation pattern, one must use an antenna array. The performances of antenna array are controlled by the relative positioning of elements and the distribution of excitations. To achieve a desired field pattern, a performance index (target function) must be properly chosen and optimized. For a wireless system planned for the transmission of either information or power, a natural performance index is the PTE between the transmitting (Tx) and receiving (Rx) antennas, which is defined as the ratio of the power delivered to the load of the receiver to the input power of the transmitter. To attain the best possible quality of wireless communication or power transfer, the PTE must be maximized. Motivated by the fact that antennas must be designed to enhance the PTE for all wireless systems, the PTE can thus be adopted as a performance index for the design of antennas. The optimization procedure provides a powerful and universal technique for the synthesis of antenna arrays of all types and can overcome the existing challenges with the conventional array synthesis methods using array factors. The technique is based on eigenmode analysis and called the method of maximum power transmission efficiency (MMPTE), which can achieve various field patterns in any complicated environment in the near- or far-field region. The conventional methods of antenna synthesis largely depend on field theory, while the MMPTE reduces the field synthesis problem into a circuit analysis problem so that circuit theory may be applied to solve the original field problem. This feature makes the design process of antenna array more accessible for those who are not very familiar with EM field theory. The circuit parameters can be acquired by simulation or measurement, and therefore the MMPTE is applicable to any complicated problem. Whenever the simulation is beyond the capability of a state-of-the-art computer, one can resort to measurement to find the circuit parameters. Another important feature of the MMPTE is that it contains the information of the environment between Tx and Rx arrays, and therefore can be made adaptive to complicated environment, guaranteeing the best possible performance of the antenna array. The MMPTE has been verified to be superior to most existing array design methods in terms of simplicity, applicability, generality, and design accuracy. It generates an optimized distribution of excitation for the antenna array to assure that the gain and efficiency of the array is maximized for a fixed array configuration and is equally applicable for both near- and far-field synthesis problems. Chapter 6 summarizes several typical array synthesis methods based on the array factors, including Schelkunoff unit circle method, Dolph–Chebyshev method, and the Fourier transform method. The main part of this chapter is the formulations of MMPTE, including the unconstrained MMPTE, weighted MMPTE, constrained MMPTE, and extended MMPTE (EMMPTE). The EMMPTE is a field method but follows a procedure similar to MMPTE. A number of applications of MMPTE and EMMPTE are demonstrated.

For the convenience of readers, three Appendices A, B, and C are included to provide the fundamentals of vector analysis, dyadic analysis, and the SI unit system. A unified theory for fields (UTF) is explored in Appendix D. The UTF unveils that an arbitrary static field (either scalar or vectorial, called an ontological field) in an inertial system (static system) will merge as two vector fields in an inertial system moving relative to the static system, which satisfy Maxwell-like equations. Therefore, the Maxwell equations are valid not only for describing the EM fields but also for any physical fields for which an ontological field exists. The UTF is based on the theory of special relativity and the Helmholtz theorem. In order to find how the vector field changes in different inertial systems, one only needs to examine how the curl and divergence of the vector field transform. As a demonstration, the Maxwell-like equations for the gravitational field are derived from the UTF, and they are also derived from the Einstein field equations in the theory of general relativity. Some universal laws of nature are shown to be derivable from the UTF.

The book can be used either for undergraduate or graduate courses on “Advanced Antenna Theory,” or as a reference for researchers and engineers in the areas of microwave, antenna, and EM compatibility. The prerequisites for the book are advanced calculus and linear algebra. After reading the book, the readers should be able to better understand antenna radiation theory and antenna analysis and synthesis from a different perspective in terms of eigenmode analysis. The SI units are used throughout the book. A $e^{j\omega t}$ time variation is assumed for time-harmonic fields. A special symbol “□” is used to indicate the end of a theorem, a remark, or an example.

The author is grateful to his family. Without their constant support and encouragement, the book would never have been completed.

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1

Eigenvalue Theory

Science is spectral analysis. Art is light synthesis.
– Karl Kraus (Austrian writer and journalist, 1874–1936)

The study of eigenvalue problems can be traced back to the eighteenth century, when Swiss mathematician and physicist Leonhard Euler (1707–1783) investigated the rotational motion of a rigid body. The word "eigen" is from German and means "own" or "belonging to," and was first used by German mathematician David Hilbert (1862–1943) to characterize eigenvalues and eigenvectors in 1904. Eigenvalue problems often arise in mathematics, physics, and engineering sciences. In linear algebra, an **eigenvector** of a linear transformation is a nonzero vector that changes by a scalar factor when the linear transformation acts on it. The scalar factor is called the **eigenvalue** corresponding to the eigenvector. Geometrically, this implies that the eigenvector is not rotated after transformation. The eigenvalue problem for a differential operator often results from the boundary value problems defined in a finite region. When the defining region is unbounded, the discrete eigenvalues become a continuum. A very useful technique for studying the eigenvalue problem is to establish the Rayleigh quotient for the eigenvalues and then use the calculus of variations to investigate the properties of eigenmodes. In physics, an **eigenmode** of a system is a possible state when the system is free of excitation, which might exist in the system on its own under certain conditions, and is also called an **eigenstate** of the system. The **method of eigenfunctions** is very similar to the Fourier series expansion in signal analysis, and will be used throughout this book. The method is based on the solution of an eigenvalue problem available from the system. An arbitrary state of the system can be expressed as a linear combination of the eigenmodes, and the expansion coefficients can then be determined from the source conditions or the initial values of the system. If only one or a few eigenmodes dominate in the linear combination, this will significantly simplify the analysis of the problem.

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The modal theory for a scatterer plays an important role in antenna theory and designs. The basic idea behind the modal theory is to introduce the fundamental field patterns, called **modes**, so that the fields outside the scatterer can be expanded into a linear combination of these modes. There have been several modal theories for studying electromagnetic (EM) radiation and scattering problems (exterior boundary value problems). The **singularity expansion method** (SEM) is based on the analysis in complex frequency domain and formulated by electric field integral equation [1, 2]. The **natural resonant frequencies** arise from the requirement that a nontrivial current distribution exists on a conducting scatterer free of incident fields. The corresponding field patterns are called **natural resonant modes**. The natural resonant frequencies and the modes in SEM are complex, which significantly increases the computational time and the difficulty in numerical implementations. The **eigenmode expansion method** (EEM) expands the currents and the radiated fields in terms of the eigenmodes of an integral operator [3, 4]. Same with the SEM, the eigenvalues and the eigenmodes in EEM are complex numbers. The EEM is based on the eigenfunctions of integral equations and lacks a solid mathematical foundation. The integral operator involved in EEM is not symmetric, and it is therefore hard to prove the existence and completeness of the eigenfunctions. A more useful method for the study of scattering problem is the **singular function expansion**, which was first introduced by the German mathematician Erhard Schmidt (1876–1959) in 1907 [5], and has been applied to study various scattering problems [6, 7]. The theory of **characteristic mode** is another interesting modal notion and is carried out in the real frequency domain [8–11], of which the characteristic values (eigenvalues) and the corresponding characteristic modes are all real. In general, the characteristic values range from $-\infty$ to $+\infty$, among which those of the smallest magnitudes are the most important for radiation and scattering problems. The external resonant modes correspond to the zero characteristic values, and can be determined approximately by sweeping the frequency. It is noted that all the abovementioned modal formulations depend not only on the properties of the scatterer but also on the operating frequency.

The eigenvalue problems discussed in this book are derived from waveguide, cavity resonator, and spherical waveguide, whose eigenfunctions are independent of frequency and can thus be used to expand the fields in either frequency or time domain. The importance of eigenvalue theory in mathematics and physics cannot be overstated. There have been various methods developed to calculate eigenvalues and eigenfunctions, with the most important one being the variational method based on the Rayleigh quotient [12]. This chapter provides the necessary background information for later chapters. The Maxwell equations and the solution methods for partial differential equations (PDEs) are briefly introduced. The emphasis is upon the eigenvalue theory for operators, including the matrix and the

Laplacian on scalar and vector fields. The properties of eigenfunctions are derived from the Rayleigh quotient, and the Ritz method for the numerical solution of the Rayleigh quotient is demonstrated. Also included in this chapter is the Helmholtz theorem, which states that any vector field can be decomposed into the sum of an irrotational vector field and a solenoidal vector field. Such a decomposition has interesting applications in the modal expansion of fields and is the theoretical basis of introducing scalar and vector potentials. The Helmholtz theorem indicates that a vector field is fully determined by its divergence and curl. Indeed, Maxwell equations are nothing but a couple of rules that regulate the divergences and the curls of electric and magnetic fields according to impressed and induced sources. As a generalization of Helmholtz theorem, the eigenfunctions of curl operator are discussed, in terms of which the plane-wave expansions for the fields as well as the dyadic Green's functions can be obtained.

1.1 Maxwell Equations

Maxwell equations are a set of PDEs that unify electricity and magnetism and describe how electric and magnetic fields, as the functions of space and time, are generated by charges and currents and altered by each other. They have been proved to be very successful in explaining and predicting a variety of macroscopic EM phenomena.

1.1.1 Wave Equations

The **generalized Maxwell equations** that include both electric and magnetic sources consist of two vector equations and two scalar equations:

$$\begin{aligned}
\nabla \times \mathbf{H}(\mathbf{r}, t) &= \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t), \\
\nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} - \mathbf{J}_m(\mathbf{r}, t), \\
\nabla \cdot \mathbf{D}(\mathbf{r}, t) &= \rho(\mathbf{r}, t), \\
\nabla \cdot \mathbf{B}(\mathbf{r}, t) &= \rho_m(\mathbf{r}, t).
\end{aligned} \tag{1.1}$$

In the above, $\mathbf{r}$ is the observation point of the fields in meter (m) and t is the time in second (s), $\mathbf{H}$ is the **magnetic field intensity** measured in amperes per meter (A/m), $\mathbf{B}$ is the **magnetic induction intensity** measured in tesla (Wb/m^2), $\mathbf{E}$ is **electric field intensity** measured in volts per meter (V/m), $\mathbf{D}$ is the **electric induction intensity** measured in coulombs per square meter (C/m^2), $\mathbf{J}$ is **electric current density** measured in amperes per square meter (A/m^2), ρ is the **electric charge density** measured in coulombs per cubic meter (C/m^3), $\mathbf{J}_m$

is **magnetic current density** in volts per square meter (V/m^2), and ρ_m is **magnetic charge density** in webers per cubic meter (Wb/m^3). The first equation is **Ampère's law**, and it describes how the electric field changes according to the current density and magnetic field. The positive sign in the first equation indicates that the directions of the magnetomotive force and the electric current are related by the right-hand rule. The term $\partial \mathbf{D}/\partial t$ was introduced by Maxwell in 1861 and is called **displacement current**, which is necessary for the existence of wave solutions. The second equation is **Faraday's law**, and it characterizes how the magnetic field varies according to the electric field and equivalent magnetic current density. The minus sign in the second equation indicates that the directions of electromotive force and the magnetic current are related by the left-hand rule, which is required by **Lenz's law**. In other words, when an electromotive force is generated by a change of magnetic flux, the polarity of the induced electromotive force is such that it produces a current whose magnetic field opposes the change, which produces it. The third equation is **Coulomb's law**, and it says that the electric field depends on the charge distribution and obeys the inverse square law. The last equation shows that the magnetic field also obeys the inverse square law and depends on the equivalent magnetic charge distribution. It should be understood that none of the experiments had anything to do with waves at the time when Maxwell derived his equations. Maxwell equations imply more than the experimental facts. The **continuity equation** can be derived from (1.1):

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \tag{1.2}$$

The electric charge density ρ and the electric current density $\mathbf{J}$ in Maxwell equations are free charge density and currents and they exclude charges and currents forming part of the structure of atoms and molecules. The bound charges and currents are regarded as material, which are not included in ρ and $\mathbf{J}$. The current density usually consists of two parts: $\mathbf{J} = \mathbf{J}_{con} + \mathbf{J}_{imp}$. Here, $\mathbf{J}_{imp}$ is referred to as external or **impressed current source**, which is independent of the fields and delivers energy to electric charges in a system. The impressed current source can be of electric and magnetic type as well as of non-electric or nonmagnetic origin. $\mathbf{J}_{con} = \sigma \mathbf{E}$, where σ is the **conductivity** of the medium in siemens per meter (S/m), denotes the **conduction current** induced by the impressed source $\mathbf{J}_{imp}$. Sometimes it is convenient to introduce an **impressed electric field** $\mathbf{E}_{imp}$ defined by $\mathbf{J}_{imp} = \sigma \mathbf{E}_{imp}$. In more general situation, one may write $\mathbf{J} = \mathbf{J}_{ind} + \mathbf{J}_{imp}$, where $\mathbf{J}_{ind}$ is the **induced current** by the impressed current $\mathbf{J}_{imp}$. The continuity equation for the magnetic current $\mathbf{J}_m$ and magnetic charges ρ_m can be derived from (1.1):

$$\nabla \cdot \mathbf{J}_m = -\frac{\partial \rho_m}{\partial t}. \tag{1.3}$$

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