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Pavel Kurasov, Ari Laptev, Barry Simon,
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From Complex Analysis to Operator Theory: A Panorama

In Memory of Sergey Naboko

Operator Theory: Advances and Applications

Volume 291

Founded in 1979 by Israel Gohberg

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From Complex Analysis to Operator Theory: A Panorama

In Memory of Sergey Naboko

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ISSN 0255-0156

ISSN 2296-4878 (electronic)

Operator Theory: Advances and Applications

ISBN 978-3-031-31138-3

ISBN 978-3-031-31139-0 (eBook)

<https://doi.org/10.1007/978-3-031-31139-0>

Mathematics Subject Classification: 01A65, 47B40, 47B28, 47A45, 47B25, 46E22, 47A55, 47B32, 35P15, 81Q12, 81Q10, 46N50, 47B80, 47A20, 81Q35, 34L25, 47F05, 35Q61, 35Q74, 34E05, 34E13, 58C40, 35J10, 37D05, 47A10, 47B36, 52C23, 58J51, 35J15, 35J70, 47B44, 35K65, 34L20, 34L40, 45P05, 37K40, 35Q53, 37K45, 35Q15, 47A48, 47A06, 47B50, 30E99, 35P20, 49R05, 81Q20, 37K15, 47B35, 33C45, 39A70, 47A40, 47B39, 34D10, 34L15, 30H20, 30I0, 47N50

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The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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 Dimitri R. Yafaev

Sergey Naboko: A Life in Mathematics

In memoriam



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Sergey Naboko (Сергей Николаевич Набоко) died in St. Petersburg on December 24, 2020, 1 week before his 71st birthday. Sergey was a talented mathematician with many remarkable results who influenced the development of analysis, especially its applications to mathematical physics—a branch of mathematics well-represented in Leningrad (St. Petersburg).

The closest relatives are his wife Ekaterina Vasilyeva (Kate) and three older siblings (Natalia, Olga, and Vasily) with families.

Sergey was born on January 2, 1950, in Vyritsa (Вырица) near Leningrad (St. Petersburg) in a family that was strongly influenced by the tradition of Sergey's maternal great-grandfather, Saint Serafim Vyritsky (преподобный Серафим Вырицкий), who was canonised by the Orthodox Church in 2000. Sergey's grandmother Eugeniya Lubarskaya (Евгения Лубарская) came from Moscow jewish *petite bourgeoisie*.

Sergey had two older sisters (Natalia and Olga) and one older brother (Vasily), all extremely close to their mother Margarita Nikolaevna (Маргарита Николаевна), granddaughter of Serafim Vyritsky. All children received an excellent education

and lived secular lives so that even the closest friends could not guess the family's background, which of course was known to competent authorities.

Academic Career

Already as a young man, Sergey showed great talent and he started primary school 1 year ahead of his contemporaries. After high school, he studied at the Department of Mathematical Physics of Leningrad University with outstanding teachers such as Vladimir Smirnov, Olga Ladyzhenskaya, Ludvig Faddeev, Mikhail Birman, and Boris Pavlov. This was an excellent environment for studying applications of analysis in mathematical and theoretical physics, especially, in connection with the spectral theory for differential operators. Sergey had close ties with the analysis group at the Faculty of Mathematics and Mechanics of Leningrad University, especially with N.K. Nikolskii and his seminar. Sergey received his PhD (кандидат наук) in 1977 with Boris Pavlov as supervisor. The dissertation, which dealt with non-Hermitian operators, was judged to be brilliant. Mark Krein even stated that the quality of the dissertation was on the level of the Russian doctoral degree (Доктор Наук). Subsequently, when evaluating new doctoral dissertations that fell into the purview of Mark Krein, one could hear from him [1]

Yes, not bad, but can it be compared with Naboko's Ph.D. thesis?

Sergey's background in Christian circles combined with Jewish origin was of course not exactly a springboard in the Soviet society, still, in 1991, Sergey became a professor at the famous Department of Mathematical Physics, where he had studied originally. Before perestroika, Sergey could not travel abroad, given his background, but in 1991 he was allowed to visit Sweden followed by visits and longer stays at many foreign countries:

Belgium, France, Germany, Ireland, Israel, Mexico, New Zealand, Norway, Poland, Portugal, Sweden, UK, and USA.

Sergey worked for extended periods as a visiting professor at the

- University of Alabama at Birmingham (2004–2009);
- University of Kent at Canterbury (2013–2015);
- Stockholm University (2017–2019).

Being an experienced tourist, he established permanent base camps at

Cardiff, Bordeaux, Krakow, Lund–Stockholm, Mexico-City

—places he visited on a regular basis.

Sergey remained faithful to his university in St. Petersburg, where he returned regularly after every trip, despite numerous proposals to accept a permanent position abroad (see, e.g., section *The Swedish Connection*). In addition to personal reasons, he felt comfortable as a prominent representative of the St. Petersburg mathematical school, attracting talented students whom he felt he could not abandon.



Fig. 1 At Birmingham (Al) with Yulia Karpeshina and Gunter Stolz

Sergey liked to visit colleagues, learn new ideas, and expand mathematical knowledge (his own and his host's). Everywhere he made close contacts with like-minded people who valued mathematical jewels higher than regular jewelry. One meets mathematicians having benefited from collaboration with Sergey everywhere in the world. He had an infectious enthusiasm and sheer endless energy for the subject. Sergey was ready to discuss mathematics at any time of the day and shared his deep knowledge with colleagues, often without requiring any acknowledgment. One could often see him explaining after somebody's lecture how the subject can be seen from a general perspective, and in which possible directions one may proceed. He frequently understood the lecture better than the lecturer himself.

Personality

In spite of mathematics having been Sergey's passion, he was interested in all other areas of culture: literature, history, cinema, music, etc. He liked to share his knowledge and preferences with colleagues and friends: each time visiting a new place his luggage contained not only kilograms of chocolates, but also books which he was distributing following individual interests of his friends. When hosting guests in St. Petersburg, Sergey was always keen to show off his city, helping organize visits to palaces, concerts, or the ballet and inviting people home for a Russian meal.

Sergey cared very much about other people and had an extraordinary ability *to understand people down to their core*—he claimed that this ability was inherited from his great-grandfather. While he could find something attractive in almost any person, he could not stand betrayal, hypocrisy, and meanness—some people who lost his trust had a difficult time gaining it back. Sergey had good relations with many people, but he did not permit people to come too close; being his close friend was a hard task due to extremely high standards he applied to people, including himself. He could easily see the needs of others and could often find a good solution. This ability had an unexpected side: if Sergey was convinced that somebody needed his help, it was absolutely impossible to convince him otherwise. It was better to let it slide. Similarly, Sergey had difficulties accepting presents despite having distributed numerous presents himself.



Fig. 2 Sergey at Gregynog Hall with Boris Pavlov (viewer's right) and Pavel Kurasov (left).
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University Professor

Sergey Naboko was an extraordinary lecturer. Even students not formally registered for the course were attracted to his lectures. Highly non-trivial material was presented in such an elegant and clear way that it was hard to see the obstacles; therefore, he used to illustrate possible difficulties via explicit examples.

Sergey always used very simple clothing, but students immediately saw the highly non-trivial and attractive underlying personality, a combination of an old-fashioned gentleman (really, a gentle man) and a modern human being interested in most facets of life.

Mathematical Research

Sergey once listed his main research interests (the references refer to the list of Sergey's publications)

- Scattering theory for self-adjoint and nonself-adjoint operators, perturbation theory for continuous spectrum. Spectral shift function and scattering matrix properties [2, 3, 6, 9, 19, 26, 32, 44, 70, 71, 102, 122]
- Embedding of point into continuous spectrum, in particular existence of embedded dense point spectrum [13, 16, 30, 31, 36, 39, 61, 79, 94, 104, 109]
- Investigation of the singular spectrum [10, 12, 14, 17, 18, 25, 29, 34, 35, 37, 84]
- Nevanlinna class operator-functions in perturbation theory [11, 12, 14, 20, 22–24, 27, 28, 33, 37, 38, 86, 113, 114]
- Dissipative operators [100, 105, 117, 121, 125]
- Liapunov stability problems, stability problems from hydrodynamics [41]
- Spectral theory of non-dissipative operators, theory of operators with almost Hermitian spectrum [4–8, 10, 14, 15, 17, 35, 64, 67, 76, 85]
- Functional models and characteristic functions of operators [4, 6, 7, 9, 46, 51, 56, 117, 121, 123]
- Wave propagation in inhomogeneous media
- Localization for Anderson models in random Schrödinger operators theory [73, 75, 78, 98]
- The spectral structure for the matrix-differential operators from magnetohydrodynamics [45, 52, 54, 62, 82]
- Spectral analysis of Jacobi matrices [7, 40, 42, 43, 47–50, 53, 55, 60, 61, 63, 65, 68, 72, 77, 80, 84, 89–91, 93, 95, 104, 108, 109, 111]
- Spectral analysis of the Boltzmann transport operators [46, 51, 56]
- Semiclassical analysis of Regge trajectories for singular and nonsingular potentials [57–59, 66, 69, 92]
- Spectral properties of Block Jacobi Operators [103, 110, 112, 116, 126, 127]
- Spectral theory of quantum graphs [74, 99, 101, 118, 125]

- Homogenization theory for self-adjoint operators with stiff and soft regimes [115, 123]

We tried to describe Sergey's mathematical achievements by inviting some of his coauthors and prominent experts to write review articles. Each of these accounts is devoted to one of the areas in mathematical analysis where Sergey's contributions were seminal:

- B. Malcolm Brown, Marco Marletta and Ian Wood, *Working with Sergey Naboko on Boundary Triples*
- Fritz Gesztesy, *Operator-Valued Nevanlinna–Herglotz Functions, Trace Ideals, and Sergey Naboko's Contributions*
- Alexander V. Kiselev and Vladimir Ryzhov, *Mathematical Heritage of Sergey Naboko: Functional Models of Non-Self-Adjoint Operators*
- Pavel Kurasov, *On Crossroads of Spectral Theory with Sergey Naboko*
- Luis O Silva and Sergey Simonov, *Sergey Naboko's Legacy on the Spectral Theory of Jacobi Operators*
- Dmitry Yakubovich, *On the Work by Serguei Naboko on the Similarity to Unitary and Self-Adjoint Operators*

These contributions are included in the current volume.

Working with Sergey

It was an honor working with Sergey. His knowledge of mathematical analysis allowed him to understand almost immediately whether a problem was trivial, or impossible to solve at this stage. He surprised colleagues by fearlessly attacking difficult problems, often not approached by other mathematicians. This ability can be traced back to his first paper [4] devoted to the number of geodesics on a fundamental domain of the modular group, related to the idea of proving the Riemann hypothesis using L.D. Faddeev's and B.S. Pavlov's scattering theory for automorphic functions, extended later by P.D. Lax and R.S. Phillips [2, 3, 7]. Sergey's approach was typical for the St. Petersburg mathematical school:

Start with the simplest possible example still possessing the main features of the problem and proceed gradually to more sophisticated and complicated cases.

Working alongside this approach can be compared with the siege of a fortress, where simple examples remind one of the falling of certain bastions without which the fortress can no longer be defended. When Sergey passed away, he left piles of papers containing calculations of numerous simple examples—they were used to get intuition on what was going on, that is, how some of the bastions were conquered.

But taking down bastions was just a beginning of a long journey, often leading to a complete solution of the problem, removing unnecessary assumptions. On the other hand, generality was never to contradict clarity of the presented mathematics.

He was never satisfied until the result could be formulated in a clear and elegant way.

Sergey liked to work on several different problems at the same time, sometimes with completely unrelated teams of collaborators. This gave him the unique opportunity to gain general insights into mathematics, but one cannot help wonder how he was able to keep control over the entire network simultaneously.

Sergey's Students

When working with students, Sergey was extremely generous. At every conference he co-organized, he invited several young lecturers, thereby helping talented mathematicians at the beginning of their career to find their way in science. After lectures, Sergey liked to provide detailed comments without requiring any acknowledgment of his help. His encyclopedic knowledge of mathematical analysis allowed him to find unexpected connections and relations. Every thesis supervised by Sergey contains non-trivial mathematical results, many of the Bachelor and Master theses led to scientific publications in reputable journals. Traveling around the world, Sergey helped his colleagues to supervise students: In fact, one can frequently find his traces in students' works, where he was not an official supervisor. Unfortunately, we failed to be able to compile a complete list of all Bachelor and Master students who worked under Sergey's supervision; picking just a few of them did not seem appropriate.

Sergey supervised a total of ten Ph.D. students (see Sergey's Curriculum Vitae). It is remarkable that most of these students remained in academia, despite sometimes difficult circumstances.

The Swedish Connection

Leningrad mathematicians traditionally had a very special relation to Scandinavia, in general, and Stockholm University, in particular. This can be explained not only by geographical reasons, but also by the cooperation agreement between the Universities of Stockholm and Leningrad signed in 1981 by Boris Pavlov, at that time the Vice-Rector of Leningrad University, an agreement that has lasted over 40 years. Therefore, it is not surprising that Sergey Naboko published his first article abroad in *Arkiv för Matematik* (a renowned journal published by the Institute Mittag-Leffler) in 1987 [5]. It should be emphasized that the manuscript was submitted already in October 1984, a few months before Gorbachev became General Secretary of the CPSU and *perestroika* started. It is not surprising that the publication took more than 2 years—the postal service between the former USSR and the West was unreliable and scientists usually asked colleagues to carry abroad manuscripts and article proofs in their luggage.



Fig. 3 At Institut Mittag-Leffler Pavel Kurasov, Stanislav Kupin, Sergey Naboko, Vladimir Mazya, Tatyana Shaposhnikova, Brian Davies, Luis Silva, Frantisek Stampach, Vladimir Sloushch, Sergey Simonov. ©Ekaterina Vasilyeva

Sergey's first visit abroad was to Stockholm University in the Spring of 1991 as a guest of Professor Jaak Peetre. Sergey valued Swedish mathematics very highly, especially mathematical analysis, which of course has a unique tradition in Sweden. Sergey arrived in Sweden when his professorship at Leningrad University was already decided, but not yet formally approved. He was a fellow at the Department of Mathematics at Stockholm University within the cooperation agreement mentioned above, but spent a lot of time at the Institut Mittag-Leffler, where Jaak Peetre together with Svante Janson organized a 1-year program *Operator Theory and Complex Analysis*. At least half of the participants in the program came from the former Soviet Union, and one could easily get an impression that Nikolskii's seminar temporarily changed its location. Sergey fell in love with Sweden, which can partially be explained by the fact that half of the month he stayed at the Institut Mittag-Leffler with its wonderful working atmosphere, rich library, beautiful surroundings, and kind administration, taking care of all the guests. One of his deepest impressions was a visit to the Royal Opera House, to which he returned many times since. Sergey did not feel constrained by the beautifully dressed public

and even met Maya Plisetskaya calling her *Our Goddess* (Наша богиня), long before the book with the same title was published [6].

During the 30 years since then, Sergey visited Sweden on a regular basis collaborating with Pavel Kurasov (Stockholm University, 1996–2001, 2011–2019, Lund University, 2001–2010), Ari Laptev, and Oleg Safronov (Royal Institute of Technology, 2001–2011). He became a true member of the Swedish mathematical community and tried to learn more about the history of Swedish mathematics, always emphasizing the contributions by Swedish mathematicians in his lectures. One of the outstanding Swedish mathematicians even encouraged him to apply for a permanent position, but Sergey did not want to leave his mother, with whom he was especially close, taking care of her when it became necessary. When Margarita Nikolaevna died, Sergey went to Sweden for the mourning period.

In 2017 Sergey received the prestigious position as a Wallenberg Guest Professor, to be spent at Stockholm University, collaborating with Pavel Kurasov. Unfortunately, serious illness first forced him to postpone the position and then slowed down his activity. It became difficult for Sergey to use all the wonderful opportunities connected with the position. Sergey liked Sweden and Stockholm so much that he used this grant, originally intended for 1 year, to stay almost 2 years. He passed away in St. Petersburg while still being a Guest Professor at Stockholm University.

In June of 2023, a Wallenberg Symposium will be organized in Sergey's memory.



Fig. 4 At seashore in San Sebastian. ©Ekaterina Vasilyeva

Malcolm Brown (1946–2022)

During the time period in which this volume was collated, Malcolm Brown, one of the editors, died on January 14, 2022. Malcolm was born in Aberdare, South Wales, on January 11, 1946. He studied in Cardiff under the supervision of Des Evans and obtained his PhD in 1975 with a thesis on deficiency indices for ordinary differential operators. Following a spell working for the National Health Service, Malcolm returned to Cardiff University in 1986. He worked in the School of Computer Science, being promoted to a chair in 2003, and was an active member of staff up until his death. Despite his position in computer science, the main focus of his research remained in spectral theory and differential equations. Malcolm had a talent for bringing people together to create fruitful collaborations. Many people will fondly remember the numerous conferences in spectral theory he organized, particularly those in Gregynog between 1993 and 2014.

Malcolm likely first met Sergey during a sabbatical he spent in Birmingham, Alabama, in 2000. They remained close over the next two decades with Sergey visiting Cardiff many times and several of Sergey's former students spending extended periods in Cardiff. Their initial collaboration was on inverse resonance problems with Rudi Weikard. Following a successful EPSRC application together with Marco Marletta, which also brought Ian Wood to Cardiff as a postdoc in 2006, much of their later collaboration focused on boundary triples. This area of research led to Sergey's EU Marie Curie Fellowship which he spent at the University of Kent 2013–2015 and is the topic of one of the articles in this volume.

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Curriculum Vitae

(based on the application to the Knut and Alice Wallenberg Foundation)

Sergey Naboko

(Vuritsa (Вьрица), Russia, January 2, 1950

—St. Petersburg (Санкт-Петербург), Russia, December 24, 2020)

Citizenship: Russian citizen.

Educational qualification:

MSc, 1972

(Dept. of Mathematical Physics, Faculty of Physics, Leningrad University)

Academic degrees:

- Ph.D. in Mathematical Physics, 1977
(Leningrad Branch of Steklov Math. Inst.)
Supervisor: Prof. B.S.Pavlov
Title: “*Functional model of nonselfadjoint operator and its application to spectral analysis and scattering theory*”
Referees:
 - Prof. A.A.Kostuchenko (Moscow University)
 - Prof. N.K.Nikolskii (Leningrad Branch of Steklov Math. Inst.)
 - Prof. V.M.Adamyan (University of Odessa)
- Senior Doctorate in Math. Analysis, 1987
(Leningrad Branch of Steklov Math. Inst.)
Title: “*Operator-valued R-functions and their applications to spectral analysis*”
Referees:
 - Acad. V.A.Marchenko (Kharkov Inst. of Low Temperatures)
 - Prof. V.M.Adamyan (University of Odessa)
 - Prof. N.K.Nikolskii (Leningrad Branch of Steklov Math. Inst.)

Professional appointments:

- 1972–1974 Junior researcher, Leningrad Branch of Steklov Math. Inst.
- 1974–1977 PhD student, Leningrad Branch of Steklov Math. Inst.
- 1977–1986 Assistant professor, Leningrad State Univ.
- 1986–1991 Associate professor, Leningrad State Univ.
- 1991–2019 Professor, St.Petersburg (Leningrad) State Univ.

Visits abroad:

- Visiting positions:
 - 2004–2009 Visiting professor at the University of Alabama at Birmingham (USA);
 - 2013–2015 Marie Curie Fellow at the University of Kent at Canterbury (UK);
 - 2019–2020 Wallenberg guest professor at Stockholm University (Sweden).
- Selected long term visits:
 - **Belgium:** Solvay Inst. for Physics and Chemistry, Brussels;
 - **France:** Bordeaux University; Ecole Polytechnique, Paris; University Paris-VII; University Paris-Sud;
 - **Germany:** University of Regensburg;
 - **Ireland:** Dublin Institute of Technology;
 - **Israel:** Weizmann Institute of Science, Rehovot;
 - **Mexico:** UNAM, Mexico City;
 - **New Zealand:** Auckland University;
 - **Norway:** NTNU, Trondheim;
 - **Poland:** Math.Inst.Acad.Sci., Warsaw,
 - **Portugal:** Lisbon Technical University;
 - **Sweden:** Lund Institute of Technology; Mittag-Leffler Institute, Djursholm, Royal Institute of Technology, Stockholm; Stockholm Univ. (reg visits 1998–2001, 2011–2019);
 - **UK:** University of Bath; Cardiff University;
 - **USA:** University of Alaska at Fairbanks; Columbia University, Columbia; Caltech, Pasadena; Georgia Tech.; Clark Atlanta University.

Research interests:

1. Scattering theory for self-adjoint and non-self-adjoint operators, perturbation theory for continuous spectrum. Spectral shift function and scattering matrix properties
2. Embedding of point spectrum into continuous spectrum. Investigation of the singular spectrum of self-adjoint operators
3. Nevanlinna class operator-functions in perturbation theory
4. Dissipative operators. Functional models and characteristic functions of operators. Spectral analysis of the Boltzmann transport operators
5. Liapunov stability problems, stability problems from hydrodynamics

6. Spectral theory of non-dissipative operators, theory of operators with almost Hermitian spectrum
7. Wave propagation in inhomogeneous media
8. Localization for Anderson models in random Schrödinger operators theory
9. The spectral structure for the matrix-differential operators from magnetohydrodynamics
10. Spectral analysis of Jacobi matrices. Spectral properties of Block Jacobi Operators
11. Inverse problems in terms of the boundary triples. Detectable subspaces and their properties
12. Semiclassical analysis of Regge trajectories for singular and nonsingular potentials
13. Spectral theory of quantum graphs
14. Homogenization theory for selfadjoint operators with stiff and soft regimes

Teaching activity:

(at St.Petersburg State Univ., Univ. of Alabama at Birmingham, Univ. of Kent at Canterbury)

- General lecture courses in various mathematical subjects, including: Mathematical Analysis, Linear Algebra, Mathematical Physics, Complex Analysis, etc.
- Special (advanced) courses of lectures given in: scattering theory, functional analysis, spectral theory of selfadjoint operators, series courses on dissipative operators: I, II, III, IV, V, perturbation theory, operator theory, linear algebra, operator-valued R-functions and their applications, spectral theory of PDO, Sobolev spaces and PDO
- Seminars given in: numerical methods and computing, mathematical analysis, ordinary differential equations, linear algebra, operator theory, integral operator theory mathematical physics
- Numerous courses in Pure Mathematics and Applications (around 20) given at University of Alabama at Birmingham (USA) in 2005–2009
- 3 courses of lectures at the University of Kent at Canterbury (UK) in 2013–2015

PhD students

1. **Michail Faddeev** MSc 1984, PhD 1993
2. **Serguei I. Iakovlev** Master 1987, PhD 1992
3. **Alexander V. Kiselev** PhD 2000
4. **Roman Romanov** PhD 1999
5. **Vladimir Ryzhov** MSc 1988, PhD 1994
6. **Luis O Silva** MSc 1996, PhD 2003
7. **Sergey Simonov** MSc 2007, PhD 2010
8. **Vladimir Veselov** MSc, PhD 1986
9. **Christoph Fischbacher** PhD 2017 (co-supervisor)
10. **Edmund Judge** PhD 2017 (co-supervisor)

Major Grants

- 1994–2017 Polish Academy of Sciences
- 1998–2004 Royal Swedish Academy of Sciences
- 2011–2017 Stockholm University
- 1997–2018 Russian Foundation of Fundamental Researches
- 2016–2018 Russian Science Foundation
- 2013–2015 EU Marie Curie Research Fellowship Grant

Editorial Board for International Journals

- Complex Analysis and Operator Theory, Associated editor
- Mathematical Physics: Analysis and Geometry (2004–20014)
- Opuscula Mathematica
- Selected issues of ‘Operator Theory: Advances and Applications’ (2003–2014)

Referee for Journals (Selected)

Journal of Functional Analysis, Proceedings of the London Mathematical Society, Transactions of AMS, Mathematical Physics: Analysis and Geometry, Journal of Approximation Theory, Constructive Approximation, Letters in Mathematical Physics, Journal of Mathematical Physics, Saint Petersburg Mathematical Journal, Journal of Physics A: Math.and Theoretical, Mathematische Nachrichten, Communications in Math.Physics, Integral Equations and Operator Theory, Journal of Operator Theory, Opuscula Mathematica, Complex Analysis and Operator Theory, etc.

Organised Conferences

- Founder (together with J. Janas) of the Workshop series” Spectral Theory and its Applications” at Banach Center (Warsaw): 1998, 1999, 2000, 2001, 2003, 2005, 2007
- Founder (together with J. Janas, P. Kurasov and A. Laptev) of the conference series: Operator Theory, Analysis and Mathematical Physics (OTAMP):
 - OTAMP2002, Bedlewo, Poland, May 11–18, 2002
 - OTAMP2004, Bedlewo, Poland, July 6–12, 2004
 - OTAMP2006, Lund, Sweden, June 15–22, 2006
 - OTAMP2008, Bedlewo, Poland, June 15–22, 2008
 - OTAMP2010, Bedlewo, Poland, August 5–12, 2010

- OTAMP2012, CRM Barcelona, Spain, June 11–14, 2012
- OTAMP2014, Stockholm Univ., Sweden, July 7–11, 2014
- OTAMP2016, Euler Institute, St. Petersburg, Russia, August 1–7, 2016
- OTAMP2020, UNAM, Mexico City, Mexico, January 8–14, 2020

List of Publications

Sergey Naboko

Research Papers

- [1] S. N. Naboko, *Estimates of the number of geodesics on a fundamental domain of the modular group*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **37** (1973), 43–46 (Russian). Differential geometry Lie groups and mechanics. MR0390966
- [2] S. N. Naboko, *The nonselfadjoint Friedrichs model*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **39** (1974), 40–58 (Russian). Investigations on linear operators and the theory of functions, IV. MR0343109
- [3] S. N. Naboko, *Analytic continuation to the second sheet of the Fredholm determinant of the resolvent of a Schrödinger operator in R^3* , Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **47** (1974), 81–89, 186, 192 (Russian, with English summary). Investigations on linear operators and the theory of functions, V. MR0382870
- [4] S. N. Naboko, *Absolutely continuous spectrum of a nondissipative operator, and a functional model. I*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **65** (1976), 90–102, 204–205 (Russian, with English summary). Investigations on linear operators and the theory of functions, VII. MR0500225
- [5] S. N. Naboko, *On the spectral analysis of nonselfadjoint operators*, Dokl. Akad. Nauk SSSR **232** (1977), no. 1, 36–39 (Russian). MR0438157
- [6] S. N. Naboko, *Wave operators for nonselfadjoint operators and a functional model*, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **69** (1977), 129–135, 275 (Russian, with English summary). Boundary value problems of mathematical physics and related questions in the theory of functions, 10. MR0477816
- [7] S. N. Naboko, *Absolutely continuous spectrum of a nondissipative operator, and a functional model. II*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **73** (1977), 118–135, 232–233 (1978) (Russian, with English summary). Investigations on linear operators and the theory of functions, VIII. MR513172
- [8] S. N. Naboko, *The separability of spectral subspaces of a nonselfadjoint operator*, Dokl. Akad. Nauk SSSR **239** (1978), no. 5, 1052–1055 (Russian). MR0482298
- [9] S. N. Naboko, *Functional model of perturbation theory and its applications to scattering theory*, Trudy Mat. Inst. Steklov. **147** (1980), 86–114, 203 (Russian). Boundary value problems of mathematical physics, 10. MR573902
- [10] S. N. Naboko, *On the singular spectrum of a nonselfadjoint operator*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **113** (1981), 149–177, 266 (Russian, with English summary). Investigations on linear operators and the theory of functions, XI. MR629838

- [11] S. N. Naboko, *Conditions for similarity to unitary and selfadjoint operators*, Funktsional. Anal. i Prilozhen. **18** (1984), no. 1, 16–27 (Russian). MR739086
- [12] S. N. Naboko, *Uniqueness theorems for operator-valued functions with positive imaginary part and the singular spectrum in the selfadjoint Friedrichs model*, Dokl. Akad. Nauk SSSR **275** (1984), no. 6, 1310–1313 (Russian). MR746376
- [13] S. N. Naboko, *Schrödinger operators with decreasing potential and with dense point spectrum*, Dokl. Akad. Nauk SSSR **276** (1984), no. 6, 1312–1315 (Russian). MR753371
- [14] S. N. Naboko, *Similarity problem and the structure of the singular spectrum of nondissipative operators*, Lecture Notes in Math **1043** (1984), 147–151.
- [15] V. F. Veselov and S. N. Naboko, *The determinant of the characteristic function of a nonselfadjoint operator*, Funktsional. Anal. i Prilozhen. **19** (1985), no. 4, 80–81 (Russian). MR820090
- [16] S. N. Naboko, *On the dense point spectrum of Schrödinger and Dirac operators*, Teoret. Mat. Fiz. **68** (1986), no. 1, 18–28 (Russian, with English summary).
- [17] V. F. Veselov and S. N. Naboko, *The determinant of the characteristic function and the singular spectrum of a nonselfadjoint operator*, Mat. Sb. (N.S.) **129(171)** (1986), no. 1, 20–39, 159, DOI 10.1070/SM1987v057n01ABEH003053 (Russian); English transl., Math. USSR-Sb. **57** (1987), no. 1, 21–41. MR830093
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- [19] S. N. Naboko, *Conditions for the existence of wave operators in the non-selfadjoint case, Wave propagation. Scattering theory* (Russian), Probl. Mat. Fiz., vol 12, Leningrad. Univ., Leningrad, 1987, pp. 132–155, 258 (Russian). MR923975
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- [21] S. N. Naboko, *Estimates in symmetrically normed ideals for the difference of powers of accretive operators*, Vestnik Leningrad. Univ. Mat. Mekh. Astronom. **vyp. 2** (1988), 40–45, 131 (Russian, with English summary); English transl., Vestnik Leningrad Univ. Math. **21** (1988), no. 2, 53–59. MR965100
- [22] S. N. Naboko, *On the boundary values of analytic operator-valued functions with a positive imaginary part*, Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) **157** (1987), no. Issled. po Linein. Operator. i Teorii Funktsii. XVI, 55–69, 179, DOI 10.1007/BF01463185 (Russian, with English summary); English transl., J. Soviet Math. **44** (1989), no. 6, 786–795. MR899274
- [23] S. N. Naboko, *Nontangential boundary values of operator R -functions in a halfplane*, Algebra i Analiz **1** (1989), no. 5, 197–222 (Russian); English transl., Leningrad Math. J. **1** (1990), no. 5, 1255–1278. MR1036844
- [24] S. Ya. Naboko, *Estimates in operator classes for the difference of functions from the Pick class of accretive operators*, Funktsional. Anal. i Prilozhen. **24** (1990), no. 3, 26–35, 96, DOI 10.1007/BF01077959 (Russian); English transl., Funct. Anal. Appl. **24** (1990), no. 3, 187–195 (1991). MR1082028
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