LEARNING MADE EASY

3rd Edition

Calculus II

for dummies®
A Wiley Brand

Solve area problems using definite and indefinite integrals

Tackle *u*-substitution, integration by parts, and partial fractions

Review Pre-Calculus and Calculus I concepts

Mark Zegarelli
Math Tutor Extraordinaire

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Contents at a Glance

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Introduction

Calculus is the great Mount Everest of math. Most of the world is content to just gaze upward at it in awe. But only a few brave souls attempt the ascent.

Or maybe not.

In recent years, calculus has become a required course not only for math, engineering, and physics majors, but also for students of biology, economics, psychology, nursing, and business. Law schools and MBA programs welcome students who've taken calculus because it demonstrates discipline and clarity of mind. High schools now have multiple math tracks that include calculus, from the basic college prep track to the AP tracks that prepare students for the Advanced Placement exam.

So perhaps calculus is more like a well-traveled Vermont mountain, with lots of trails and camping spots, plus a big ski lodge on top. You may need some stamina to conquer it, but with the right guide (this book, for example!), you're not likely to find yourself swallowed up by a snowstorm half a mile from the summit.

About This Book

You *can* learn calculus. That's what this book is all about. In fact, as you read these words, you may well already be a winner, having passed a course in Calculus I. If so, then congratulations and a nice pat on the back are in order.

Having said that, I want to discuss a few rumors you may have heard about Calculus II:

- Calculus II is harder than Calculus I.
- Calculus II is harder, even, than either Calculus III or Differential Equations.
- Calculus II is more frightening than having your home invaded by zombies in the middle of the night and will result in emotional trauma requiring years of costly psychotherapy to heal.

Now, I admit that Calculus II is harder than Calculus I. Also, I may as well tell you that many — but not all — math students find it to be harder than the two semesters of math that follow. (Speaking personally, I found Calc II to be easier than Differential Equations.) But I'm holding my ground that the long-term psychological effects of a zombie attack far outweigh those awaiting you in any one-semester math course.

The two main topics of Calculus II are integration and infinite series. *Integration* is the inverse of differentiation, which you study in Calculus I. (For practical purposes, integration is a method for finding the area of unusual geometric shapes.) An *infinite series* is a sum of numbers that goes on forever, like $1 + 2 + 3 + \ldots$ or $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$. Roughly speaking, most teachers focus on integration for the first two-thirds of the semester and infinite series for the last third.

This book gives you a solid introduction to what's covered in a college course in Calculus II. You can use it either for self-study or while enrolled in a Calculus II course.

So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section in case you want to refresh yourself on the basics.

Here are two pieces of advice for math students (remember them as you read the book):

- **Study a little every day.** I know that students face a great temptation to let a book sit on the shelf until the night before an assignment is due. This is a particularly poor approach for Calc II. Math, like water, tends to seep in slowly and swamp the unwary!

  So, when you receive a homework assignment, read over every problem as soon as you can and try to solve the easy ones. Go back to the harder problems every day, even if it's just to reread and think about them. You'll probably find that over time, even the most opaque problem starts to make sense.

- **Use practice problems for practice.** After you read through an example and think you understand it, copy the problem down on paper, close the book, and try to work it through. If you can get through it from beginning to end, you're ready to move on. If not, go ahead and peek, but then try solving the problem later without peeking. (Remember, on exams, no peeking is allowed!)

Conventions Used in This Book

Throughout the book, I use the following conventions:

- *Italicized* text highlights new words and defined terms.
- **Boldfaced** text indicates keywords in bulleted lists and the action parts of numbered steps.
- `Monofont` text highlights web addresses.
- Angles are measured in radians rather than degrees, unless I specifically state otherwise. (See Chapter 2 for a discussion about the advantages of using radians for measuring angles.)

What You're Not to Read

All authors believe that each word they write is pure gold, but you don't have to read every word in this book unless you really want to. You can skip over sidebars (those gray shaded boxes) where I go off on a tangent, unless you find that tangent interesting. Also feel free to pass by paragraphs labeled with the Technical Stuff icon.

If you're not taking a class where you'll be tested and graded, you can skip paragraphs labeled with the Tip icon and jump over extended step-by-step examples. However, if you're taking a class, read this material carefully and practice working through examples on your own.

Foolish Assumptions

Not surprisingly, a lot of Calculus II builds on topics introduced in Calculus I and Pre-Calculus. So here are the foolish assumptions I make about you as you begin to read this book:

- If you're a student in a Calculus II course, I assume that you passed Calculus I. (Even if you got a D-minus, your Calc I professor and I agree that you're good to go!)
- If you're studying on your own, I assume that you're at least passably familiar with some of the basics of Calculus I.

I expect that you know some things from Calculus I, Algebra, and even Pre-Algebra, but I don't throw you in the deep end of the pool and expect you to swim or drown. Chapter 2 contains a ton of useful Algebra and Pre-Algebra tidbits that you may have missed the first time around. And in Chapter 3, I give you a review of the most important topics from Calculus I that you're sure to need in Calculus II. Furthermore, throughout the book, whenever I introduce a topic that calls for previous knowledge, I point you to an earlier chapter or section so you can get a refresher.

Icons Used in This Book

Here are four useful icons to help you navigate your way through the book:

TIP

Tips are helpful hints that show you the easy way to get things done. Try them out, especially if you're taking a math course.

REMEMBER

This icon points out key ideas that you need to know. Make sure you understand these ideas before reading on.

TECHNICAL STUFF

This icon points out interesting trivia that you can read or skip over as you like.

WARNING

Warnings flag common errors that you want to avoid. Get clear where these traps are hiding so you don't fall in.

EXAMPLE

Examples walk you through a particular math exercise designed to illustrate a particular topic. Practice makes perfect!

Beyond the Book

In addition to the introduction you're reading right now, this book comes with a free, access-anywhere Cheat Sheet containing information worth remembering about Calculus II. To get this Cheat Sheet, simply go to `www.dummies.com` and type **Calculus II For Dummies Cheat Sheet** in the Search box.

Where to Go from Here

You can use this book either for self-study or to help you survive and thrive in a course in Calculus II.

If you're taking a Calculus II course, you may be under pressure to complete a homework assignment or study for an exam. In that case, feel free to skip right to the topic that you need help with. Every section is self-contained, so you can jump right in and use the book as a handy reference. And when I refer to information that I discuss earlier in the book, I give you a brief review and a pointer to the chapter or section where you can get more information if you need it.

If you're studying on your own, I recommend that you begin with Chapter 1, where I give you an overview of the entire book, and then read the chapters from beginning to end. Jump over Chapters 2 and 3 if you feel confident about your grounding in the math leading up to Calculus II. And, of course, if you're dying to read about a topic that's later in the book, go for it! You can always drop back to an easier chapter if you get lost.

1

Introduction to Integration

IN THIS PART . . .

See Calculus II as an ordered approach to finding the area of unusual shapes on the *xy*-graph

Use the definite integral to clearly define an area problem

Slice an irregularly shaped area into rectangles to approximate area

Review the math you need from Pre-Algebra, Algebra, Pre-Calculus, and Calculus I

IN THIS CHAPTER

» Measuring the area of shapes by using classical and analytic geometry

» Using integration to frame the area problem

» Approximating area using Riemann sums

» Applying integration to more complex problems

» Seeing how differential equations are related to integrals

» Looking at sequences and series

Chapter 1

An Aerial View of the Area Problem

Humans have been measuring the area of shapes for thousands of years. One practical use for this skill is measuring the area of a parcel of land. Measuring the area of a square or a rectangle is simple, so land tends to get divided into these shapes.

Discovering the area of a triangle, circle, or polygon is also relatively easy, but as shapes get more unusual, measuring them gets harder. Although the Greeks were familiar with the conic sections — parabolas, ellipses, and hyperbolas — they couldn't reliably measure shapes with edges based on these figures.

René Descartes's invention of analytic geometry — studying lines and curves as equations plotted on the *xy*-graph — brought great insight into the relationships among the conic sections. But even analytic geometry didn't answer the question of how to measure the area inside a shape that includes a curve.

This bit of mathematical history is interesting in its own right, but I tell the story in order to give you, the reader, a sense of what drove those who came up with the concepts that eventually got bundled together as part of a standard Calculus II course. I start out by showing you how *integral calculus* (*integration* for short) was developed from attempts to answer this basic question of measuring the area of weird shapes, called the *area problem.* To do this, you will discover how to approximate the area under a parabola on the *xy*-graph in ways that lead to an ordered system of measuring the exact area under any function.

First, I frame the problem using a tool from calculus called the *definite integral*. I show you how to use the definite integral to define the areas of shapes you already know how to measure, such as circles, squares, and triangles.

With this introduction to the definite integral, you're ready to look at the practicalities of measuring area. The key to approximating an area that you don't know how to measure is to slice it into shapes that you do know how to measure — for example, rectangles. This process of slicing unruly shapes into nice, crisp rectangles — called finding a *Riemann sum* — provides the basis for calculating the exact value of a definite integral.

At the end of this chapter, I give you a glimpse into the more advanced topics in a basic Calculus II course, such as finding volume of unusual solids, looking at some basic differential equations, and understanding infinite series.

Checking Out the Area

Finding the area of certain basic shapes — squares, rectangles, triangles, and circles — is easy using geometric formulas you typically learn in a geometry class. But a reliable method for finding the exact area of shapes containing more esoteric curves eluded mathematicians for centuries. In this section, I give you the basics of how this problem, called the *area problem,* is formulated in terms of a new concept, the definite integral.

The *definite integral* represents the area of a region bounded by the graph of a function, the *x*-axis, and two vertical lines located at the *bounds of integration.* Without getting too deep into the computational methods of integration, I give you the basics of how to state the area problem formally in terms of the definite integral.

Comparing classical and analytic geometry

In *classical geometry*, you discover a variety of simple formulas for finding the area of different shapes. For example, Figure 1-1 shows the formulas for the area of a rectangle, a triangle, and a circle.

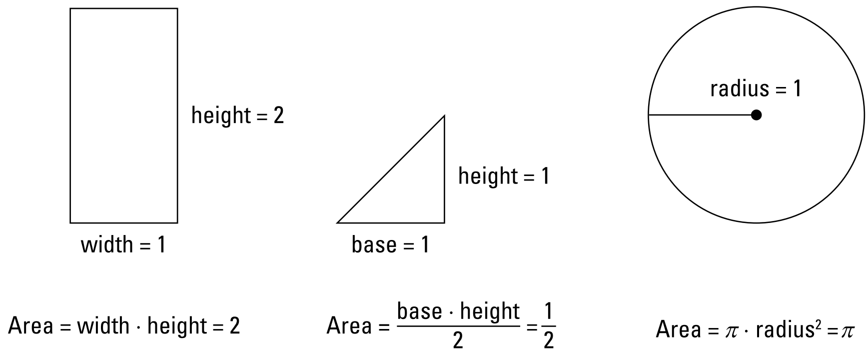

FIGURE 1-1: Formulas for the area of a rectangle, a triangle, and a circle.

On the *xy*-graph, you can generalize the problem of finding area to measure the area under any continuous function of x. To illustrate how this works, the shaded region in Figure 1-2 shows the area under the function $f(x)$ between the vertical lines $x = a$ and $x = b$.

The area problem is all about finding the area under a continuous function between two constant values of x that are called the *bounds of integration*, usually denoted by a and b. This problem is generalized as follows:

$$\text{Area} = \int_a^b f(x)\,dx$$

WISDOM OF THE ANCIENTS

Long before calculus was invented, the ancient Greek mathematician Archimedes used his *method of exhaustion* to calculate the exact area of a segment of a parabola. He was also the first mathematician to come up with an approximation for π (pi) within about a 0.2% margin of error.

Indian mathematicians also developed *quadrature* methods for some difficult shapes before Europeans began their investigations in the 17th century.

These methods anticipated some of the methods of calculus. But before calculus, no single theory could measure the area under arbitrary curves.

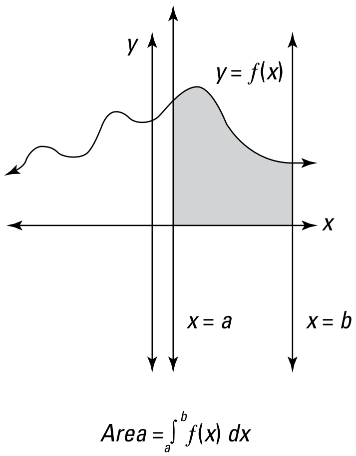

FIGURE 1-2: A typical area problem.

In a sense, this formula for the shaded area isn't much different from the geometric formulas you already know. It's just a formula, which means that if you plug in the right numbers and calculate, you get the right answer.

For example, suppose you want to measure the area under the function x^2 between $x = 1$ and $x = 5$. (You can see what this area looks like by flipping a few pages forward to Figure 1-5.) Here's how you plug these values into the area formula shown previously:

$$\text{Area} = \int_1^5 x^2 \, dx$$

The catch, however, is how exactly to calculate using this new symbol. As you may have figured out, the answer is on the cover of this book: calculus. To be more specific, *integral calculus,* or *integration.*

REMEMBER

Most typical Calculus II courses taught at your friendly neighborhood college or university focus on integration — the study of how to solve the area problem. So, if what you're studying starts to get confusing (and to be honest, you probably will get confused somewhere along the way), try to relate what you're doing to this central question: "How does what I'm working on help me find the area under a function?"

Finding definite answers with the definite integral

You may be surprised to find out that you've known how to integrate some functions for years without even knowing it. (Yes, you can know something without knowing that you know it.)

For example, find the rectangular area under the function $y = 2$ between $x = 1$ and $x = 4$, as shown in Figure 1-3.

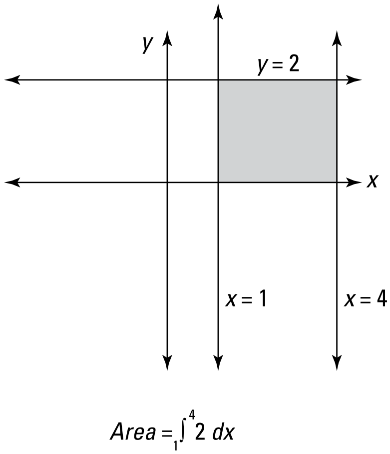

FIGURE 1-3: The rectangular area under the function $f(x) = 2$, between $a = 1$ and $b = 4$ equals 6.

This is just a rectangle with a base of 3 and a height of 2, so its area is 6. But this is also an area problem that can be stated in terms of integration as follows:

$$\text{Area} = \int_1^4 2\, dx = 6$$

As you can see, the function I'm integrating here is $f(x) = 2$. The bounds of integration are 1 and 4 (notice that the greater value goes on top). You already know that the area is 6, so you can solve this calculus problem without resorting to any scary or hairy methods. But you're still *integrating,* so please pat yourself on the back, because I can't quite reach it from here.

The following expression is called a *definite integral:*

$$\int_1^4 2\, dx$$

For now, don't spend too much time worrying about the deeper meaning behind the $\int$ symbol or the *dx* (which you may fondly remember from your days spent differentiating in Calculus I). Just think of $\int$ and *dx* as notation placed around a function — notation that means *area.*

What's so definite about a definite integral? Two things, really:

» **You definitely know the bounds of integration** (in this case, 1 and 4). Their presence distinguishes a definite integral from an indefinite integral, which you find out about in Chapter 5. Definite integrals always include the bounds of integration; indefinite integrals never include them.

- **A definite integral definitely equals a number** (assuming that its limits of integration are also numbers). This number may be simple to find or difficult enough to require a room full of math professors scribbling away with #2 pencils. But, at the end of the day, a number is just a number. And, because a definite integral is a measurement of area, you should expect the answer to be a number.

TECHNICAL STUFF

When the limits of integration *aren't* numbers, a definite integral doesn't necessarily equal a number. For example, expressions such as k and $2k$ might be used as limits of integration to stand in for constants. In such cases, the answer to a definite integral may include the letter k. Similarly, a definite integral whose limits of integration are $\sin\theta$ and $2\sin\theta$ would most likely equal a trig expression that includes θ. To sum up, because a definite integral represents an area, it always equals a number — though you may or may not be able to compute this number.

As another example, find the triangular area under the function $y = x$, between $x = 0$ and $x = 8$, as shown in Figure 1-4.

This time, the shape of the shaded area is a triangle with a base of 8 and a height of 8, so its area is 32 (because the area of a triangle is half the base times the height). But again, this is an area problem that can be stated in terms of integration as follows:

$$\text{Area} = \int_0^8 x\,dx = 32$$

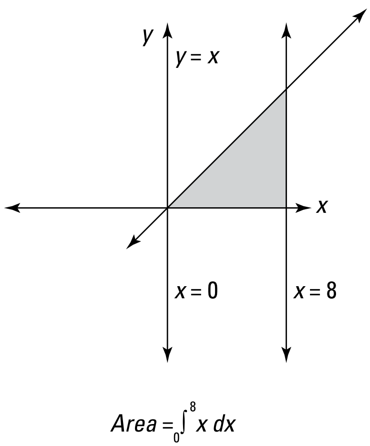

FIGURE 1-4: The triangular area under the function $y = x$, between $x = 0$ and $x = 8$ equals 32.