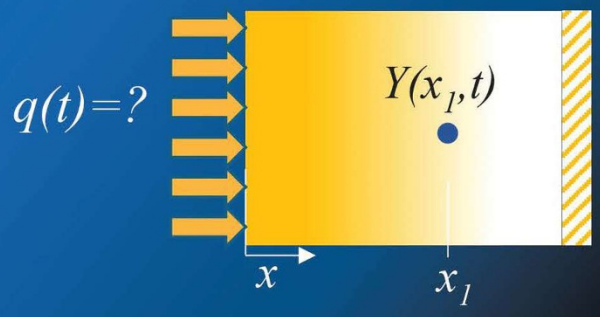
Second Edition

Inverse Heat Conduction

Ill-Posed Problems

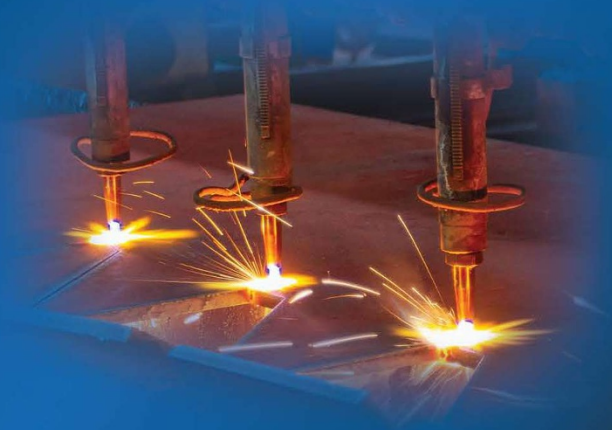
Keith A. Woodbury • Hamidreza Najafi
Filippo de Monte • James V. Beck

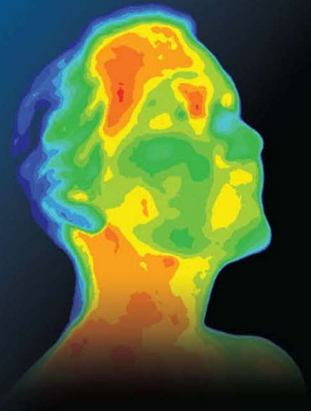


q=FY

T=Xq

Y= T+ε





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To engineers and researchers who thirst for knowledge and pass this way, with undying gratitude for those who came before and illuminated the path, and with special appreciation to my colleague, mentor, and co-author: James V. Beck.

K. A. W.

To my wife, Mehrasa; and to my mother and father, Kobra and Mohammadsadegh.

H. N.

To my wife Emanuela and my children Federica Flora, Matilde, and Fabrizio Nicola; to my sisters Saveria and Alessia.

F. d. M.

To my wife, Barbara; children, Sharon and Douglas; and father and mother, Peter and Louise Beck

J. V. B.

In Memorium

James V. Beck (1930–2022)

Prof. Beck was the inspirational and motivational force in the development of the two editions of Inverse Heat Conduction: Ill-posed problems. Sadly, he passed away during the last stages of copyediting of this second edition. His spirited enthusiasm and boundless energy for scientific research will not soon be forgotten.

Contents

List of Figures

Nomenclature

$\mathbf{1}$	vector of all ones
a	a constant
A	counting integer for number of decimal places of precision in building block solution (−)
$\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$	coefficient matrices from numerical discretization of heat conduction equation
b	a constant
Bi	Biot number, hL/k (−)
c	specific heat capacity (J $kg^{-1}K^{-1}$)
C	volumetric heat capacity, ρc (J $m^{-3}K^{-1}$)
ceil(.)	MATLAB function that rounds the argument to the nearest integer greater than or equal to it (−)
d	deviation distance (m)
$\tilde{d}$	dimensionless deviation distance, d/L (−)
D_M^2	square of deterministic error for a single heat flux (Eq. 6.13)
E	relative error based on maximum temperature variation (−)
$E(.)$	expected value operator
E	depth, $0 < E < L$ (m)
$E_{q,\mathrm{bias}}^2$	square of the expected value of the bias error
$E_{q,\mathrm{rand}}^2$	square of the expected value of the random (variance) error
$E_{q,\mathrm{RMS}}$	root mean squared (RMS) error
f	filter coefficient
$f(.)$	arbitrary function
F	eigenfunction (−)
$\mathbf{F}$	filter matrix
Fo	Fourier number, $\alpha t/L^2$ (−)
g	filter coefficients (Chapter 5)
$g(.)$	arbitrary function
G	one-dimensional Green's function (m^{-1})
$\mathbf{G}$	filter matrix (X12 case in Chapter 5)
h	heat transfer coefficient (W $m^{-2}K^{-1}$)
$H(.)$	Heaviside unit step function (−)
H	temperature correction kernel (Chapter 10) (−)
$\mathbf{H}$	vector of temperature correction kernel values (−)
$\mathbf{H}_i$	discrete Tikhonov regularization matrix ($i = 0,1,2$)
i	counting integer for time steps (−)
$\mathbf{I}$	identity matrix
j	counting integer for space steps (−)
k	thermal conductivity (W $m^{-1}K^{-1}$)
J	number of sensors
K	gain coefficients in Future Times regularization
$\mathbf{K}$	Kalman gain matrix

L	length (m)
$\mathcal{L}(.)$	Laplace transform operator
m	counting integer in the x–direction (−)
m_f	number of required data points from future time steps for filter form solutions
$m_{\max}$	maximum number of terms in summations along x (−)
m_p	number of required data points from previous time steps for filter form solutions
M	M-th time step and maximum number of time steps (−); constant in Eq. (4.7)
n	counting integer in the y–direction; also number of data points (−)
n_p	number of pulses
$n_{\max}$	maximum number of terms in summations along y (−)
N	N-th space step; also maximum number of steps or times (−)
p	power-law exponent (−)
P	constant in Eq. (4.8)
$\mathbf{P}$	covariance matrix in Kalman filter solution
q	heat flux (W m^{-2})
$\tilde{q}$	dimensionless heat flux, q/q_0 (−)
$\mathbf{q}$, $\mathbf{q}_0$	vector of surface heat-flux components/values (W m^{-2})
Q^2	process noise variance
r	number of future times
R^2	measurement noise variance
R_q^2	mean squared error for vector of heat flux estimates (Eq. 6.9)
R_q	root mean squared error of heat flux estimates, $R_q = \sqrt{R_q^2}$
R_T^2	mean squared error for vector of measured temperature estimates (Eq. 6.36)
R_M^2	mean squared error of a single heat flux value
s	an integer
s_q	standard error
S	total number of terms in the computational analytical solution
S	sum of squared errors, e.g. $\mathbf{(Y–T)}^T\mathbf{(Y–T)}$
$\mathbf{S}$	measurement covariance in Kalman filter solution
t	time (s)
$\tilde{t}$	dimensionless time, $\alpha t/L^2$ (−)
T	temperature (K)
$\tilde{T}$	dimensionless temperature, $(T - T_{in})/(T_0 - T_{in})$ or $(T - T_{in})/(q_0 L/k)$ in Chapter 2 (−)
$\mathbf{T}$	vector of temperatures (K)
$tr[.]$	trace of a matrix
u	cotime, $t - \tau$ (s)
u_i	random error with zero mean and unity standard deviation
$\mathbf{v}$	measurement noise vector
$V(.)$	variance operator
V_α	the GCV function (Eq. 6.49)
$\mathbf{w}^k$	descent direction for iteration k in conjugate gradient method
$\mathbf{w}$	process noise vector
W	width (m)
$\tilde{W}$	aspect ratio, W/L (−)
W_0	width of “active” (heated or cooled) boundary (m)
$\tilde{W}_0$	dimensionless width of “active” boundary, W_0/L (−)
x	rectangular space coordinate (m)
$\tilde{x}$	dimensionless rectangular space coordinate, x/L (−)
x'	dummy variable for space (m)
X	sensitivity coefficient

$X_{M,i}$ heat flux-based sensitivity coefficient, $\partial T_M(x)/\partial q_{0,i}$ (°C m^2 W^{-1})
$X_{M,i,j}$ heat flux-based sensitivity coefficient, $\partial T_M(x, y)/\partial q_{0,i,j}$ (°C m^2 W^{-1})
$\mathbf{X}$ matrix of sensitivity coefficients $X_{M,i}$ (1D case) and $X_{M,i,j}$ (2D case) (°C m^2 W^{-1})
$\mathbf{X}_0$ vector of sensitivity coefficients $X_{M,0} = \partial T_M/\partial q_{0,0}$ for $q_{0,0}$ (Chapter 3) (°C m^2 W^{-1})
$\mathbf{X}_0$ matrix of sensitivity coefficients evaluated at surface $x = 0$ (Chapter 10) (°C m^2 W^{-1})
y rectangular space coordinate (m)
$\tilde{y}$ dimensionless rectangular space coordinate, y/L (−)
Y measured temperature (°C)
$\mathbf{Y}$ vector of measured temperatures (°C)
$\mathbf{y}$ vector of measured temperatures by a second sensor on the remote boundary (°C)
Z_h step sensitivity for heat transfer coefficient (Chapter 9) (m^2 °C^2 W^{-1})
$\mathbf{Z}$ vector of step sensitivity coefficients for h (m^2 °C^2 W^{-1})
$\mathbf{Z}$ sensitivity matrix for measured temperature boundary condition at remote boundary (Chapter 5)

Greek Symbols

α thermal diffusivity, k/C (m^{2}s^{-1})
α_i Tikhonov regularization coefficient ($i = 0,1,2$)
α_T Tikhonov regularization parameter for the time term (Chapter 8)
α_s Tikhonov regularization parameter for the space term (Chapter 8)
β eigenvalue in the x−direction; also a parameter (−)
β ratio of thermal properties (Eq. 10.20)
$\boldsymbol{\beta}$ vector of parameters
$\delta(.)$ Dirac delta function
δ level of error in measured temperatures for Morozov discrepancy principle
γ temperature rise per unit linear-in-time increase of surface heat-flux at $x = 0$ (with $x = L$ kept insulated) over the time $t_{ref} = \Delta t$, that is, $\tilde{T}_{\text{X22B20T0}} \times L/k$ with $\tilde{t}_{ref} = \Delta\tilde{t}$ (°C m^2 W^{-1})
Γ input gain in Kalman filter
Δ variation of a function (e.g. ΔT is the variation of the temperature function)
$\Delta\gamma_i$ forward difference of the γ temperature at time $i\Delta t$, $\gamma_{i+1} - \gamma_i$ (°C m^2 W^{-1})
Δt time step (s)
$\Delta\tilde{t}$ dimensionless time step, $\alpha\Delta t/L^2$ (−)
Δy space step along y (m)
$\Delta\tilde{y}$ dimensionless space step along y, $\Delta y/L$ (−)
$\Delta\phi_i$ forward difference of the ϕ temperature at time $i\Delta t$, $\phi_{i+1} - \phi_i$ (°C m^2 W^{-1})
$\Delta\varphi_i$ forward difference of the φ temperature at time $i\Delta t$, $\varphi_{i+1} - \varphi_i$ (−)
$\Delta(\Delta\gamma)_i$ central difference of the γ temperature at time $i\Delta t$, $\Delta\gamma_i - \Delta\gamma_{i-1}$ (°C m^2 W^{-1})
$\Delta_i\lambda$ forward difference at time $i\Delta t$ of the λ temperature, $\lambda_{i+1} - \lambda_i$ (°C m^2 W^{-1})
$\Delta_j\lambda$ backward difference at the j−th grid point of the λ temperature, $\lambda_j - \lambda_{j-1}$ (°C m^2 W^{-1})
$\Delta_i\Delta_j(\lambda)$ second cross difference in space (Δ_j) and time (Δ_i) at the j−th grid point and at time $i\Delta t$ of the λ temperature, $\Delta_j(\lambda_{i+1}) - \Delta_j(\lambda_i) = \Delta_j\Delta_i(\lambda)$ (°C m^2 W^{-1})
$\Delta\mu_i$ forward difference of the μ temperature at time $i\Delta t$, $\mu_{i+1} - \mu_i$ (−)
$\Delta(\Delta\mu)_i$ central difference of the μ temperature at time $i\Delta t$, $\Delta\mu_i - \Delta\mu_{i-1}$ (−)
ε_a absolute error (°C)
ε_r relative error (−)
ε_i a single random error
$\boldsymbol{\varepsilon}$ vector of random error
η eigenvalue in the y−direction (−)
∇ gradient operator

$\nabla_{\mathbf{q}}$	gradient with respect to the components of vector $\mathbf{q}$
θ_0	surface temperature rise, $T_0(t) - T_{in}$ (K)
$\boldsymbol{\theta}_0$	vector of surface temperature-rise components/values (°C)
λ	temperature rise per unit of surface heat flux of the X22B(y1pt1)0Y22B00T0 heat conduction case, that is, $\tilde{T}_{\text{X22B(y1pt1)0Y22B00T0}} \times L/k$ (°C m^2 W^{-1})
Λ_j	heat flux-based sensitivity coefficient, $\partial T(x,y,t)/\partial q_{0,j}$ (°C m^2 W^{-1})
$\boldsymbol{\Lambda}$	vector of sensitivity coefficients Λ_j (°C m^2 W^{-1})
μ	temperature rise per unit linear-in-time increase of surface temperature at $x = 0$ being $x = L$ insulated over the time $t_{ref} = \Delta t$, that is, $\tilde{T}_{\text{X12B20T0}}$ with $\tilde{t}_{ref} = \Delta\tilde{t}$ (−)
ρ	density (kg m^{-3})
ρ^k	descent direction on iteration k for conjugate gradient method
σ	standard deviation
τ	dummy variable for time (s)
ϕ	temperature rise per unit step change of surface heat flux at $x = 0$ with $x = L$ kept insulated, that is, $\tilde{T}_{\text{X22B10T0}} \times L/k$ (°C m^2 W^{-1})
$\phi(.)$	generic influence function – response in domain due to step change in surface condition
$\boldsymbol{\Phi}$	state transition matrix
φ	temperature rise per unit step change of surface temperature at $x = 0$ being $x = L$ insulated, that is, $\tilde{T}_{\text{X12B10T0}}$ (−)
ψ	Lagrange multiplier
$\Psi_{M,i}$	temperature-based sensitivity coefficient, $\partial T_M(x)/\partial\theta_{0,i}$
$\boldsymbol{\Psi}$	matrix of sensitivity coefficients $\Psi_{M,i}$ for $\boldsymbol{\theta}_0$ (−)
$\boldsymbol{\Psi}_0$	vector of sensitivity coefficients $\Psi_{M,0} = \partial T_M(x)/\partial\theta_{0,0}$ for $\theta_{0,0}$ (−)

Subscripts

0	boundary surface at $x = 0$; also, initial time $t = 0$
∞	undisturbed fluid
α	pertaining to regularization (Tikhonov or other)
b	body
c	computational
c	constant
ct	complementary transient
d	deviation
δ	second deviation
e	exact
f	final time
i	i-th time step; also, i-th component in time of surface temperature or surface heat flux
j	j-th space step ($y_N = W$); also, j-th component in space of surface heat flux
in	initial (time $t = 0$)
L-curve	pertaining to the L-curve (Section 6.6)
m	counting integer in the $x-$direction
M	M-th time step
n	counting integer in the $y-$direction
n_s	number of singular values removed in TSVD
N	N-th space step
opt	optimal
p, pen	penetration time (Eq. 4.1)
p	"look-ahead" time, $p = r_{opt}\Delta t$ (Chapter 7) (s)
p	sensor measurement location (Chapter 10)
qs	quasi steady
ref	reference

ss	steady state
us	unsteady
w	wire
x	along the *x*−direction
X12	heat conduction along *x* with BCs of the 1st and 2nd kind
X22	heat conduction along *x* with BCs of the 2nd kind on both sides
XI0BKT0	heat conduction along *x* for a semi-infinite body with BC of the 1st kind (I = 1) or of the 2nd kind (I = 2); $K = 1$ or 2 for a constant or linear-in-time BC
y	along the *y*−direction
Y	pertaining to measurement

Superscripts

$\tilde{}$, $^+$	dimensionless quantity
$\bar{}$	mean or average
$\bar{}$	Laplace transform of function (Chapter 10)
$\hat{}$	estimated value
^{T}	transpose of a matrix
(*A*)	counting integer for number of decimal places of precision in solution
(*L*)	large-time form
(*S*)	short-time form

Acronyms

1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
BC	boundary condition
BEM	boundary element method
CD	central difference
CGM	conjugate gradient method
DHCP	direct heat conduction problem
FD	forward difference
FR	Fletcher-Reeves conjugate gradient method
FS	function Specification
GF	Green's function
GFSE	Green's function solution equation
IHCP	inverse heat conduction problem
KF	Kalman filter
LHS	left-hand side
LT	Laplace transform
RHS	right-hand side
SD	Steepest Descent iteration method
SOV	separation of variables
SVD	singular value decomposition
TSVD	truncated singular value decomposition
TR	Tikhonov regularization
USEM	unsteady surface element method

Preface to First Edition

This book presents a study of the inverse heat conduction problem (IHCP), which is the estimation of the surface heat flux history of a heat conducting body. Transient temperature measurements *inside* the body are utilized in the calculational procedure. The presence of errors in the measurements as well as the ill-posed nature of the problem lead to "estimates" rather than the "true" surface heat flux and/or temperature.

This book was written because of the importance and practical nature of the IHCP; furthermore, at the time of writing, there is no available book on the subject written in English. The specific problem treated is only one of many ill-posed problems but the techniques discussed herein can be applied to many others. The basic objective is to estimate a function given measurements that are "remote" in some sense. Other applications include remote sensing, oil exploration, nondestructive evaluation of materials, and determination of the Earth's interior structure.

The authors became interested in the IHCP over two decades ago while employed in the aerospace industry. One of the applications was the determination of the surface heat flux histories of reentering heat shields.

This book is written as a textbook in engineering with numerical examples and exercises for students. These examples will be useful to practicing engineers who use the book to become acquainted with the problem and methods of solution. A companion book, *Parameter Estimation in Engineering and Science* by J. V. Beck and K. J. Arnold (Wiley 1977), discusses the estimation of certain constants or parameters rather than functions as in the IHCP. Though many of the ideas relating to least squares and sensitivity coefficients are present in both books, the present book does not require a mastery of parameter estimation.

The book is written at the advanced BS or the MS level. A course in heat conduction at the MS level or courses in partial differential equations and numerical methods are recommended as prerequisite materials.

Our philosophy in writing this book was to emphasize general techniques rather than specialized procedures unique to the IHCP. For example, basic techniques developed in Chapter 4 can be applied either to integral equation representations of the heat diffusion phenomena or to finite difference (or element) approximations of the heat conduction equation. The basic procedures in Chapter 4 can treat nonlinear cases, multiple sensors, nonhomogeneous media, multidimensional bodies, and many equations, in addition to the transient heat conduction equation.

The two general procedures that are used are called (a) function specification and (b) regularization. A method of combining these (the trial function method) is also suggested. One of the important contributions of this book is the demonstration that all of these methods can be implemented in a sequential manner. The sequential method in some case gives nearly the same result as whole domain estimation and yet is much more computationally efficient.

One of our goals was to provide the reader with an insight into the basic procedures that provide analytical tools to compare various procedures. We do this by using the concepts of sensitivity coefficients, basic test cases, and the mean squared error. The reader is also shown that optimal estimation involves the compromise between minimum sensitivity to random measurement errors and the minimum bias.

Preliminary notes have been used for an ASME short course and for a graduate course at Michigan State University.

There are many people who have helped in the preparation of this text and to whom we express our appreciation. These include D. Murio, M. Raynaud, other colleagues, and students who have read and commented on the notes. Thanks are also due to Judy Duncan, Phyllis Murph, Terese Stuckman, Alice Montoya, and Jeana Pineau, who have aided in typing the manuscript.

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August 1985

Preface to Second Edition

The first edition of this textbook was published in 1985. At that time, the IBM PC had been available for about four years, and release of the IBM PS/2 was still two years in the future. These historical milestones frame the thinking of the times about computing in the mid-1980s. It is often said, and is demonstrably true, that any common pocket cell phone of the 2020s has significantly more computation and storage capability than early scientific computing machines. While the first edition of the book continues to be used by researchers and professionals as a major reference on inverse heat conduction problems (IHCPs), with the major advancements in solution techniques for IHCPs and the vast progress in use of computer programs for solving engineering problems over the past few decades, the need for writing the second edition of the book was long due.

Over the past 30 years, the authors of this second edition have collaborated on an array of research projects on exact solutions to heat conduction problems and solution of inverse heat conduction problems (IHCPs), and much of that research forms the basis for this book.

This text aims to present the methodology of many IHCP solution techniques (Chapter 4) and outline approaches for optimizing the degree of regularization in an IHCP solution (Chapter 6). These discussions will be presented primarily in the framework of the filter matrix concept presented in Chapter 5.

Almost 40 years have passed since publication of the first edition of this book, and much research on IHCPs and their solutions has been performed since that time. Chapter 1 presents an extensive literature review of IHCP work over the past decades in the areas of manufacturing, aerospace, biomedical, electronics cooling, instrumentation, nondestructive testing, and other areas of engineering applications. Throughout the text, additional references are offered to guide the interested reader to sources for additional information.

Because computing power today is great, many (most?) direct heat conduction problems are solved using commercial software based on finite element or finite difference methodology. However, analytical solutions provide highly accurate results that can be computed efficiently. This second edition continues the emphasis from the first edition on utilization of exact solutions for linear heat transfer problems – those with thermophysical properties that are independent of temperature. Chapter 2 presents exact solutions to several fundamental problems, and superposition of these solutions provides a gateway to solution of a wide array of problems. Also discussed in Chapter 2 is the concept of the computational analytical solution which provides efficient and accurate computation of the solution.

Chapter 3 outlines approximate solution methods for linear problems which form the basis for the IHCP solutions in the remainder of the book. These concepts are presented using ideas of superposition of exact solutions; however, a more rigorous treatment using Green's functions is given in Appendix.

Chapter 4 retains detailed description of the Stolz method, the Function Specification (Beck's) Method, and Tikhonov regularization. New to this second edition are descriptions of the Conjugate Gradient Method and Singular Value Decomposition. Additionally, two methods that are amenable to nonlinear problems are explained: the Adjoint Method, which is used in conjunction with the Conjugate Gradient Method, and the Kalman Filter approach to solving the IHCP.

Chapter 6 is new to the second edition and focuses on selection of the optimal degree of regularization in solution of IHCPs. Because IHCPs are inherently ill-posed, some form of regularization is necessary. Regularization decreases sensitivity to noise in measurements at the expense of introducing bias into the estimates. Striking the appropriate balance between these competing factors is an important, and often overlooked, part of analyzing an IHCP.

Because of the relative costliness of computer resources at the time, a significant focus of the first edition was computational efficiency of IHCP algorithms. In this second edition, more attention is devoted to comparing accuracy of IHCP solution methods than to their computational efficiency. Chapter 7 contains head-to-head comparison of many IHCP solution methods using a common suite of test problems.

The filter coefficient concept is used throughout the text. Chapter 5 explores the nature of the filter matrix for many IHCP methods presented in Chapter 4. The character of many of these filter matrices is such that a truncated filter vector can be extracted from the matrix to use in sequential online (near real-time) estimation. Additionally, Chapter 5 illustrates how linear IHCP concepts can be applied to problems with temperature-dependent thermophysical properties. Chapter 8 outlines application of the filter concepts to two-dimensional transient IHCP problems with multiple unknown heat fluxes.

Chapter 9 addresses the problem of estimating the heat transfer coefficient, h. Much of the chapter focuses on the case of lumped capacitance bodies (with negligible internal temperature gradients) and outlines several approaches for estimating h under those conditions. Application to bodies with temperature gradients is also discussed.

Chapter 10 is new to this edition and addresses bias in temperature measurements caused by sensor installation. Two situations are explored: correction of measured temperature to remove the bias and utilization of the biased measurements directly in the IHCP solution. Both situations typically require a computational model which incorporates the sensor in the domain and can be used to quantify the bias.

The second edition of the book also comes with additional materials that are available on a companion website. These materials include MATLAB codes for the examples that are solved in the book. Furthermore, course slides, additional problems, and suggested course syllabus will be available for instructors.

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