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3rd Edition

# Trigonometry

for  
**dummies**<sup>®</sup>  
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Understand the basics of  
sines, cosines, and tangents

Learn how to use trigonometry  
outside of the classroom

Graph functions and  
solve tough formulas

**Mary Jane Sterling**

Master Pi Maker





# Trigonometry

3rd Edition

**by Mary Jane Sterling**

for  
**dummies**<sup>®</sup>  
A Wiley Brand

## Trigonometry For Dummies®, 3rd Edition

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# Introduction

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**M**any of the exciting and, yes, practical accomplishments of early humans were performed using trigonometry. Even before it was formalized into a particular topic to study or used to solve problems, trigonometry helped people to sail across large bodies of water, build gigantic structures, plot out land, and measure heights and distances — even to the stars.

We still use trigonometry for all these reasons and more. For example, if you're going to get your pilot's license, you'll need trigonometry. Trigonometry is also the basis for many courses in mathematics — starting in grade school with geometric shapes and map reading, and moving on through calculus. Trig is all over the place.

You can get as deeply into this topic or as little into it as you want, and you'll still come out of it thinking, “Gee, I didn't realize that trigonometry was used to do this! Wasn't that just loads of fun!” Well, maybe I'm pushing it a bit — loads may be a slight exaggeration.

Whether you're pursuing trigonometry so that you can go on to calculus or prepare for architecture or drafting or do that piloting thing, or you're just curious, you'll find what you need here. You can get as technical as you want. You can skip through the stuff you don't need. Just know that you'll be on the same adventure as those early humans — you'll just have the advantage of a few more tools.

## About This Book

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So, what's in it for you? What's in a book on trigonometry that'll ring your bell or strike your fancy or just make you pretty happy? Where do I begin?

You can start anywhere in the book, jump around, and just go any direction you please. If you're really excited about triangles or circles and how they can be used to your advantage, check them out. Everything you'd want to know about them is here.

What if you've got another angle? Or, maybe you didn't have one to begin with but wish you did. If you're looking for angles, you've come to the right place. There are big angles and little angles, all named depending on their situation.

They're measured in degrees or radians. "What's a radian?" you ask. You can find out in this book, that's for sure.

You may be very analytical. If so, you'll find that your favorite spot is among all the identities and equation-solving procedures. Hop right to it. They're waiting for you.

And if drawing pictures is your bag, go to the chapters on graphing to see what can be done with simple trig graphs, complicated trig graphs, and everything in between. There are even explanations on what the function equations mean, why they're used in an application, and how they're related to the graphs.

You'll find many sidebars throughout this book. Sidebars are those fun, little anecdotes that don't necessarily contain a lot of math content but present interesting little tidbits — fun things to read. Neither the sidebars nor the items marked with the Technical Stuff icon are necessary for your understanding of the material. Think of them as little diversions for your reading pleasure.

Within this book, you may note that some web addresses break across two lines of text. If you're reading this book in print and want to visit one of these web pages, simply key in the web address exactly as it's noted in the book, pretending as though the line break doesn't exist. If you're reading this as an e-book, you've got it easy — just click the web address to be taken directly to the web page.

## Foolish Assumptions

How foolish of me to assume that you're reading *Trigonometry For Dummies* because it looks more interesting than the latest bestseller! I'm really not that foolish! To be honest, trig wouldn't be my first choice for a fun read. I'm just going to assume that you really want to do this. While writing this book, I made a few other assumptions about you as well:

- » You have a goal in mind. You want to conquer some of the topics in this book so you're prepared for a course of study.
- » You have a pretty solid grasp on algebra and can solve a simple algebraic equation without falling completely apart.
- » You're planning on being on *Jeopardy*, and you need to bone up on the possible trig questions.



# Icons Used in This Book

Icons are easy to spot. They could be called *eye-cons*, because they catch your eye. Here are the ones I use in this book:



TIP

This icon points out handy hints and shortcuts that make your life easier when it comes to solving trig problems.



TRIG  
RULES

Of course, trig rules — it's fun! But taken another way, this icon is used to point out to you when particular equations or expressions or formulas are used in trigonometry that you should be paying attention to. They're important. This icon helps you find them again, if you need them.



REMEMBER

This icon refers back to information that I think you may already know. It needs to be pointed out or repeated so that the current explanation makes sense.



TECHNICAL  
STUFF

What about trigonometry isn't technical? Actually, there's quite a bit, but this icon points out the rules or absolutely unchangeable stuff that you may need in order to understand the situation.



WARNING

There are always things that are tricky or confusing, or problems that just ask for an error to happen. This icon is there to alert you, hoping to help you avoid a mathematical pitfall.

## Beyond the Book

In addition to the material in the print or e-book you're reading right now, this product also comes with some access-anywhere goodies on the web. No matter how diligent you are about reading through this material, you'll likely come across a few questions where you don't have a clue. Check out the free online Cheat Sheet for helpful information, all provided in a concise, quick-access format. Simply go to [www.dummies.com](http://www.dummies.com) and type **Trigonometry For Dummies Cheat Sheet** in the search box.

# Where to Go from Here

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Back when I was in college, my friend, Judy Christopher, once consoled me with, “Life is like a sine curve. It has its ups and downs. If you’re feeling like you’re really down, then just remember that you’ll be going up that same amount someday soon.” So, if you’re in the dumps, maybe you want to start with the graphs of the sine curves and other trig curves. Make of them what you will.

Or maybe, like me, you’re a puzzle buff. I can’t wait to tackle a crossword or other fun type of puzzle in the Sunday paper or online. You have to call up bits and pieces of information and make them all fit into something logical. If that’s what you’re interested in today, then go to proving identities and solving equations. That’s a great challenge for a rainy Sunday afternoon’s pleasure.

Are you into angles and directions and plans? You may want to start with the ways that angles are measured and how they all fit together in the big picture. The basics are always a good place to start when you’re investigating a topic.

No matter where you start with this book, be ready to flip the pages front to back or back to front. Think of it as an adventure that can take you many interesting places. Enjoy!

# 1

## Getting Started with Trigonometry

## **IN THIS PART . . .**

Become acquainted with angle measures and how they relate to trig functions.

Discover formulas that provide lengths of segments, midpoints, and slopes of lines.

Become familiar with circles and the relationships between radii, diameters, centers, and arcs.

Find a simple conversion method for changing from degrees to radians and vice versa.

Observe the properties of special right triangles and use the Pythagorean Theorem to formulate the relationships between the sides of these right triangles.

- » Identifying angles and their names
- » Understanding trig speak
- » Finding trig applications in the basics

## Chapter **1**

# Taking On Trig Technicalities

**H**ow did Columbus find his way across the Atlantic Ocean? How did the Egyptians build the pyramids? How did early astronomers measure the distance to the moon? No, Columbus didn't follow a yellow brick road. No, the Egyptians didn't have LEGO instructions. And, no, there isn't a tape measure long enough to get to the moon. The common answer to all these questions is trigonometry.

*Trigonometry* is the study of angles and triangles and the wonderful things about them and that you can do with them. For centuries, humans have been able to measure distances that they can't reach because of the power of this mathematical subject.

## Taking Trig for a Ride: What Trig Is

“What's your angle?” That question isn't a come-on such as, “What's your astrological sign?” In trigonometry, you can measure angles in both degrees and radians. You can position the angles into triangles and circles and make them do special things. Actually, angles drive trigonometry. Sure, you have to consider

algebra and other math to make it all work. But you can't have trigonometry without angles. Put an angle measure into a trig function, and out pops a special, unique number. What do you do with that number? Read on, because that's what trig is all about.

## Sizing up the basic figures

Segments, rays, and lines are some of the basic forms found in geometry, and they're just as important in trigonometry. As I explain in the following sections, you use those segments, rays, and lines to form angles and triangles and other geometric and trig forms.

### Drawing segments, rays, and lines

A *segment* is a straight figure drawn between two endpoints. You usually name it by its endpoints, which you indicate by capital letters. Sometimes, a single letter names a segment; this single letter is positioned at about the middle of the segment. For example, in a triangle, a lowercase letter may refer to a segment opposite the angle labeled with the corresponding uppercase letter.

A *ray* is another straight figure that has an endpoint on one end, and then it just keeps going forever in some specified direction. You name rays by their endpoint first and then by any other point that lies on the ray. You indicate that the other end goes on forever by using an arrow point.

A *line* is a straight figure that goes forever and ever in either direction. You only need two points to determine a particular line — and only one line can go through both of those points. You can name a line by any two points that lie on it.

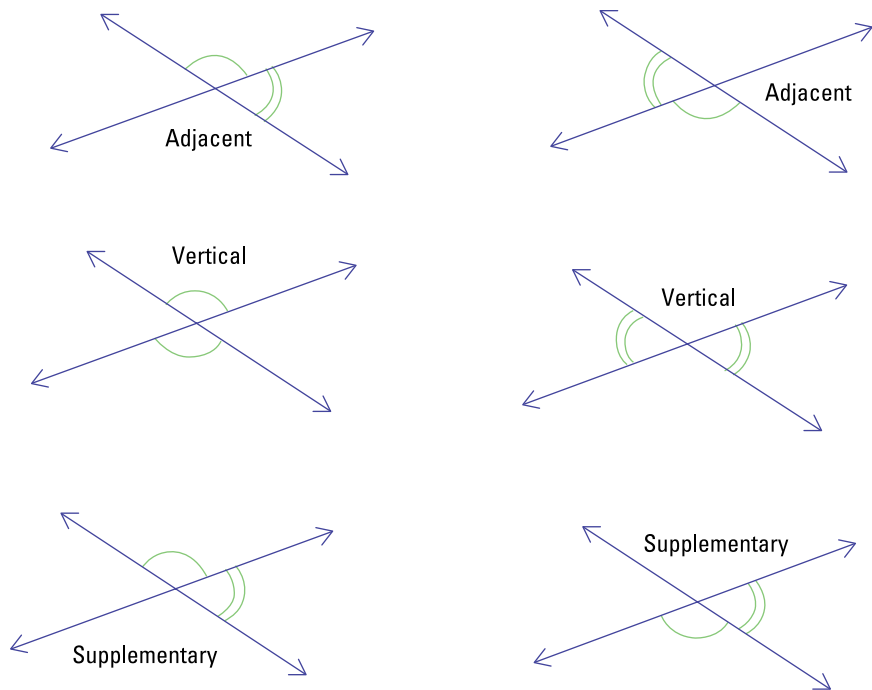
Figure 1-1 shows a segment, ray, and line and the different ways you can name them using points.



**FIGURE 1-1:**  
Segment AB, ray  
CD, and line EF.

## Intersecting lines

When two lines intersect — if they do intersect — they can only do so at one point. They can't double back and cross one another again. And some curious things happen when two lines intersect. The angles that form between those two lines are related to one another. Any two angles that are next to one another and share a side are called *adjacent angles*. In Figure 1-2, you see several sets of intersecting lines and marked angles. The top two figures indicate two pairs of adjacent angles. Can you spot the other two pairs? The angles that are opposite one another when two lines intersect also have a special name. Mathematicians call these angles *vertical angles*. They don't have a side in common. The two middle pairs in Figure 1-2 are vertical angles. Vertical angles are always equal in measure.



**FIGURE 1-2:** Intersecting lines form adjacent, vertical, and supplementary angles.

Why are these different angles so special? They're different because of how they interact with one another. The adjacent angles here are called *supplementary angles*. The sides that they don't share form a straight line, which has a measure of 180 degrees. The bottom two figures show supplementary angles. Note that these are also adjacent.

## Identifying angles and their names

When two lines, segments, or rays touch or cross one another, they form an angle or angles. In the case of two intersecting lines, the result is four different angles. When two segments intersect, they can form one, two, or four angles; the same goes for two rays.

These examples are just some of the ways that you can form angles. Geometry, for example, describes an angle as being created when two rays have a common endpoint. In practical terms, you can form an angle in many ways, from many figures. The business with the two rays means that you can extend the two sides of an angle out farther to help with measurements, calculations, and practical problems.

Describing the parts of an angle is pretty standard. The place where the lines, segments, or rays cross is called the *vertex* of the angle. From the vertex, two sides extend.

## Naming angles by size

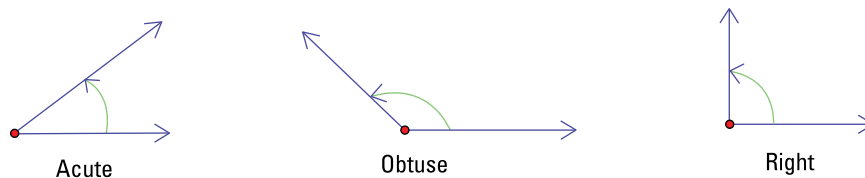
You can name or categorize angles based on their size or measurement in degrees and radians. For more on radian measures, go to Chapter 4. Figure 1-3 shows examples of each of the following angles.

- » **Acute:** An angle with a positive measure less than 90 degrees (less than  $\frac{\pi}{2}$  radians).
- » **Obtuse:** An angle measuring more than 90 degrees but less than 180 degrees (between  $\frac{\pi}{2}$  and  $\pi$  radians).
- » **Right:** An angle measuring exactly 90 degrees (or  $\frac{\pi}{2}$  radians).
- » **Straight:** An angle measuring exactly 180 degrees (a straight line or  $\pi$  radians).
- » **Oblique:** An angle measuring more than 180 degrees (more than  $\pi$  radians).

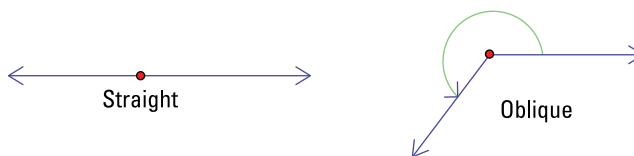
## Naming angles by letters

How do you name an angle? Why does it even need a name? In most cases, you want to be able to distinguish a particular angle from all the others in a picture. When you look at a photo in a newspaper, you want to know the names of the different people and be able to point them out. With angles, you should feel the same way.



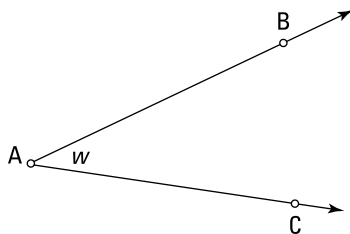


**FIGURE 1-3:**  
Types of  
angles — acute,  
obtuse, right,  
straight, and  
oblique.



You can name an angle in one of three different ways.

- » **By its vertex alone:** Often, you name an angle by its vertex alone because such a label is efficient, neat, and easy to read. In Figure 1-4, you can call the angle  $A$ . You only use this type of name if there aren't any angles adjacent at the vertex  $A$ . It has to stand alone.
- » **By a point on one side, followed by the vertex, and then a point on the other side:** For example, you can call the angle in Figure 1-4 angle  $BAC$  or angle  $CAB$ . This naming method is helpful if someone may be confused as to which angle you're referring to in a picture. **Remember:** Make sure you always name the vertex in the middle.
- » **By a letter or number written inside the angle:** Usually, that letter is Greek; in Figure 1-4, however, the angle has the letter  $w$ . Often, you use a number for simplicity if you're not into Greek letters or if you're going to compare different angles later.



**FIGURE 1-4:**  
Naming an angle.

# Taking on triangles and their angles

All on their own, angles are certainly very exciting. But put them into a triangle, and you've got icing on the cake. Triangles are one of the most frequently studied geometric figures. When angles are part of a triangle, they have many characteristics.

## Angles in triangles

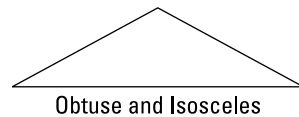
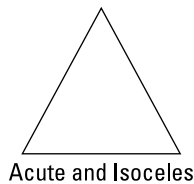
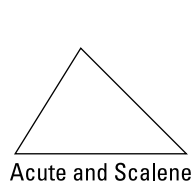
A triangle always has three angles. The angles in a triangle have measures that always add up to 180 degrees — no more, no less. A triangle named  $ABC$  (often written  $\triangle ABC$ ) has angles  $A$ ,  $B$ , and  $C$ , and you can name the sides  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$ , depending on which two angles the side is between. The angles themselves can be acute, obtuse, or right. If the triangle has either an obtuse or right angle, then the other two angles have to be acute.

## Naming triangles by their shape

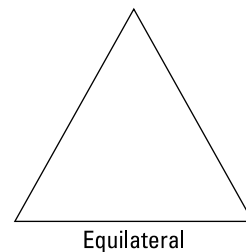
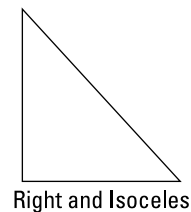
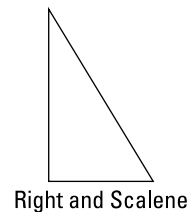
Triangles can have special names based on their angles and sides. They can also have more than one name — a triangle can be both acute and isosceles, for example. Here are their descriptions, and check out Figure 1-5 for the pictures.

» **Acute triangle:** A triangle where all three angles are acute.

» **Right triangle:** A triangle with a right angle (the other two angles must be acute).



**FIGURE 1-5:** Triangles can have more than one name, based on their characteristics.



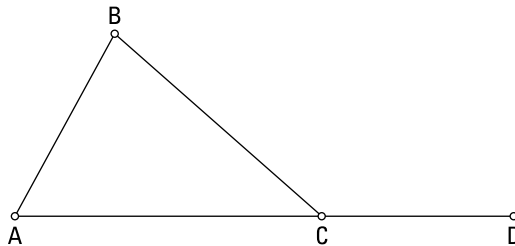
- » **Obtuse triangle:** A triangle with an obtuse angle (the other two angles must be acute).
- » **Isosceles triangle:** A triangle with two equal sides; the angles opposite those sides are equal, too. The equal angles have to be acute.
- » **Equilateral triangle:** A triangle where all three side lengths are equal, and all the angles measure 60 degrees.
- » **Scalene triangle:** A triangle with no angles or sides of the same measure.

## Going outside the triangle

Another angle that comes up frequently in trigonometry is an exterior angle. When one side of a triangle is extended on both sides of its vertex, an exterior angle is formed. An angle and its exterior angle share the vertex, and the measure of the exterior angle is quickly deduced (as shown in Figure 1-6)! Here are some characteristics of an exterior angle:

- » An exterior angle to a triangle is always supplementary to the angle it's adjacent to in the triangle. This means that the sum of the angle and its exterior angle is always 180 degrees.
- » An exterior angle to a triangle has a measure that is equal to the sum of the two non-adjacent angles in the triangle.

**FIGURE 1-6:**  
Angle  $BCD$  is adjacent to angle  $BCA$ .

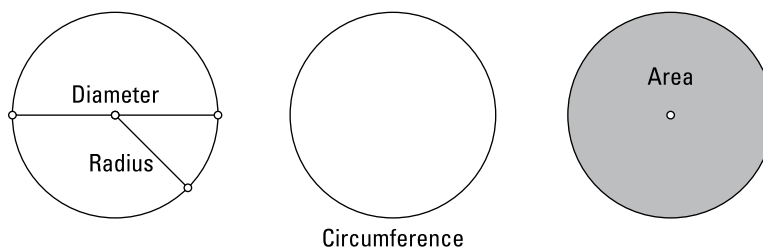


## Making a circle work from every angle

A *circle* is a geometric figure that needs only two parts to identify it and classify it: its *center* (or middle) and its *radius* (the distance from the center to any point on the circle). Technically, the center isn't a part of the circle; it's just a sort of anchor or reference point. The circle consists only of all those points that are the same distance from the center.

## Radius, diameter, circumference, and area

After you've chosen a point to be the center of a circle and you know how far that point is from all the points that lie on the circle, you can draw a fairly decent picture. With the measure of the radius, you can tell a lot about the circle: its *diameter* (the distance from one side to the other, passing through the center), its *circumference* (how far around it is), and its *area* (how many square inches, feet, yards, meters — what have you — fit into it). Figure 1-7 shows these features. A *chord* of a circle is a segment that joins any two points on the circle. A chord can be a *diameter*.



**FIGURE 1-7:**  
The different  
features  
of a circle.

Ancient mathematicians figured out that the circumference of a circle is always a little more than three times the diameter of the circle. Since then, they narrowed that “little more than three times” to a value called *pi* (pronounced “pie”), designated by the Greek letter  $\pi$ . The decimal value of  $\pi$  isn't exact; in fact, it goes on forever and ever. However, most of the time, people refer to it as being approximately 3.14 or  $\frac{22}{7}$ , whichever form works best in specific computations.

The formula for figuring out the circumference of a circle is tied to  $\pi$  and the diameter:



TRIG  
RULES

$$\text{Circumference of a circle } C = \pi d = 2\pi r$$

The  $d$  represents the measure of the diameter, and  $r$  represents the measure of the radius. The diameter is always twice the radius, so either form of the equation works.

Similarly, the formula for the area of a circle is tied to  $\pi$  and the radius:



TRIG  
RULES

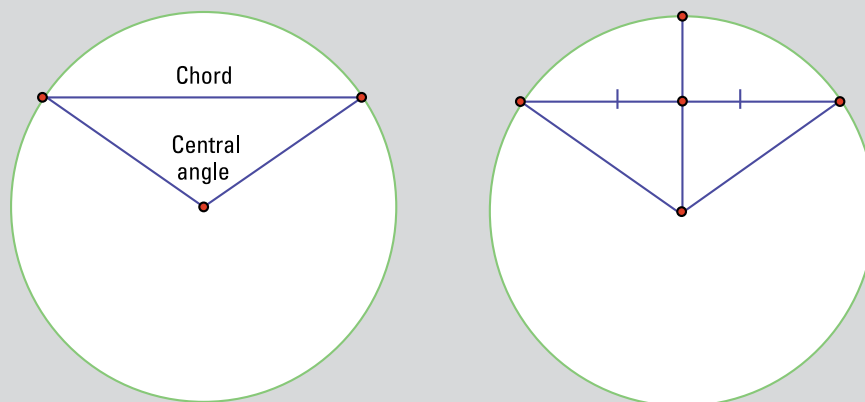
$$\text{Area of a circle } A = \pi r^2$$

This formula reads, “Area equals pi are squared.” (And here I thought that *pies are round*.)

Example: Find the radius, circumference, and area of a circle if its diameter is equal to 10 feet in length.

## DON'T GIVE ME THAT *JIVA*

The ancient Greek mathematician Ptolemy was born some time at the end of the first century. Ptolemy based his version of trigonometry on the relationships between the chords of circles and the corresponding central angles of those chords. Ptolemy came up with a theorem involving four-sided figures that you can construct with the chords. (See the section, "Finding Trig Applications in the Basics," later in this chapter.) In the meantime, mathematicians in India decided to use the measure of *half* a chord and *half* the angle to try to figure out these relationships. Drawing a radius from the center of a circle through the middle of a chord (halving it) forms a right angle, which is important in the definitions of the trig functions. These half-measures were the beginning of the sine function in trigonometry. In fact, the word *sine* actually comes from the Hindu word *jiva*.

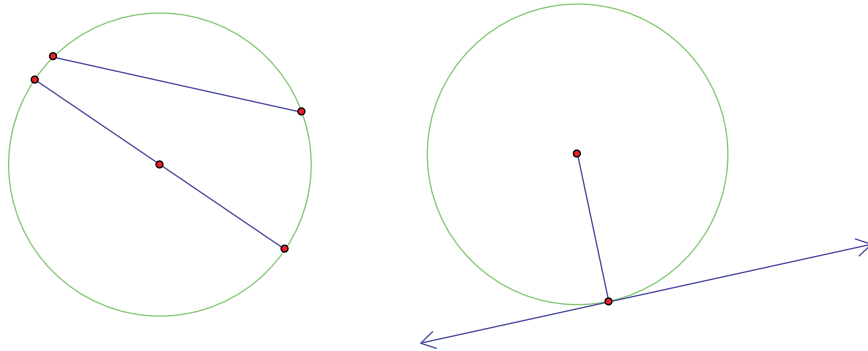


If the diameter ( $d$ ) is equal to 10, you write this value as  $d = 10$ . The radius is half the diameter, so the radius is 5 feet, or  $r = 5$ . You can find the circumference by using the formula  $C = \pi d = \pi \cdot 10 \approx 3.14 \cdot 10 \approx 31.4$ . So, the circumference is about  $31\frac{1}{2}$  feet around. You find the area by using the formula  $A = \pi r^2 = \pi \cdot 5^2 = \pi \cdot 25 \approx 3.14 \cdot 25 \approx 78.5$ , so the area is about  $78\frac{1}{2}$  square feet.

### Chord versus tangent

You show the diameter and radius of a circle by drawing segments from a point on the circle either to or through the center of the circle. But two other straight figures have a place on a circle. One of these figures is called a chord, and the other is a tangent.

- » A *chord* of a circle is a segment that you draw from one point on the circle to another point on the circle (see Figure 1-8). A chord always stays inside the circle. The largest chord possible is the diameter — you can't get any longer than that segment.
- » A *tangent* to a circle is a line, ray, or segment that touches the outside of the circle at exactly one point, as shown in Figure 1-8. It never crosses into the circle. A tangent can't be a chord, because a chord touches a circle in two points, crossing through the inside of the circle. Any radius drawn to a tangent is perpendicular to that tangent.



**FIGURE 1-8:**  
Chords and  
a tangent  
of a circle.

## Looking at angles in a circle

There are several ways of drawing an angle in a circle, and each has a special way of computing the size of that angle. The four different types of angles are central, inscribed, interior, and exterior. In Figure 1-9, you see examples of these different types of angles.

### Central angle

A *central angle* has its vertex at the center of the circle, and the sides of the angle are two radii of the circle. The measure of the central angle is the same as the measure of the arc that the two sides cut out of the circle.

### Inscribed angle

An *inscribed angle* has its vertex on the circle, and the sides of the angle are two chords of the circle. The measure of the inscribed angle is half that of the arc that the two sides cut out of the circle.