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Makoto Takahashi · Yasuhiro Omori · Toshiaki Watanabe

Stochastic Volatility and Realized Stochastic Volatility Models

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Preface

The aim of this book is to introduce recent developments in stochastic volatility models for asset returns such as returns of stocks, foreign currencies, and interest rates. It is intended for researchers who are interested in Bayesian analysis of financial time series. In financial markets, it is well known that financial time series often exhibit a behavior called volatility clustering and that the volatility of the asset return changes randomly with high persistence. The stochastic volatility model describes such dynamic structures of the unobserved volatilities. We first introduce the basic stochastic volatility model and then extend it to various volatility models. Noting that realized volatility computed from high-frequency data has recently been used to estimate true volatility, we also discuss joint modeling of the asset return and realized volatility, which is called realized stochastic volatility.

Chapter 1 summarizes the topics covered in this book. We describe the various stochastic volatility models covered in this book including the basic model, the asymmetric stochastic volatility model, the stochastic volatility model with generalized hyperbolic (GH) skewed Student's t error, and joint modeling of stochastic volatility and realized volatility.

Chapter 2 discusses the basic stochastic volatility model and describes an estimation method using Markov chain Monte Carlo (MCMC) simulation. In stochastic volatility models, there are as many latent volatilities as asset returns, and we need to integrate them out to compute the likelihood function. Since it is difficult to obtain the likelihood numerically, we use the Bayesian approach and implement MCMC simulation to facilitate statistical inference of model parameters. The simple MCMC sampling algorithm for the latent log volatilities is called a single-move sampler, which samples one variable given other variables. However, it is known to be inefficient in the sense that it produces highly autocorrelated MCMC samples. To overcome this difficulty, we describe two major efficient MCMC estimation methods: the mixture sampler and the multi-move (or block) sampler. The former transforms the model into a linear state-space model and approximates the error distribution using a mixture of normal distributions. Given the mixture normal distribution, the volatilities are generated all at once. The latter method generates a block of state variables given other parameters and latent variables. Using real data in empirical studies, we will show that they are highly efficient.

Chapter 3 introduces the asymmetric stochastic volatility model, which extends the basic model to allow for correlation between the return and log volatility errors. It has long been recognized in stock markets that a decrease in today's return is followed by an increase in the tomorrow's volatility. This phenomenon is called the "leverage effect" or "asymmetry." The efficient MCMC algorithms given in Chap. 2 are extended to incorporate such a negative correlation between the errors for the leverage effect.

Chapter 4 discusses the stochastic volatility model with GH skew Student's t error. Financial time series data such as stock returns and foreign exchange returns are known to have several properties that depart from a normality assumption. Major characteristics of return distributions for financial variables are skewness, heavy-tailedness, and volatility clustering with leverage effects. These properties are crucial not only for describing return distributions but also for asset allocation, option pricing, forecasting, and risk management. Further, to visualize a leverage effect or volatility asymmetry, we introduce a news impact curve in the context of the stochastic volatility models, which measures how the new information affects the return volatility.

In Chap. 5, we consider an additional piece of information, namely the realized volatility, for the measurement equation. The realized volatility is the sum of squared intraday returns over a certain interval such as a day; it has recently attracted the attention of financial economists and econometricians as an accurate measure of true volatility. In the real market, however, the presence of non-trading hours and market microstructure noise in transaction prices may cause bias in the realized volatility. On the other hand, daily returns are less subject to noise and may therefore provide additional information about the true volatility. From this view point, modeling realized volatility and daily returns simultaneously based on the well-known stochastic volatility model is proposed. The GH skew Student's t error is considered, as in Chap. 4, and we investigate the predictive performance of the realized stochastic volatility model with respect to one-day-ahead volatility, value at risk, and expected shortfall forecasts.

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Chapter 1
Introduction

Abstract This chapter provides a brief background on the developments in stochastic volatility models, including Markov chain Monte Carlo simulation for the estimation of model parameters, and summarizes the topics covered in the subsequent chapters.

1.1 Research Background

In finance and related fields, the term *volatility* indicates a dispersion of financial asset returns and represents a financial risk. It is considered as either standard deviation or variance of returns but cannot be directly observed because it changes stochastically over time. Thus, modeling and forecasting time-varying financial volatility is one of the most important topics in finance for the prediction of return distribution and risk management.

Time-varying volatility used to be estimated and forecasted via two classes of time series models using financial returns: the generalized autoregressive conditional heteroskedasticity (GARCH) methodologies [15, 25] and the stochastic volatility (SV) model [26, 46, 51]. These models are consistent with volatility clustering (high persistence in volatility) and have been extended to accommodate the phenomenon called volatility asymmetry or the leverage effect, i.e., the negative correlation between today's return and tomorrow's volatility observed in stock markets [14, 19]. Early works related to such extensions in the GARCH framework include the exponential GARCH [41], GJR [27], and asymmetric power GARCH [23] models, while those for the SV model are [31, 39, 54].

Among various econometric models in the literature, the SV model has been one of the most popular with the GARCH models. Past empirical studies have shown that the SV model outperforms other models (including GARCH models) with respect to goodness of fit, forecasting volatilities, and evaluating the value at risk (a measure widely used for risk management). However, unlike the GARCH models, it is difficult to evaluate the likelihood of the SV model analytically and hence to estimate the model parameters using the maximum likelihood method. Thus, Bayesian methods for estimating the parameters using the Markov chain Monte Carlo (MCMC) technique have been developed in the literature.

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There are two major efficient MCMC estimation methods, in the sense that they substantially reduce autocorrelations of MCMC samples. One method is the mixture sampler [33, 44], which transforms the model into a linear state-space model and approximates the error distribution using a mixture of normal distributions. The mixture sampler is fast and highly efficient, but its use is limited to models that can be transformed into a linear state-space form. The other method is the multi-move (or block) sampler [45, 47, 55], which generates a block of state variables given other parameters and latent variables. In contrast to the mixture sampler, it can be applied to the model directly, without first transforming it into a linear state-space form.

Along with development regarding efficient estimation methods, SV models have been extended to capture the characteristics observed in financial returns such as daily equity returns and exchange rates. The empirical return distribution is more peaked and has heavier tails than the normal distribution. Such a heavy tail or leptokurtosis can be partly explained by time-varying volatility, but the return distribution conditional on the volatility may still be leptokurtic. To describe the heavy tail in the SV context, Student's t distribution is often used for the error distribution [13, 18, 37, 44, 56]. Moreover, the return distribution may also be skewed, and several types of skew Student's t distributions, such as the generalized hyperbolic (GH) skew Student's t distribution [1], have been used for the error distribution [36, 40].

Recently, thanks to the availability of high-frequency data that includes prices and other trade information within a day, a model-free volatility estimator, called realized volatility (RV), has become popular in finance. Basically, a daily RV is defined as the sum of squared intraday returns over a day. Early studies have shown its theoretical and empirical properties such as consistency under some conditions and strong persistence [3, 5–7, 10, 11].

To forecast time-varying volatility for financial risk management, one must model the dynamics of RV. Many researchers, including [5–7], have documented that daily RV may follow a long-memory process, so they use autoregressive fractionally integrated moving average (ARFIMA) models (see [12] for long-memory and ARFIMA models). A more widely used model for RV dynamics is the heterogeneous autoregressive (HAR) model proposed by [20]. It is not a long-memory model but is known to approximate the long-memory process well, and several extensions have been proposed [4, 21, 22].

Although the ARFIMA and HAR models have been shown to significantly improve volatility forecasts relative to GARCH and SV models, the RV is subject to a bias caused by market microstructure noise and non-trading hours. To mitigate or correct the bias, various RV measures have been proposed [8, 9, 32, 57, 58]. Such RV measures have been analyzed extensively in the literature [2, 38, 53].

For modeling and forecasting volatility while taking into account the bias in RV, two classes of hybrid models have been proposed. One is the realized stochastic volatility (RSV) model based on the SV model [24, 35, 48], and the other is the realized GARCH (RGARCH) model based on the GARCH model [28, 29]. By jointly modeling the RV and daily return, which is less subject to microstructure noise, these hybrid models can consider the bias in RV as a model parameter. Further, accounting for other empirical characteristics in RVs and daily returns, several extensions of the RSV model [42, 43, 49, 52] and the RGARCH model [16, 28] have been proposed.

These volatility models' forecasting abilities have been compared within each class, e.g., GARCH [30], RGARCH [17], and RSV [49], but a comprehensive comparison across different classes of volatility models is rare. A few exceptions are [34] and [50]. In particular, Takahashi et al. [50] documented that models with RV outperforms those without it and that the RSV model performs better than the HAR and REGARCH models.

1.2 Summary of Topics

This book introduces recent developments in stochastic volatility models. For the estimation of model parameters, we focus on Bayesian methods using MCMC simulation. The models and estimation methods are further illustrated in the applications to exchanges rates and stock indices.

Chapter 2 describes a basic SV model without the leverage effect and an efficient Bayesian estimation method using MCMC. Several efficient sampling algorithms for latent volatilities, such as the mixture sampler and the multi-move sampler, are described in detail. Parameter estimation is illustrated in an empirical study using daily returns of the Japanese yen/US dollar exchange rate.

In Chap. 3, the basic SV model is extended to accommodate the leverage effect and an efficient Bayesian estimation method is described. In particular, the mixture sampler and the multi-move sampler for latent volatilities are developed for the extension. The empirical study presents the estimation results for daily returns of the US stock index, Dow Jones Industrial Average (DJIA).

Chapter 4 further extends the SV model to incorporate the heavy tail and skewness. In particular, the SV model with the GH skew Student's t error is described, and its efficient MCMC estimation method is presented. To visualize the leverage effect as a function of multiple parameters in the extended model, we also introduce a simulation method to calculate a news impact curve in the context of SV models. The extended model and estimation method are applied to daily returns of the Japanese stock index, Nikkei 225 (N225), as well as to the DJIA.

In Chap. 5, the RSV model is introduced with a brief description of RV. Similar to the SV model, the RSV model is extended with the GH skew Student's t error. An efficient MCMC estimation method is also developed to estimate the models. In the empirical study, the SV and RSV models, as well as the EGARCH and REGARCH models, are applied to the DJIA and N225 and compared with respect to one-day-ahead volatility, VaR, and expected shortfall (a measure to assess a financial tail risk) forecasts.