Advances in Mathematics Education

Ole Skovsmose

Critical Mathematics Education

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The chapters in this book have the format of articles. Some of them are reworked versions of previously published articles, which I acknowledge in a note following such chapters. I thank the publishers for their kind permission to allow me to use such previous writings.

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December 2022 Ole Skovsmose

Introduction

In 1994, I visited São Paulo for the first time in my life. I had brought with me a small Danish travel guide, which recommended taking a walk from Praça da Sé to Praça da República, and I followed the recommendation.

For a couple of days, I was staying in the flat of Ubiratan D'Ambrosio and his wife Maria José and enjoying their hospitality. One morning, D'Ambrosio brought me in his car to Praça de Sé. We had agreed that, in the afternoon at 16:00, he would pick me up close to Praça da República at the corner of Avenida Ipiranga and São João. This corner was clearly shown in my Danish tourist guide. Later, I was told that this corner had become famous due to the song and music "Sampa," composed in 1978 by Caetano Veloso. In the "Sampa," Veloso wanted to express the creative energy that he associated with São Paulo.

I started walking around at Praça da Sé. There were so many people, and music emanated from all directions. With loud vices, street sellers were announcing their special offers. One could buy water, soft drinks, pastries, fruit, small figurines, T-shirts, caps; all kinds of touristy things. Next to the Praça, I saw many fountains where children were playing. Later I came to find out that these fountains attracted many street children, some of them coming from far away to get an opportunity to take a shower. I could have stayed the whole morning around the Praça filled with colors, sounds, and movements, but after a while I started my walk.

The streets seemed packed with people. There was much to look at and also to listen to. Music rang out from many of the shops. The music became mixed with loud voices announcing the shops' special offers. My walk led me across a long pedestrian bridge with street sellers on both sides. Deep below the bridge ran a highway, which I later came to pass many times. I found a place to sit and have a cup of coffee. The Brazilian coffee was strong to me, but soon I came to appreciate precisely this taste of coffee. Later I found a small restaurant and enjoyed my lunch there. In the afternoon I found Praça da República, and D'Ambrosio found me.

Since then I have visited Brazil many times. I fell in love with Miriam there, and we have now been married for almost 20 years. We visited São Paula many times, and now we live there. Over the years, we have walked around many different neighborhoods. We came to like Vila Madalena with its restaurants and galleries.

Once I had an exhibition there showing a collection of my paintings. Many times we have walked along Avenida Paulista, a main avenue framed with huge modern concrete-glass buildings. Occasionally we have seen old, still buildings with stucco and architectural styles that have survived from the past. Looking at them, one can travel back in time and imagine how the whole Avenida was once surrounded by such buildings. Over the last few years, Avenida Paulista has also changed appearance due to the growing number of homeless people settling there in small tents or behind shelters made of cardboard and plastic. Together with my daughter, I have walked down Oscar Freire, which has many fashionable shops and nice coffee places, and now also has its share of homeless people.

Miriam and I live in a flat located in Aclimação, a neighborhood next to Liberdade and close to the hospital where I had my surgeries and am now being treated in. When slavery was abolished in Brazil, many of the liberated people came to live in Liberdade, and this is the reason that the neighborhood got this name. Later, many people with Japanese origins came to live there, and in Liberdade, one finds excellent Japanese restaurants. São Paulo encompasses many different villages; it seems to be composed of a patchwork of townscapes.

One can take many different routes when walking in São Paulo. One can naturally pass the same place on different walks and can come to see a location from different perspectives. One can pass by rather fast, but also take one's time. One can look straight ahead when walking, but also look up and around. Over time, places are changing. Old buildings with history are demolished to make way for new, tall buildings shimmering with glass façades. When walking around, one can see buildings from a way off. One could see the Edifício Copan, designed by Oscar Niemeyer and located in walking distance of Praça da República. Like a huge but all too short snake, Copan seems to have found a comfortable position in between the sharp corners of streets and houses. At night, the tall Altino Arantes Building, also called the Farol Santander, is illuminated in purple colors and can be seen from far away, including from the balcony of our flat in Aclimação. Many neighborhoods in São Paulo remain unknown to me. The metropolis includes huge *favelas*, which I have only seen when driving by.

One can think of critical mathematics education as a metropolis, not made up of streets, places, architectures, and neighborhoods, but by ideas, examples, theories, and practices. In this book, I will present different routes around critical mathematics education through eighteen chapters. These routes, like my walks around São Paulo, will be crossing each other several times, and as a reader, you might come to see the same observation several times, although from different perspectives. Like São Paulo, critical mathematics education has also undergone many changes over time, and we will come to see how some ideas and concerns are rooted back in history.

The chapters in the book take the form of articles. All of the chapters can be read in isolation, or in any order one might prefer. I have numbered the chapters and grouped them in four separate parts, but this is only one suggestion for organizing walks around critical mathematics education.

In *Part I: Learning and Landscapes*, I address the notion of social justice, which is vitally important for formulating the concerns and hopes of critical mathematics

education. Landscapes of investigation establish learning environments different from those formed by the school mathematics tradition. Such landscapes make it possible for students and teachers in cooperation to identify and challenge cases of social injustice. This could be the exploitation of workers, patterns of structural racism, or erosions of democracy. Landscapes could also turn climate change into focus. Finally, I bring together a circle of concepts for interpreting learning processes, and in particular for understanding students' possible motives, or lack of motives, for learning mathematics. This cycle tends to capture low achievement in mathematics as a socioeconomic phenomenon rather than as a personal failure.

In *Part II: Crises and Formatting*, I explore the notion of crisis and the formatting power of mathematics. It has been advocated that, by means of mathematics, one can provide reliable pictures or models of reality, and also that mathematics is the language of science, ensuring objectivity and neutrality. This understanding of mathematics is challenged when we recognize the formatting power that can be acted out through mathematics. A crisis might concern economy, politics, security, hunger, sustainability, or pandemics. At times, mathematics provides a formatting of such crises by shaping how we see and react to them. A "banality of mathematical expertise" appears when mathematics is brought into action without considering the possible formatting power of such actions. This banality locates mathematics in an ethical vacuum, which might occur both in mathematics research and mathematics education. It is crucial to critical mathematics education to challenge this banality.

In *Part III: Critique and Dialogue*, a conception of mathematics as glorious, harmless, and innocent is confronted with a conception of mathematics as indefinite. Mathematics is indefinite with respect to concepts, proofs, culture, applications, and power. Mathematics might come to serve any kind of interest, which makes a critique of mathematics necessary. Critique has been conceived of as methodology (as inspired by René Descartes), as hammering (as practised by Friedrich Nietzsche), and as dialogue (as I am going to propose). I want to formulate a conception of critique that does not assume the absolutist features connected to critique as the right methodology without embracing the absolute relativism connected to critique as hammering. Seeing critique as dialogue is crucial for the further development of critical mathematics education.

In *Part IV: Width and Depth*, I illustrate how the significance of critical mathematics education reaches much further than an educational project addressing students at social risks socially. It is important for all groups of students to address cases of social injustice. Critical mathematics education concerns ways of doing research, and it tries deliberately to prevent assuming stereotypes common in mathematics education research. Insight provided by critical mathematics education has relevance not only for critical mathematics education itself, but for any kind of social theorizing. This insight might contribute to an understanding of the scope of social structuration, the emergence of risk society, the formation of life worlds, and characteristics of structural violence. In the final walk around in critical mathematics education, we recognize uncertainty as a universal human condition. This turns critique into a necessary human activity.

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Part IV Width and Depth

Part I
Learning and Landscapes

Chapter 1
Concerns and Hopes

Abstract Critical mathematics education is characterised in terms of concerns and hopes: concerns about what is taking place in the classroom, and about the students and their future opportunities in life. Concerns about socio-economic exploitation, racism, sexism, homophobia, xenophobia, and violence. Concerns about our environment. Critical mathematics education is also made up of concerns about mathematics itself, not only as a school subject but also as applied in technology, natural science, social science, economy, and management. Hoping is more than an individual or local undertaking; it also concerns out shared future. Hope represents a socio-political utopia, which might turn hope into political acts. Critical mathematics education engages students in the very formulation of what social justice could mean. I see the articulations of concerns and hopes as driving forces in the construction of conceptions of social justice. Such constructions can take place in the mathematics classroom.

Keywords Concern · Hope · Exploitation · Racism · Social justice · Environment · Social construction · Utopia

What is critical mathematics education? One could try to answer this question by describing how it might be acted out in the classroom; the nature of the problems that are addressed; and the ways students and teachers are invited to interact. One could also try to characterise critical mathematics education through its theoretical references, what philosophical ideas it relates to, and what conceptions of society and of learning it assumes. Previously, I have tried both such approaches, but I have never found either of them quite satisfactory.

Many times I have experienced teachers engaging their students in fascinating projects which I would consider exemplary for a critical mathematics education. However, the teacher might not have heard about critical mathematics education, and might describe the ideas of the projects in quite different terms. This is perfectly fine. I am not interested in trying to ringfence critical mathematics education, but rather in the concerns and hopes that are acted out through educational ideas and practices.

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After addressing concerns related to critical mathematics education, I will explore the notion of hope more carefully. Finally, I will interpret the notion of social justice as being under construction and illustrate this with a classroom example.

1.1 Concerns

Critical mathematics education is concerned about *what is taking place in the classroom*. Looking at explicitly formulated aims for mathematics education, everything appears obvious: mathematics education is the principal social institution for establishing mathematical knowledge among students. However, the actual roles of social institutions need not be what they are claimed to be. A paradigmatic recognition of this possibility was expressed by Karl Marx, when he labelled religion as being opium of the people. The church was an institution with a quite different function than commonly assumed. Could mathematics education be a socio-political instrument for making people submissive and easy to dominate? Such a possibility could be associated with the exercise paradigm dominating this education. Engaging students in investigative processes is an attempt to create learning environments that provide mathematics education with a different socio-political role.

Critical mathematics education is concerned about the *students*: not only their learning in the classroom, but their status as future citizens. An education for citizenship could mean preparing students to fit into the given social order, but it could also be an education for autonomy, turning students into critical citizens. It has been common in critical mathematics education to pay particular attention to students facing social risks. This could be students living in poor or in violent environments; it could be students of colour subjected to racist patterns of exclusion; it could be students doomed through caste systems or similar structures of oppression; and it could be immigrant students arriving into hostile environments. It is important for critical mathematics education to operate in solidarity with students at social risks. However, all groups of students are important to critical mathematics education, including students in comfortable positions. With this expression, I refer to students from economically affluent environments, about whom statistics suggest they will most likely go on to further education, obtain good jobs, and become economically well situated. However, it is important also to explore cases of social and economic injustice together with students in comfortable positions. It is important to draw their attention to the extreme inequalities of the world. Another group of students also important for critical mathematics education is that of university students in mathematics, who will come to represent mathematical expertise and to bring mathematics into action. They will come to exercise power through applications of mathematics.[1]

[1] See Chap. 15, “All Students”, for further elaborations of this point.

Critical mathematics education is concerned about the *exploitation of workers and their families*. This concern was the principle one for the emergence of the critical approach as formulated by Karl Marx. Sharp observations on the exploitation of workers during the initial period of industrialisation were made by Friedrich Engels (1993) in *The Condition of the Working Class in England*, first published in German in 1845. Here he documented, in descriptions and in numbers, the horrible conditions that the English working class was subjected to. In *Germinal*, published in 1885, Émile Zola (1954) provided a literary description of working-class conditions in the coal districts in Northern France. In *The Road to Wigan Pier*, first published in 1937, George Orwell (1937) provided an update of Engels' description of English working-class conditions. The accounts by Engels, Zola, and Orwell are different, but the concerns they raise are shared by many political and social movements, and also by critical mathematics education. Inspired by Marx, focus has been put on the exploitation of workers in industrialised countries. However, in a globalised world exploitations reach much further, also manifesting new forms of colonisation. It is a concern of critical mathematics education to address any form of exploitation.

Critical mathematics education is concerned about *racism*. Colonisation was an explicit and brutal form of exploitation. Colonies were supposed to serve as natural resources for the colonising powers. Colonisation was accompanied by slavery, and racism developed as a pathological justification of slavery. Racist ideologies – through which colonial abuse and cruelty sought legitimisation – normalised slavery. The British Empire was a paradigmatic case of worldwide exploitation, combining violent colonisation with the development of different versions of racism, including orientalism.[2] Colonisation means oppression, which marks the individual. In the two books *Black Skin, White Masks* and *The Wretched on the Earth*, first published in French in 1952 and 1961, Franz Fanon (2004, 2008) shows how the brutality of colonialism structures people's experiences and self-conceptions. The coloniser becomes elevated to a superior level, and this causes a perverse pressure on the colonised. Fanon points out that, in order to be accepted in a colonised world, the colonised needs to imitate the coloniser. This observation is condensed into the title of the book: *Black Skin, White Masks*. Fanon helps us to recognise that colonial exploitation operates at all levels: at the macro socio-political level as well as at the micro personal and experiential level. Racism is a devastating all-inclusive phenomenon. Today, racism has been directly confronted by many different movements, and also by critical mathematics education.

Critical mathematics education is concerned about our *environmen*t. Traditional critical positions, as directly inspired by Marx, did not express concern about the natural environment. The focus was first of all on the miserable social conditions for the working class in industrialised societies. Nature was considered an infinite resource that needed to be dominated abd used for cresting welfare. A profound preoccupation with the environment, however, developed during the 1960s and 1970s. Since then, a deep concern about our environment has been broadly

[2] For a discussion of orientalism, see Said (1979).

articulated. Formerly considered an unrestricted resource, nature has now been recognised as fragile. Human enterprises might have a devastating impact on nature. Pollution is recognised to have a tremendous impact on our living conditions. The oceans, once considered bottomless and used as a global waste bin, now need to be cleaned up. Preoccupation with our environment has become part of critical mathematics education.

Critical mathematics education is concerned about *mathematics* itself, not only as a school subject but as applied is a variety of contexts. Mathematics has been celebrated as a unique phenomenon, providing unquestionable truths. However, it is a concern of critical mathematics education to leave aside any such glorification of mathematics and to formulate a critical conception of it instead. It is a concern to present a critical awareness of the impact of bringing mathematics into action. Mathematics is a powerful tool for creating new technological possibilities. However, such possibilities might come to serve any kind of political and economic interests, also of the most questionable kind.

1.2 Hopes

To be concerned about something means hoping for something to be different. The notions of concern and hope are intrinsically connected, but they are also different. While the notion of concern is part of everyday discourse, hope has been addressed philosophically, and is recently turned into a political conception as well.

When we look through the history of philosophy, we find some notions appearing again and again, like "knowledge", "truth", and "justice". These belong to the crown jewels of philosophy. The notion of hope has also been addressed in philosophy, but it has not played any significant role. In the dialogue *Timaeus*, written 360 years before Christ, Plato points out that "hope easily led astray", as it mixes with irrationality.[3] Throughout his whole philosophy, Plato celebrates knowledge, ranking it much higher than belief, while hope, like belief, is seen as an expression of lack of knowledge. With the emergence of philosophies dedicated to Christianity, the discussion of hope took a new direction. Hope became connected to religious faith; it not only concerned a better life on earth, but first of all salvation and eternal life after death.

René Descartes and several other philosophers, both from the rationalist and the empiricist traditions, make observations about hope. Hope was associated with the notion of belief, and therefore considered less important than knowledge. In *Meditations on First Philosophy*, first published in Latin in 1641, Descartes (1993) concentrates on the principal theme of his philosophy: to establish the foundation of certain knowledge. In this work, one does not find references to the notion of hope, nor to the related notion of faith. However, in *The Passions of the Soul*, completed

[3] See Plato, *Timaeus*. The Internet Classics Archive, p. 26.

in 1649, Descartes addresses the notion of hope, although with some reservations. Compared to knowledge, the concept of hope remains in an inferior position.[4]

Through the work of Immanuel Kant, the notion of hope obtained a new position in philosophy. In *Critique of Pure Reason*, first published in German in 1781, Kant (1929) formulates the question "What may I hope?" in parallel with the two questions "What can I know?" and "What ought I do?" (p. 635). This means that a philosophy of hope runs parallel to epistemology and ethics, and so one may consider Kant the founder of a philosophy of hope.

In several places in his authorship, Søren Kierkegaard reflects on the notion of hope, although nowhere does he provide a systematic analysis of the concept. Contrary to Kant, Kierkegaard cuts all possible connections between hope and knowledge. To him, hope does not contain epistemic residues, but first of all religious worries. Hope concerns faith and is a strictly personal affair. According to Kierkegaard, hope of salvation is an entirely irrational affair. This cannot be different; this is a condition of life. By making such a claim, Kierkegaard introduces a new direction for existentialist philosophy.[5]

In radical opposition to Kierkegaard's position, Friedrich Nietzsche claims that religious belief does not make any sense. He sees Christianity as an expression of a deplorable slave morality. Hope associated to faith is not only pointless, it is the cause of human suffering. In *Beyond Good and Evil*, first published in German in 1886, Nietzsche (2009) claims all forms of hope in a "hidden harmony, in future blessedness and justice" to be pointless (§55, p. 60), and in *Human, All Too Human*, Nietzsche (1986) refers to hope as "the most evil of evils because it prolongs man's torment" (§71, p. 58).

In Ernst Bloch's (1995) monumental work *The Principle of Hope*, the philosophy of hope gets its own home ground. Bloch was German and of Jewish origins, and when the Nazis came to power, he and his wife fled from Germany, and via Switzerland, Austria, France, and Czechoslovakia, they finally got to the USA. Here, during the period 1938–1947, he drafted *Das Prinzip Hoffnung*, published in three volumes in 1954, 1955, and 1959, later translated as *The Principle of Hope*.

In the introduction, Bloch states that in the book he has made an extensive attempt "to bring philosophy to hope" (Volume I, p. 6). Contrary to Descartes, Bloch gives the notion of hope his full attention. He shares with Kant the conviction that hope is a crucial notion that needs to be investigated philosophically; however, he disagrees with most of what – until then – philosophy had contributed to its clarification. In particular, Bloch does not make any association between hope and religious ideas. Hope has nothing to do with faith or salvation, but is a completely mundane affair. Bloch's interpretation contrasts directly with the one provided by

[4] *The Passions of the Soul* has always been considered a peripheral work in Descartes' philosophy. It was dedicated to the Swedish queen Christina, who in 1649 had brought Descartes to Sweden in order to teach her philosophy. See *The Passions of the Soul,* https://TheVirtualLibrary.org

[5] Careful expositions of Kierkegaard's conception of hope are found in Bernier (2015), Fremstedal (2012), and Sweeney (2016).

Kierkegaard. Bloch might, in fact, share Nietzsche's conviction that hope, in the form of religious faith in salvation after death, is most dangerous.

In the introduction to *The Principle of Hope*, Bloch states that hope concerns the "dreams of a better life" (Volume I, p. 11), and Bloch has a better life *on earth* in mind. Bloch considers himself a Marxist, and throughout *The Principle of Hope* he makes many references to Marx. However, he is far from being an orthodox Marxist, and his work has only been half-heartedly embraced by other Marxists.

To Bloch, hope is formed through political visions and aspirations. Hope concerns what does not exist – at least, not yet. It concerns socio-political utopias. Bloch highlights that hope points towards possible states of affairs that might not be properly grasped in advance; it refers to vaguely anticipated alternatives. Bloch does not claim that, by departing from a Marxist outlook, one becomes able to specify the content of hope. Such content cannot be deduced from any theoretical or political positions. Still, Bloch does not think of hope as a free-floating fantasy. He locates hope in solid distance from fantasies, as well as from economic determinism that, according to orthodox Marxism, leads to a classless society. Instead, Bloch talks about true hope as being "mediated in terms of history and tendency" (Volume 3, p. 1372).

Bloch expresses his ideas in complex formulations. He talks about the act-content of hope, which I find an important concept for articulating the political dimension of hope. In Bloch's words:

> … the *act-content* of hope is, as a consciously illuminated, knowingly elucidated content, the *positive utopian function*; the *historical content* of hope, first represented in ideas, encyclopaedically explored in real judgements, is *human culture referred to its concrete-utopian horizon*. (Volume I, p. 146. Italics in original).

The act-content of hope is a utopian matter – not a utopia formed by religious fantasies, but by visions formed through "real judgements" elaborated within a "concrete-utopian horizon". In other places, Bloch refers to this as "true hope", which is a political hope concerning a better life on earth, contrasting a "false hope" about an eternal life after death.

One can think of hoping as an individual undertaking. A person can have many hopes about what they may achieve in the future. Hoping is a common human state of mind. Hopes can also be shared. The whole family can hope for the best for the children and the best for the neighbours. However, hope includes much more than individual or local undertakings. Hope also concerns future socio-political states of affairs. One can hope for a better society. Hope represents a shared socio-political utopia.

In *Pedagogy of Hope*, Freire (2014) pays particular attention to the notion of hope. He does not make any reference to Bloch's work, but he seems to be in harmony with Bloch's position: hope forms an integral part of a struggle for a better society.[6] In Freire's words: "I do not understand human existence, and the struggle needed to improve it, apart from hope and dream. Hope is an ontological need"

[6] For comments about Freire's and Bloch's conceptions of hope, see Streck et al. (2018).

(p. 8). Freire makes a similar point to Bloch, namely that one cannot rely on any economic determinism. The fulfilment of socio-political hopes cannot be taken as a result of certain economic dynamics. Freire states that "in the struggle to improve the world, as if that struggle could be reduced to calculated acts alone, or a purely scientific approach, is a frivolous illusion" (p. 9). And he adds: "Without a minimum of hope, we cannot so much as start the struggle. But without the struggle, hope, as an ontological need, dissipates, loses its bearings, and turns into hopelessness" (p. 9). Any struggle for a better society needs hope.[7]

For a period, the Brazilian philosopher Mário Sérgio Cortella worked closely with Freire, and Cortella (2015) has condensed Freire's conception of hope in the following way: "It is necessary to have hope, but to have hope from the verb to hope, because there are people who have hope from the verb to wait. And hope of the verb to wait is not hope, it is waiting" (p. 22, my translation).[8] Furthermore, Cortella highlights: "What I learned most from Paulo Freire was the idea of active hope, which is not pure waiting, but hope that searches, constructs, looks for, and knows that teaching, above all, is not just a job, it is a source of life. Teaching is a source of life, in which hope is our refusal of biocide, our refusal of the failure of life and, therefore, our way of existing and hope" (Cortella, 2017, p. 14, my translation).[9] It is important not to let hoping slide into waiting, but to turn hoping into an active search for what to do. Active hoping is important for Freire, for Bloch, and for critical mathematics education.

Daniela Alves Soares (2022) investigates dreams and hopes as formulated by young people living in disadvantaged situations. She reveals the deep complexities of such hopes, which combine personal and social elements. Students are formulating hopes with respect to their own lives, with respect to their families, with respect to their neighbourhoods, with respect to society in general. With her study, Soares illustrates what it could mean to establish hopes within a "concrete-utopian horizon".

The conception of hope as a socio-political force – and as an actual search for what to do – is crucial for critical mathematics education. Hopes concerns what could come to take place in the classroom, but also the direction social changes might take.

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[7] That hope is part of political activism has been underlined by Solnit (2016). She points out that even when the future looks dark, hope is possible. The title of her book is precisely that: *Hope in the Dark.*

[8] The original Portuguse version is: "É preciso ter esperança, mas ter esperança do verbo esperançar, porque tem gente que tem esperança do verbo esperar. E esperança do verbo esperar não é esperança, é espera."

[9] The original Portuguse version is: "O que mais aprendi com Paulo Freire foi a ideia de esperança ativa, que não é de pura espera, mas é a esperança que procura, constrói, busca e sabe que a atividade docente, acima de tudo, não é somente um emprego, é fonte de vida. A docência é uma fonte de vida, em que a esperança é a nossa recusa ao biocídio, a nossa recusa à falência da Vida e, portanto, o nosso modo de existir e esperançar."

1.3 Social Justice as Construction

In the literature on mathematics education for social justice, we find fascinating examples of classroom activities; however, it is more difficult to find thorough interpretations of the very notion of social justice. To work for social justice seems to be an intrinsic good thing to do. Who would ever start propagating an education for social injustices? Critical mathematics education is an expression of a concern for social justice, and it is relevant to consider more carefully the notion of *social justice*.

Social justice is a contested concept, meaning that it can be interpreted in different, and also contradictory, ways. Only if one assumes that a concept has a kind of genuine meaning that can be discovered, maybe by some hermeneutic approaches, then one might feel comfortable using the notion to designate some desirable states of affairs. But if we assume that the meaning of a concept becomes defined by how it is bought into operation in diverse discourses, the situation is rather different. The notion of social justice will be without a solid semantic kernel.

While the notion of "justice" has a long history in philosophy, the expression "social justice" first appears in the nineteenth century. It was mixed with religious interpretations, indicating that the meaning of social justice could be clarified through careful studies of the *Bible*. The notion of social justice appears in the title of the book *The Constitution Under Social Justice* (*Costituzione Secondo la Giustizia Sociale*) published in Italian in 1848 by Rosmini-Serbati (2007). He relates the notion to a range of political ideas circulating at the time.

In *A Theory of Justice*, first published in 1971, John Rawls (1999) performs a profound philosophical analysis of the conceptions of justice, drawing on inspirations from the liberal tradition, including works by John Stuart Mill, but at the same time he adds his own philosophic ideas to the discussion. At times, Rawls has addressed the question of how to identify justice in society in terms of a thought experiment. To him, such an identification can be reached through a process of dialogue among a group of people if two conditions are fulfilled: First, that everybody from the group is going to live in the society. Second, that nobody from the groups would know in advance what position they are going to obtain: being rich or poor, ill or health, man of woman, black or white, etc. In other words, Rawls presents idealised (as well as impossible) conditions for establishing a constructive process of the concept of justice.

Let us pay attention to the idea that, in order to define social justice, one is *not* going to be engaged in any analytical process of trying to discover the meaning of "justice", but in a process of social construction.[10] I move on from Rawls' thought experiment and consider the formation of a conception of social justice as taking place – not in any imagined congregation – but in real-life contexts. One such context is the classroom. I think of education for social justice in terms of processes that

[10] At times I have suggested ethical constructivism: seeing ethical conceptions, including conceptions of social justice, as an expression of social constructions. See Skovsmose (2012, 2018).

engage students in the very formulation of what social justice could mean, and not as an education informing students about justices and injustices. A principal step is to engage students in identifying and articulating what they find to be just or unjust, and also to be ready to challenge their opinions.

Looking along the production line for a given product like a T-shirt, it is possible to locate many cases of injustices. In "Sweatshop Accounting", Larry Steele (2013) presents a project involving high school students "with the hope to be future business leaders" (p. 78).[11] The project starts with the students taking a look at the labels in their own clothes: "I was made in Vietnam" (p. 78), one of the students observed. The students marked the location with a yellow post-it note at a world map, and soon the yellow notes are shown in China, the Philippines, Malaysia, Indonesia, and many other countries.

More information was provided for the students in the form of videos, references, and papers. They received more specific information with respect to the production of certain fashion sports shoes, which in the shops at that time did cost 50 USD. At the other end of this production line, one finds the factory workers whose salaries for producing the pair of shoes were 0.40 USD. Between these two extreme points on the production line, we find, for instance, the production cost to be 1.60 USD; the profit of the factory to be 2 USD; and the cost of advertising and publicity to be 13.50 USD. The image one gets is that the total price of the pair of shoes is added up incrementally. Further information is provided with respect to the living conditions of a worker at a garment factory in Nicaragua. The salary at the time is 0.31 USD per hour, which gives them 14.88 USD per week. Costs for transport to the factory adds up to 15.04 USD per week. Costs for necessities – like electricity, food, and wood for an outdoor stove – bring the total costs up to 29.34 USD per week. Many costs, such as buying clothes, have not been incorporated in the calculations. How could a worker and their family possible survive?

By engaging students in a project addressing what takes place along a production line in our globalised world economy, one might identify different forms and degrees of exploitations. The students acquire a space for expressing what to consider reasonable working conditions, reasonable salaries, and reasonable shop prices. It is possible for the students to argue why one finds something to be unjust; one might also come up with counterarguments. Both arguments and counterarguments might find support in graphs, statistics, calculations, and numerical estimations. The construction of conceptions of social justice is far from being only an epistemic endeavour; it also encompasses experiences and emotions. The teachers as well as the students can express their opinions, which is an important component of the constructive process.

In "Sweatshop Accounting", the students had not experienced the deplorable working conditions that they became aware of. However, for formulating conceptions of social justice, it is important that they become engaged, not only in

[11] In Chap. 2, "Landscapes and Racism", I address the notion of landscape of investigation, and "Sweatshop Accounting" is an example of such a landscape.

situations they might be familiar with, but also in situations they come to learn about. It is important to act against any normalisation of injustices. Such normalisation might bring about resignations or indifferences: resignations among those who suffer injustices and indifferences among those who are spectators to injustices. In "Sweatshop Accounting", a possible indifference was challenged: the indifference that we all might subscribe to when buying a pair of sport shoes focusses on the price, and ignoring the exploitations and sufferings taking place along its production lines.

I have been talking about conceptions of social justice in plural, as one cannot assume that social constructive processes will converge towards a uniform clarification of social justice. We are dealing with contested constructions, driven forwards by articulations of concerns and of hopes. Such processes can take place in the mathematics classroom.

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Chapter 2
Landscapes and Racism

Abstract It is a concern of critical mathematics education to construct landscapes of investigation. Such landscapes create possibilities for engaging students in investigative processes, for establishing dialogic interactions in the classroom, and for reflecting critically on socio-political issues. It is a concern of critical mathematics education to address racism in all its forms and appearances and to contribute to the general struggle against racism. Different forms of racism will be discussed, including the phenomenon of structural racism. A landscape of investigation will be presented, entitled *Media and Racism*, which addresses everyday features of racism. A Bias Index will be defined, which provides a powerful way of capturing expressions of structural racism. By means of this index, one can also address ways in which sexism might be acted out. We can consider biases with respect to any kind of gender issues. Any such applications of the Bias Index might evolve into rich landscapes of investigation.

Keywords Landscapes of investigation · Prescription readiness · Racism · Structural racism · Degree of visibility · Bias-Index

Mathematics education takes place all over the world: in schools, faculties, universities. It is a well-established social institution with an apparently well-defined function: to make students learn mathematics. Considering all the economic resources put into this activity, it seems impossible to question the social relevance of mathematics education. However, the socio-political role of mathematics education is far from transparent.

On several occasions, Alexandre Pais has highlighted that we cannot forget that mathematics education takes place in a society where capitalist structures and accompanying ideologies permeate whatever is taking place. As an illustration, he considers the broadly celebrated slogan "mathematics for all". According to Pais (2012), this slogan covers up an obscenity of a school system that "year after year throws thousands of people into the garbage bin of society under the official discourse of an inclusionary and democratic school" (p. 58). The crude reality is that "in order for some to succeed others have to fail" (p. 58). The task of identifying

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those to be thrown into the garbage bin – and the task of performing this very act – becomes concealed beneath respectable looking aims and metaphors like "mathematics for all". This obscenity forms an integral part of the socio-political role of mathematics education.

In *Travelling Through Education* (Skovsmose, 2005), I highlighted that, if one reads aloud all the exercises with which students are presented during their schooling, it would sound like a long sequence of commands. To follow orders is an important functional "quality" of attentive students as well as of an obedient workforce. I have referred to this obedience as *prescription readiness* (Skovsmose, 2008).

For critical mathematics education, it is crucial to challenge the role that mathematics education might play in preparing students to become part of a submissive and prescription-ready workforce, assuming a given political and economic order of things. One step towards doing so is to invite students to engage in investigative processes. A second step is to address controversial socio-political problems as an integral part of learning mathematics. I see the creation of landscapes of investigation as a practical attempt to create learning environments that make such activities possible.

In what follows, I first present more carefully the concept of landscapes of investigation. I then concentrate on the problem of racism. It is a concern of critical mathematics education to confront racism in all its forms and to contribute to the general struggle against racism. I elaborate in more detail on a landscape of investigation called *Media and Racism*, which addresses some everyday features of racism. I conclude the chapter by presenting a Bias Index that might be a useful tool for pointing out cases of structural racism.

2.1 Landscapes of Investigation

By landscapes of investigation, I refer to learning environments that open the learners up to a broader spectrum of problems. This could include problems related to mathematics and its applications. It could include problems concerning social issues, maybe as experienced by the students, maybe as addressed in public discourses, maybe as presented by the teachers. It could include problems concerning economic exploitation, social marginalisation, sexism, pollution, climate changes, any cases of social injustice. The students can explore landscapes in groups and through project work.

Landscapes of investigation also establish patterns of classroom communication different from those patterns characteristic of the school mathematics tradition. Dialogues can be established among students and between students and teachers. This generates resources for critical reflection, both with respect to mathematical issues and with respect to socio-political issues.[1]

[1] The concept of landscape of investigation has been developed over a longer period. For a broad presentation of landscapes of investigation, see Penteado and Skovsmose (2022).