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Preface

In the process of research on target tracking and information fusion, the authors were deeply impressed by the great influence of the random finite set (RFS) theory. As a scientific top-down method, the RFS theory provides a unified theoretical description framework and solution for target detection, tracking and identification, situation assessment, sensor management and other problems involved in information fusion, differing from the traditional methods which decompose these problems into independent sub-problems to be solved separately.

The RFS theory, started at the end of the twentieth century, was created and pushed forward by Ronald Mahler et al. However, due to its abstraction and complexity, the RFS theory was not highly regarded by the academic community for a time. With the pioneering work of B-N Vo, such as the sequential Monte Carlo (SMC) and Gaussian mixture (GM) implementations of probability hypothesis density (PHD) filter, the realizations of the RFS theory were opened up. The above implementation methods were adopted with terms and symbols commonly used in the field of target tracking and information fusion, which greatly promotes the development of the RFS theory. Based on these, the implementation methods of cardinalized probability hypothesis density (CPHD) and multi-Bernoulli (MB) filter were proposed in a few years. In particular, in 2013, the labeled RFS filters such as the generalized labeled multi-Bernoulli (GLMB) filter were put forward, which made the RFS theory more and more perfect, and attracted widespread attention from famous domestic and foreign scholars in the field of target tracking and information fusion, resulting in an explosion of research results. To some extent, the RFS theory has become a new development direction of target tracking and information fusion. However, currently, there are only a few books that introduce this theory in a systematic manner—two books written by Mahler were published. As mentioned earlier, these books mainly focused on the theory, which can be difficult for beginners to understand. How can we popularize the RFS theory in a way that can be easily accepted by more people and promote the vigorous development of RFS theory? That is the purpose of this book.

This book adopts terms and symbols that are friendly to the academic world of target tracking and data fusion, with a focus on the systematic introduction of the specific implementations of the RFS theory in the target tracking field. It covers most of the current research results in this field: PHD, CPHD, MB, labeled MB (LMB), GLMB, δ-GLMB and marginalized δ-GLMB (Mδ-GLMB) filters. These filters are the latest technologies in the target tracking field and have provided new ideas and effective solutions for target tracking. After a systematic introduction to the above filters, their extensions and popular applications are described in detail, including maneuvering target tracking, target tracking for Doppler radars, track-before-detect of dim targets, target tracking with non-standard measurements and distributed multi-sensor target tracking. This book is well organized with systematic and comprehensive contents keeping up with popular development of advanced technology, making it suitable for beginners, graduate students, engineering and technical personnel in the field. According to the authors' learning experience, the authors suggest that readers can read this book first, and then study Mahler's original work, so that the books can complement each other and improve the learning efficiency of readers.

Upon the publication of this book, the author recollected that the draft had been revised numerous times since the end of 2015 in order to continuously absorb the latest research results. Although the book is completed, the author is deeply gratified by the fact that there are still more to learn. In addition, the authors have a lot to thank for. Thanks to the pioneering work of Ronald Mahler, B-N Vo and others, the authors have been able to feed on and draw upon their extensive work in writing; thanks to the National Natural Science Foundation of China (No. 61601510), the Young Talent Support Project of China Association for Science and Technology (No. 18-JCJQ-QT-008) and National Defense Industry Press for their funding and strong support; thanks to the editors of National Defense Industry Press for their hard work; thanks to the reviewers as well as the experts and scholars who care about the publication of this book.

This book provides a systematic and comprehensive introduction to the application of the RFS theory in the field of target tracking, and its application in the field of higher-level information fusion also has exciting and broad prospects. The authors sincerely hope that this book can be used to promote the further development of the RFS theory. Although we have made the greatest efforts to write this book, inadequacies are hard to avoid due to the limit of our knowledge, so we welcome any criticisms and corrections from readers.

Guiyang, China
October 2022

Weihua Wu

Contents

Abbreviations

CBMeMBer	Cardinality balanced MeMBer
CI	Covariance intersection
C-K	Chapman–Kolmogorov
CKF	Cubature Kalman filter
CPEP	Circular position error probability
CPHD	Cardinalized PHD
CRLB	Cramer–Rao lower bound
DA	Data association
DBZ	Doppler blind zone
DLI	Distinct label indicator
EAP	Expected a posterior
EK-CPHD	Extended Kalman CPHD
EKF	Extended Kalman filter
EK-PHD	Extended Kalman PHD
EMD	Exponential mixture density
ET	Extended target
FISST	Finite set statistics
GCI	Generalized covariance intersection
GGIW	Gamma Gaussian inverse Wishart
GIF	Generalized indicator function
GIW	Gaussian inverse Wishart
GLMB	Generalized labeled multi-Bernoulli
GM	Gaussian mixture
GMTI	Ground moving target indicator
GSF	Gaussian sum filter
IID	Independent and identically distributed
IIDC	IID cluster
IMM	Interacting multiple model
IPDA	Integrated PDA
ITS	Integrated track splitting
IW	Inverse Wishart

JIPDA	Joint IPDA
JM	Jump Markov
JPDA	Joint PDA
KDE	Kernel density estimation
KF	Kalman filter
KLA	Kullback–Leibler average
KLD	Kullback–Leibler divergence
LG	Linear Gaussian
LGJM	Linear Gaussian jump Markov
LGM	Linear Gaussian multi-target
LMB	Labeled multi-Bernoulli
MAP	Maximum a posterior
MB	Multi-Bernoulli
MCMC	Markov chain Monte Carlo
Mδ-GLMB	Marginalized δ-GLMB
MDV	Minimum detectable velocity
MeMBer	Multi-target multi-Bernoulli
MHT	Multiple hypothesis tracking
MM	Multiple model
MoE	Multi-object exponential
MS	Multi-sensor
MTT	Multi-target tracking
NWGM	Normalized weighted geometric mean
OSPA	Optimal sub-pattern assignment
PDA	Probabilistic data association
PDF	Probability density function
PF	Particle filter
PGF	Probability generating function
PGFl	Probability generating functional
PHD	Probability hypothesis density
PMF	Probability mass function
RD	Renyi divergence
RFS	Random finite set
SMC	Sequential Monte Carlo
SNR	Signal-to-noise ratio
TBD	Track-before-detect
UK-CPHD	Unscented Kalman CPHD
UKF	Unscented Kalman filter
UK-PHD	Unscented Kalman PHD
UT	Unscented transformation
δ-GLMB	δ-Generalized labeled multi-Bernoulli

Chapter 1
Introduction

1.1 Basic Concepts of Target Tracking and Random Finite Sets

Target tracking is a process carried out to estimate target states based on sensor measurements. A target is usually an object of interest, e.g. a vehicle, a ship, an aircraft, or a missile. The single-target refers to one target while multi-target refers to more-than-one targets. The target state refers to the unknown but interest information about the target. The typical target state includes the location and speed in the Cartesian coordinate system and it may also include other target characteristics such as identity, attribute, amplitude, size, shape or similarity. To estimate the unknown state of a target, sensor measurements are needed, which typically include time stamp, range, azimuth, elevation, Doppler, and amplitude information. The scope of measurement differs from sensors to sensors. For an active radar, the measurement mainly includes time stamp, range and azimuth; for a three-dimensional radar, elevation is also included; for an airborne Doppler radar, Doppler (or radial speed) information is also contained; and for the passive sensors such as IR and electronic support measurement (ESM) sensors, angle information such as azimuth and elevation is usually measured. Sensor measurements may originate from targets of interest, or from targets of no interest or clutters. Even if a sensor measurement come from a target of interest, the target may be detected with certain errors, or the target even missed due to limited observing capability of a sensor. Besides, sensor measurements may be collected by a single sensor or multiple homogeneous or heterogeneous sensors.

Generally speaking, the Bayesian filter is often used in target tracking. The mainstream Bayesian filter includes the Kalman filter (KF), extended KF (EKF), unscented KF (UKF), cubature KF (CKF), Gaussian sum filter (GSF) and particle filter (PF). The Bayesian filter usually involves "two models" and "two steps". The two models are the motion model (also known as dynamic model) and the measurement model, both of which are collectively referred to as the state-space model. The motion model describes the evolution of the target state over time. Common motion models

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include constant velocity (CV), constant acceleration (CA), and coordinate turning (CT). The measurement model describes the linear or nonlinear relationship between the measurement and the target state. The two steps are prediction and update. The prediction step utilizes the motion model to predict the target state, while the update step utilizes the collected measurement to update the target state according to the measurement model.

The above filters are mainly used to reduce the impact of noise and, generally, only applied to ideal conditions (e.g. a single target is always present and there is no clutter). In fact, due to the limitation of sensor detection capabilities, in addition to noise interference, missed detection of targets occurs from time to time, and is inevitably accompanied by interference from non-target items (e.g. clutter) and other targets. The real challenge of target tracking is multi-target tracking (MTT) in clutter environment. Multi-target tracking refers to the estimation of unknown and time-varying number of targets and their tracks based on sensor observations. While the terms "multi-target tracking" and "multi-target filtering" are often interchangeable, there are still subtle differences between the two. Multi-target filtering involves estimation of unknown and time-varying number of targets and their independent states based on sensor observations; but for multi-target tracking, target tracks (track labels are required in the actual multi-target tracking system to distinguish different targets) are also of interest [1]. Therefore, multi-target tracking is essentially multi-target filtering that can provide target track estimation. Strictly speaking, multi-target tracking should be referred to as multi-target tracking filter.

Compared with the (single-target) Bayesian filtering algorithm, the main difficulty of multi-target tracking is the further increase of uncertainty. In addition to the uncertainty caused by the measurement noise and missed detection of targets, there is also the association uncertainty caused by the corresponding relationship between the measurement, clutter and each target. To overcome the association uncertainty, data association (DA) is usually carried out before Bayesian filtering in multi-target tracking to determine whether a measurement is from a clutter or to which target the measurement belongs. Through data association, multi-target tracking is thus decomposed into multiple single-target tracking problems. Famous DA-based MTT algorithms mainly are the joint probabilistic data association (JPDA) and multiple hypothesis tracking (MHT). However, the DA-based MTT problem is a non-deterministic polynomial (NP) problem, which requires a lot of computation and has the problem of combination explosion. In addition, it is difficult for these algorithms to give satisfactory results when multiple targets are close to each other and behaviors such as newborn, spawning, merging and death of targets are considered. Essentially, in these algorithms, the target state is modeled as a random variable (or random vector). Because of the birth[1] and death processes of each target, the number and states of moving targets are time-varying and unknown, hence it is difficult to model the finite and time-varying targets and measurements mentioned above with random variables. In order to track multiple targets with time-varying numbers, a complete and practical multi-target tracking should include track management (such

[1] Target birth includes target newborn and target spawning.

as track initiation and track termination) for behaviors such as target birth, merging and death, in addition to data association. Therefore, in a certain sense, the DA-based MTT algorithms adopt a divide-and-conquer, bottom-up approach.

In recent years, a type of tracking algorithm based on random finite sets (RFS) has emerged and attracted great attention. The RFS approach provides a multi-target Bayesian formula for the problem of multi-target filtering/tracking, where a set of target states (referred to as the multi-target state) is regarded as a finite set. Both random variables and RFS are random, except that the number of elements (referred to as the "cardinality") in the RFS is random and out of order. In particular, random variables take values according to a certain probability distribution in a certain space, while the RFS is a set. Unlike the traditional concept of a set, the number of elements in the RFS is uncertain, namely, the cardinality of the set is a random variable, and every element of the set is also random, which means it may or may not exist. If it does exist, its value follows a certain distribution. Therefore, an RFS is a set-valued random variable, which is the generalization of the concept of the random variable in probability theory. In fact, an RFS is actually a set in which elements and their number are also random variables. A random variable is used for solving random point functions while the RFS is used for solving random set-valued functions. The RFS theory is a generalization of point variable (vector) statistics to "set variable" statistics (finite set statistics). The RFS theory is also known as the point process theory, or more accurately, simple point process theory.[2]

In conclusion, different from the DA-based tracking algorithms, the RFS-based tracking algorithm models the multi-target state and the multi-target measurement as RFSs, and naturally incorporates the mechanism for track initiation and track termination. Hence, it is a top-down scientific approach and can realize the simultaneous estimation of the number of targets and their states. In addition to the MTT application, it also provides a unified theoretical framework and solution for target detection, tracking and identification, situation assessment, multi-sensor (MS) data fusion and sensor management [2].

Due to the systematic and scientific features of the RFS theory, it has developed into the 4th generation filter (Here, the 2nd, 3rd, and 4th generation filters refer to the cardinalized probability hypothesis density (CPHD) [3], multi-Bernoulli (MB) [4, 5], and generalized labeled multi-Bernoulli (GLMB) filters, respectively; see the following for details) in just over 10 years since the proposal of the 1st generation probability hypothesis density (PHD) [6, 7] filter, and has rapidly penetrated into various applications in the tracking field, showing its tenacious vitality. On top of the detailed introduction of the four generations of filters, this book also respectively introduces the main extensions and applications of each filter, including maneuvering target tracking, target tracking with Doppler radars, track-before-detect (TBD) for dim targets, target tracking with non-standard measurements, and distributed multi-sensor target tracking.

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[2] A set for simple finite point process does not allow repeated elements, and only contains a limited number of elements.

1.2 Research Status of Target Tracking

Target tracking covers a wide range of contents. From the perspective of the number of targets, it can be classified into single target tracking and multi-target tracking; from the perspective of motion models, it contains constant velocity, constant acceleration and coordinate turning models; from the perspective of targets' environment, it can be classified into target tracking with clutter and target tracking without clutter; from the perspective of the number of sensors, it can be classified into single-sensor target tracking and multi-sensor target tracking; from the perspective of sensor property, it contains active sensors and passive sensors; and from the perspective of the spatial dimensions, it can be classified into two-dimensional target tracking and three-dimensional target tracking. The main process of target tracking generally includes data pre-processing, track initiation, filtering/tracking, and track termination. A variety of implementation methods are available for each process, especially for filtering/tracking. In addition, the integration of target tracking and detection, recognition, sensor management and decision-making is also the current research focus. In recent years, the emerging RFS approach provides a unified theoretical framework for the development of integration, and it is developing vigorously.

The following will briefly introduce the research status of single-target tracking and classical multi-target tracking [8–13], and then focus on the development status of RFS-based multi-target tracking [14, 15].

1.2.1 Single Target Tracking

The mainstream single target tracking filters mainly include: KF [16, 17], EKF [18, 19], converted measurement KF (CMKF) [20, 21], UKF [22, 23], CKF [24, 25], and PF [26, 27]. The KF is the optimal solution for linear Gaussian (LG) systems, which has made important contributions to the development of filtering theory, and the subsequent advanced filters are derived and developed from it to some extent. However, almost all practical systems are nonlinear and non-Gaussian. For example, measurements such as slant distance, azimuth and Doppler in the tracking problem are nonlinear functions of unknown states. Generally, the optimal solution for non-linear filtering can not be obtained. The EKF had once become the "standard solution" for nonlinear systems. It obtains the sub-optimal solution by linearizing the nonlinear system. However, the Jacobian matrix needs to be solved in the EKF method, which limits its scope of application. Compared with first-order EKF, the performance improvement for high-order EKF is small, but its computational complexity increases greatly. The converted measurement KF (CMKF), as the name implies, is to reconstruct the linear measurement equation after the measurement is converted, and to derive the corresponding covariance of converted measurement errors, and then execute the KF. It has been proved that the performance (estimation accuracy, robustness, consistency, etc.) of the CMKF is better than that of the EKF for nonlinear measurement systems [20, 21]. Based on the unscented transformation (UT) [28], the UKF approximates the statistical properties of random variables with a finite number of parameters. Unlike the EKF, which approximates the nonlinear dynamic and/or nonlinear measurement models by linearization, the UKF approximates the probability density function (PDF) of the state vector. The UKF is widely used because it does not need to derive and calculate the complex Jacobian matrix or higher order Hessian matrix, but its estimation can reach the 2nd order accuracy. The CKF is a new filtering algorithm based on the principles of cubature numerical integration. It has the advantages of high numerical accuracy and strong robustness, and its estimation can reach the 3rd order accuracy for the nonlinear Gaussian system. All the above algorithms assume that the PDF approximately complies with Gaussian or Gaussian mixture (GM) distribution. For (nonlinear) non-Gaussian systems, the PF method, which is also known as sequential Monte Carlo (SMC) [29] method, is required. A large number of improved methods have emerged, which have promoted the significant development of the PF algorithm, but its computational amount is significantly increased, compared with the aforementioned filtering algorithms.

The above filtering algorithms are mainly for single target non-maneuvering models. When a target maneuvers, it will lead to the mismatch between the motion model used in the filtering algorithms and the actual motion model of the target, causing the filter to diverge. For maneuvering target tracking, the algorithms adopted can be classified into tracking algorithms with maneuvering detection and adaptive tracking algorithms. The former category mainly includes tunable white noise model [30], variable-dimensional filtering [31], and input estimation algorithm [32]. However, these algorithms have problems such as detection delay and detection reliability; and the latter category mainly includes the first-order temporal correlated noise model [33], current statistics (CS) model, Singer model, Jerk model [34], multiple model (MM), and interacting MM (IMM) [35]. The IMM is a suboptimal filter with good cost-effectiveness ratio. In addition, it has strong extensibility and can be easily combined with other algorithms, such as IMM probabilistic data association (PDA) [36], IMM-JPDA [37], and IMM-MHT [38]. Due to its excellent performance, the IMM has gradually become the mainstream for maneuvering target tracking.

1.2.2 Classical Multi-target Tracking

Due to the complexity of the actual environment and since the tracking performance is affected by factors such as clutter, sensor performance (such as missed detection) and multiple targets, it is difficult for pure filtering algorithms to achieve effective target tracking. A classic tracker generally consists of two steps: data association (DA) and filtering. Filtering is meaningful only if it is based on correct data association. Data association refers to the process of determining the one-to-one correspondence between measurements and targets. Through DA processing, multi-target tracking is decomposed into multiple single-target tracking tasks. Therefore, data association, as the core of DA-based MTT algorithms, is mainly classified into two categories:

maximum likelihood data association and Bayesian data association. The former includes the track splitting method [39], the joint maximum likelihood method, and 0–1 integer programming method. They are usually achieved by batch processing, and their basic ideas are to maximize the likelihood function. The latter includes PDA [40], global nearest neighbor (GNN) [41], S-dimensional (S-D) assignment [42], integrated PDA (IPDA) [43], joint probabilistic data association (JPDA) [44], joint IPDA (JIPDA) [45], integrated track splitting (ITS) [46, 47], belief propagation (BP) method [48, 49] based on sum-product algorithm [50, 51], Markov Chain Monte Carlo (MCMC) data association [52, 53], and MHT [54]. The advantage of these methods is that they have recursive forms and can obtain the state estimation in real time. As mentioned earlier, the DA-based MTT problem is an NP problem, which requires a lot of computation and has the problem of combination explosion. In tracking algorithms, 60–90% of the computation time is consumed by data association [11], among which, the maximum likelihood data association is usually larger than the Bayesian data association. Therefore, one of the important research contents of classic MTT algorithms is to reduce the calculation amount and improve the real-time performance. For example, the m-best S-D assignment algorithm [55] and K-best MHT method [56] constrain the number of hypotheses by limiting the optimal m (or K) number of association hypotheses.

1.2.3 RFS-Based Multi-target Tracking

The first to use the RFS theory to systematically process multi-sensor multi-target filtering is Mahler's FISST (FInite Set STatistics) [14, 15]. The FISST is a systematic and unified method for multi-sensor multi-target detection, tracking and information fusion. It can realize the Bayesian unification of detection, classification, tracking, decision-making, sensor management, group target processing, expert system theories (fuzzy logic, DS theory, etc.), and performance evaluation for multi-platform, multi-source, multi-evidence, multi-target, and multi-group problems. It has the following advantages: it is an explicit, comprehensive and unified statistical model based on multi-sensor multi-target systems; it can combine the two independent purposes of multi-target tracking, namely target detection and state estimation, into a single, seamless, Bayesian optimal step; it serves as a rich soil for cultivating new methods of multi-source multi-target tracking and information fusion, and has promoted new multi-target tracking algorithms, such as the PHD, CPHD and MB filters. Although these new algorithms do not require data association between measurements and tracks, their tracking performance (in terms of tracking accuracy and execution efficiency) is comparable to or even better than that of conventional multi-target tracking algorithms [2].

The core of Mahler's RFS theory is multi-target Bayesian filter [57]. This filter, similar to single target Bayesian filter, is also composed of prediction and update. Although the form is elegant and simple, the RFS-based optimal multi-target Bayesian filter is not practical due to the combinatorial nature of multi-target density and the multiple integration of infinite-dimensional multi-target state space. To this end, Mahler derived a variety of principle approximations, such as the PHD, CPHD, and MB filters. Specifically, the PHD filter is obtained by the first-order moment approximation of the optimal multi-target Bayesian filter. Based on the classic point process theory, another derivation method of the PHD filter is presented in [58]. Then, the Poisson hypothesis condition for the number of targets is further relaxed, and the CPHD filter, the second-order moment approximation of the optimal multi-target filter, is derived. It not only propagates the intensity, but also propagates the cardinality distribution (the probability distribution of the number of targets). At the cost of increasing the computation amount, both of its filtering accuracy and estimation accuracy for number of targets are better than those of the PHD filter. In addition, Mahler also proposed the MB filter, also known as the multi-target multi-Bernoulli filter (MeMBer) [14, 59–61]. Different from the PHD and CPHD filters, which approximate the first- and second-order moments of the optimal filter, the MB filter is the probability density approximation of the optimal filter.

Despite the approximate processing, there are still complex operations such as multiple integrals in recursive expressions of the above three filters, resulting in no analytical solutions under general conditions. To this end, Vo developed an SMC implementation (also known as particle implementation) method [62] for the PHD filter under general conditions, denoted as the SMC-PHD filter. Then, under the linear Gaussian condition, the analytic form of the PHD recursion was derived, and the Gaussian mixture (GM) implementation of the PHD filter was developed, denoted as the GM-PHD filter [63]. By using linearization and unscented transformation, the closed-form recursive formula applicable only to linear models can be extended to moderate nonlinear models. Based on this, Vo et al. developed two CPHD implementation methods [64], namely the SMC-CPHD and GM-CPHD filters. Vo et al. also found that the MeMBer filter proposed by Mahler was biased in estimating the number of targets. Therefore, a cardinality balanced MeMBer (CBMeMBer) filter was proposed, and two corresponding versions were also developed, namely the SMC-CBMeMBer [4] and GM-CBMeMBer [5] filters. The above approximation algorithms have been proved to have fine convergence properties [65–68]. Nevertheless, these filters are not multi-target trackers in principle and cannot provide track labels [62]. In order to solve the problem of target track output, the concept of labeled RFS was introduced in [69, 70], in which the first GLMB analytical implementation of multi-target Bayesian filter was derived by using the conjugacy of the GLMB family (with respect to the standard measurement model), and the GLMB and δ-GLMB filters (also referred to as Vo-Vo filters) were put forward. Simulation results showed that the δ-GLMB filter is superior to the approximations of multi-target Bayesian filter. However, the δ-GLMB filter requires a large amount of computation. Inspired by the PHD and CPHD filters, the labeled multi-Bernoulli (LMB) [71] and marginalized δ-GLMB (Mδ-GLMB) [72] filters were developed respectively. Among them, the LMB filter only matches the first-order statistical moment of the δ-GLMB posterior, while the Mδ-GLMB filter matches the first-order moment of the δ-GLMB posterior and cardinality distribution. In addition to these filters, multi-target trackers incorporating the label in the target state also include the particle marginal Metropolis-Hasting tracker [73]. The monographs [14, 15] and Refs. [74, 75] have provided systematic and comprehensive introductions to this field. It is worth pointing out that, under the harsh conditions of high false alarm rate, high missed detection rate and high uncertainty of measurement source (because targets are very close to each other), Ref. [76] first verified that, the GLMB filter was able to track multiple targets with a peak value of 1 million on ordinary commercial computers. The work above has laid a solid foundation for the research of RFS-based tracking algorithms.

In short, as the RFS-based tracking algorithm becomes more and more mature, its scope of application becomes more and more extensive, such as sonar image tracking [77, 78], audio signal tracking [79], video tracking [80, 81], robot simultaneous localization and mapping (SLAM) [82, 83], traffic surveillance [84], ground moving target indication (GMTI) tracking [85], track-before-detect [86], multi-station passive radar tracking [87], angle-only tracking [88], multiple input multiple output (MIMO) radar tracking [89], sensor networks and distributed estimation [90–92], as well as target tracking with non-standard measurement models, such as tracking of unresolved targets (or tracking with merging measurement) [93, 94], extended target (ET) tracking [95], and group target tracking [96, 97].

1.2.3.1 Probability Hypothesis Density Filter

The PHD filter recursively propagates the posterior intensity (first-order moment) of the multi-target state set, and projects the posterior probability density of the multi-target state set onto the single-target state space with "minimum loss" [6]. In this way, the PHD filter only needs to implement recursion in single-target state space, which significantly reduces the complexity of computation. However, calculating the posterior probability by Bayes rule requires the integration of the product of prior and likelihood functions to obtain the normalization constant. Practically, it is still very difficult to implement multi-dimensional integration, and there is generally no closed analytic form available under nonlinear non-Gaussian conditions. Therefore, Mahler and Vo et al. studied the implementation of the PHD filter, and presented the SMC-PHD filter [62] for nonlinear non-Gaussian conditions and the GM-PHD filter [63] for linear Gaussian conditions.

In order to improve the effectiveness of particle implementation in the SMC-PHD filter, an auxiliary random variable was introduced in [98] to incorporate the measurement information into the importance function, and the auxiliary particle implementation of the PHD filter was proposed as well. In [99], the authors used the Gaussian mixture expression to approximate the importance function and predicted density function through the unscented information filter, and then put forward the Gaussian mixture unscented SMC-PHD filter. Reference [100] called the Rao-Blackwellisation to achieve a more effective SMC implementation for some specific formal models. In order to avoid the degradation and not be limited to the Gaussian system, considering that any curve can be represented by splines, the spline PHD (SPHD) filter was proposed in [101]. However, this algorithm is only suitable for tracking a few valuable targets under severe conditions because of its high complexity. In addition, considering that the statistical characteristics of noise are usually unknown in most of the actual systems, the robustness of a filter is very important. Taking into account the unknown characteristics of nonlinear models and uncertain noise, the H_∞ filter is more robust than the Kalman filter in terms of model errors and noise uncertainty. Therefore, based on the H_∞ filter, a new GM implementation for the PHD recursion was proposed in [102]. In order to solve the problem of close target occlusion in multi-target tracking, considering the "one-to-one correspondence" assumption of measurement-target in the GM-PHD filter, a competitive GM-PHD method is proposed in [103] by calling a re-normalization scheme to reset the weight assigned to each target in the GM-PHD recursion. In [104, 105], the PHD filter is applied to joint detection, tracking and classification (JDTC) of multiple targets. The goal of JDTC is to simultaneously estimate the time-varying number, kinematic states, and class labels of targets.

State extraction is a necessary step for PHD filters. For the SMC-PHD filter, state extraction is usually carried out according to the spatial distribution of particles and by *k*-mean method [106, 107] or clustering technology based on the finite mixture model (FMM) [108]. Reference [107] compared the state extraction performances of *k*-mean clustering and the expectation maximization (EM)-based FMM, and the results indicated that compared with the EM method, *k*-mean algorithm significantly reduced the complexity of computation. In fact, the EM is a deterministic method and is not suitable for estimating parameters of complex multi-mode distribution. Therefore, the random method of Markov Chain Monte Carlo (MCMC) was used for FMM parameter estimation in [108]. In addition to the spatial distribution of particles, the weight information of particles can also be used to better extract the states of close or near targets [109, 110]. Different from the SMC-PHD filter, the GM-PHD filter can easily extract state estimation without clustering (which requires a lot of computation and may lead to inaccurate estimation), but it is still limited to the linear Gaussian system. To this end, a hybrid GM/SMC implementation method of the PHD filter was proposed in [111]. Reference [112] proposed a new particle implementation algorithm for the PHD filter based on the GM-PHD filter, which can not only extract target states without clustering technology, but also be applicable to highly nonlinear non-Gaussian models.

Standard PHD filters cannot provide the track information of targets [62]. One way to solve this problem is to use estimation-track association [106, 113]. The idea is to take the multi-target state estimation output by the PHD filter as the "measurement" input of another data association-based multi-target tracker, and then implement the estimation-track association through the tracker to obtain each target track; another approach is to use the PHD filter as a clutter filter to eliminate the clutter in the measurement set that are unlikely to originate from targets, and then input the remaining measurements into the tracker [106]. It can be considered a gate operation at the global level, which eliminates most clutter measurements. Both methods have reduced the number of measurements used for data association, nevertheless, the track information is output by the tracker, and the PHD filter itself does not utilize the track information of targets. Besides the estimation-track association method, another method called labeling is also commonly used for track output. It has be used for the GM-PHD [114] and SMC-PHD [106, 107, 115] filters. The labeling method can effectively provide track information. However, when targets are passing across or close to each other, the labeling method is prone to misjudgment of targets. For this reason, in [114], the estimation-track association method was used for state association when targets were passing across or close to each other, and the labeling method was used to carry out state association when targets were far away. Additionally, [116] combined the PHD filter with multi-frame association to further reduce the false association rate.

1.2.3.2 Cardinalized Probability Hypothesis Density Filter

In the PHD filter, since the mean and variance of the Possion distribution are equal, when the number of targets is high, it is easy to cause strong fluctuations in the estimation of the number of targets under the condition of missed detections or high false alarm density, which makes the estimation unreliable and leads to the missing measurement problem [64]. In response to this problem, Ref. [117] pointed out that for the PHD filter, not only the propagation of the first-order multi-target moment, but also the propagation of the higher-order number of targets is required. Based on this, a CPHD filter with the cardinality distribution is proposed in [3] using FISST tools such as the probability generating functional (PGFl) or functional derivative. The CPHD is a general version of the PHD. It not only propagates the posterior intensity of the multi-target state set, but also propagates the posterior cardinality distribution of the set at the same time. Its recursion formula is more complex and has the cubic complexity, but it still has better performance than the JPDA with the non-polynomial complexity [64]. Different from [3], Ref. [118] obtained a new derivation of the PHD and CPHD filters by directly performing the Kullback–Leibler divergence (KLD) minimization on predicted and posterior multi-target densities. Due to the introduction of cardinality distribution, the CPHD improves the accuracy and stability of the estimation of the number of multiple targets and their states, but the response speed of the CPHD to target birth and target death is not as fast as that of the PHD [64]. In addition, although the update formula of the overall CPHD cardinality distribution is accurate, when targets are missed, it will exhibit a local singular behavior—the "spooky effect" [119], i.e., the weight of the PHD will shift from the missed part to the detected part, no matter how far apart the two parts are, resulting in a significant underestimation of the number of local targets in the vicinity of the missed measurements [119]. To this end, the surveillance area was divided into different regions in [119], and then the CPHD was applied to each region in turn. However, the clutter density needs to be modified after the division, which in turn may increase the uncertainty of the cardinality estimation. Reference [120] reduced the influence of weight drift and estimation error through a dynamic reweighting scheme.

Unlike the PHD filter, in the standard CPHD filter [64], the prediction step does not include the target spawning model. Although the CPHD filter for birth targets can be used to solve the target spawning issue, it is obviously more appropriate to use a spawning model that can consider specific issues. Taking resident space object (RSO) as an example, the natural and artificial earth orbiting satellites are composed of space shuttles, scrapped payloads, and debris. If there is no spawning model, the best option is to use diffused birth regions; however, this requires a large birth area to cover the corresponding spatial volume. To improve the performance of the CPHD filter when tracking the spawning RSO, a spawning model with accurate description of the physical process of generating a birth RSO is proposed in [121], which can obtain better accuracy and faster confidence time for birth targets. In [122], Poisson or Bernoulli spawning models are incorporated into the CPHD filter, while in [123], the CPHD prediction equation suitable for any spawning process is derived through partial Bell polynomials [124]. Moreover, for the three specific models (Poisson, zero-inflated Poisson, and Bernoulli models), a GM-CPHD filter applicable to spawning targets can be obtained without additional approximations. Other literatures on CPHD filters considering spawning targets also include [125].

1.2.3.3 Multi-Bernoulli Filter

Unlike the PHD (CPHD), which recursively propagating the moment (and cardinality distribution) of the posterior multi-target density, the Bernoulli RFS models the target track [14] through two parameter pairs, the probability of target existence and the PDF when the target exists, which has some similarity with the IPDA filter [43] that simultaneously estimates the existence probability of a single target and its state. In [126], the Bernoulli RFS was used to derive the optimal Bayesian solution to the single-target detection and tracking problem, and the Bernoulli filter, also known as the joint target detection and tracking (JoTT) filter [14, 127–131], was obtained. The JoTT here refers to the joint estimation of the number of targets and their states from sensor measurements.

The JoTT filter in the multi-target background is called the multi-Bernoulli (MB) filter. As the name implies, the MB RFS is the union of multiple Bernoulli RFSs. In essence, the MB filter propagates the parameters for approximating the MB distribution of the posterior multi-target RFS density. The multi-target multi-Bernoulli (MeMBer) update equation proposed in [14] has a significant bias (overestimation) in the estimation of the number of targets, and this bias disappears only when the detection probability is 1 [4]. For this reason, literature [4] deduced the analytical expression of the "cardinality bias" in the MeMBer filter, calculated the updated existence probability by using the accurate probability generating functional, eliminated the cardinality bias problem by correcting the measurement updated track parameters, and gave an unbiased cardinality balanced MeMBer (CBMeMBer) filter, as well as two implementation methods: SMC-CBMeMBer and GM-CBMeMBer. However, in order to calculate the effective spatial PDF, the filter makes strict assumptions about the target detection probability. For this reason, Ref. [132] eliminated this bias by introducing a fake Bernoulli target without any strict assumption, and proposed an improved MeMBer filter. Reference [59] balanced the posterior cardinality distribution by multiplying the probability of missed detection. The new algorithm has better performance than the CBMeMBer method. However, the approximation of the missed detection probability in the new algorithm is based on the assumption that the tracks are separable. In order to solve the problem of close targets, Ref. [133] extended [134] and presented a principled, highly efficient approximation method to find the MB distribution that minimizes the KLD with the full RFS distribution. To overcome the problem that the parameters such as clutter intensity, detection probability and sensor field-of-view (FoV) need to be known a priori in the MB filter, Ref. [60, 61] respectively proposed the MeMBer filters under the conditions of unknown detection probability and clutter intensity, as well as unknown non-uniform clutter intensity and sensor FoV.

For approximation algorithms such as the PHD, CPHD and CBMeMBer, the GM-CPHD algorithm has the best performance in the linear Gaussian case, while the GM-CBMeMBer has the similar performance as the GM-PHD, not showing advantages. However, under highly nonlinear non-Gaussian conditions, the MB filter should be a better option. Unlike the particle realization of the PHD/CPHD filter, which needs particle clustering to extract target states, requiring a large amount of computation and being unreliable, the SMC-CBMeMBer filter, however, does not require additional clustering operation, and can directly extract the multi-target state estimation. Under the condition of high signal-to-noise ratio (SNR), the SMC-CBMeMBer filter not only has lower computation but also has better performance compared with the SMC-PHD/CPHD filter. Additionally, the MB filter can also provide the existence probability of targets. As a result, the GM-CPHD filter has the best performance under linear Gaussian conditions, while the SMC-CBMeMBer filter has obvious advantages under nonlinear non-Gaussian conditions.

In terms of the amount of computation, Ref. [135] has verified the real-time performance of the RFS algorithm using actual data. In general, the complexity of algorithms such as the PHD, CPHD, and MeMBer are $\mathcal{O}(mn)$, $\mathcal{O}(m^3n)$, and $\mathcal{O}(mn)$, respectively, where m and n represent the number of measurements and targets respectively. In other words, the MeMBer and PHD filters have the same linear complexity, while the CPHD filter has a higher cubic complexity. By reducing the cardinality of the measurement set, the calculation amount of the algorithm can be reduced. In [136, 137], the computational amount was reduced without any significant performance loss by incorporating the ellipsoid gate technique used in conventional tracking algorithms. In addition, Ref. [138] proposed a CPHD filter with linear complexity based on a relatively simple clutter model.

1.2.3.4 Labeled Random Set Filters

It should be noted that the filters mentioned above are not multi-target trackers in nature, because the target states are unresolved, which is one of the reasons why the RFS framework was once criticized: the algorithms derived from the RFS framework cannot obtain target labels. Besides, they are all approximate filters, and even assuming special observation models, such as the standard point target observation model, they are not closed-form solutions of the optimal Bayesian filter. Therefore, in [69], the concept of labeled RFS is introduced to solve the problem of target track and its uniqueness, and a new RFS distribution class called GLMB distribution was proposed. The GLMB distribution is conjugated with respect to the multi-target observation likelihood, and is closed with respect to the multi-target transition kernel under the multi-target Chapman-Kolmogorov (C-K) equation, thus providing an analytical solution for the multi-target inference and filtering problems, namely the δ-GLMB filter, which exploits the conjugation of the GLMB family to accurately forward propagate the (labeled) multi-target filtering density over time. It is an exact closed-form solution of multi-target Bayesian recursion, yielding the joint estimation of states and labels (or tracks) in the presence of clutter, missed detection, and association uncertainties, and is the first tractable RFS-based multi-target tracking filter that can generate track estimation in a principled way, refuting the view that the the RFS methods cannot generate track estimates. Reference [139] further extended the GLMB filter to be suitable for spawning target conditions, Refs. [140, 141] extended it to multi-frame sliding window processing, and Ref. [142] extended it to be suitable for related multi-target systems. In view of the complex and lengthy derivation process in [69], the probability generating functional (PGFl) method was proposed in [143] to provide a simplified derivation of the GLMB filter, and another tractable multi-target tracker, namely labeled multi-Bernoulli mixture (LMBM) filter, with an accurate closed form was derived using the PGFl method. The LMBM filter may be more practical because the LMB mixture is simpler to calculate than the GLMB distribution.

Nevertheless, the specific implementation of the δ-GLMB filter was not given in [69] and [143]. Therefore, an efficient and highly parallel implementation of the δ-GLMB filter was given in [70], which complements the theoretical contribution of [69] with practical algorithms. Specifically speaking, each iteration of the δ-GLMB filter involves multi-target predicted density and filtering density, both of which are weighted sums of multi-object exponentials (MoE). Despite these weighted sums have a closed form, the number of component terms in the posterior grows super-exponentially over time due to the explicit data association in the δ-GLMB filter. It is obviously infeasible to adopt the pruning strategy, which first exhaustively calculates all items of the multi-target density and then discards the secondary components. Hence, a pruning strategy that does not need to exhaustively calculate all components of the multi-target density is proposed in [70], in which the multi-target predicted density and filtering density are pruned using the K shortest path and ranked assignment algorithms [144] respectively. Meanwhile, the relatively low-computational PHD filter derived from the same RFS framework is used as a look ahead strategy to significantly reduce the number of calls of the K shortest path and ranked assignment algorithms.

The two-step implementation proposed in [70] is intuitive and highly parallel, specifically, in the prediction step, pruning is realized by solving two different K shortest path problems, one for the existing tracks and the other for the birth tracks; in the update step, pruning is achieved by solving the ranked assignment problem for each predicted δ-GLMB component. However, the two-step implementation is structurally inefficient, because the pruning of the predicted and updated δ-GLMB components is carried out separately, so a large proportion of the predicted components may produce updated components with negligible weights. As a result, a large amount of computation is wasted on solving a large number of ranked assignment problems, each of which has at least a cubic complexity with the number of measurements. Therefore, Refs. [145, 146] pruned the GLMB filtering density by combining the prediction and update steps into a single step, and using a random Gibbs sampler with a linear complexity with the number of measurements and an exponential convergence rate based on the Markov chain Monte Carlo (MCMC) [147] method. There is no need to discard samples during the burn-in phase in the pruning application, so it is not necessary to wait for samples from a stable distribution. The stochastic solution has two advantages over the deterministic ranked assignment (in the order of non-increasing weights) for pruning strategies: first, it eliminates the unnecessary computations caused by component sorting and reduces the cubic complexity with the number of measurements to the linear complexity; second, it automatically adjusts the number of generated significant components by using the statistical characteristics of component weights, thus resulting in a more efficient implementation of the GLMB filter, significantly improving the running speed without affecting the filtering performance. It should be pointed out that the recommended Gibbs sampler also provides an effective solution to the data association problem or the more general ranked assignment problem. In conclusion, the new implementation method is an online multi-target tracker, which has a linear complexity with the number of measurements and a quadratic complexity with the number of hypothetical tracks. It can be applied to complex scenarios such as nonlinear dynamic and measurement models, non-uniform survival probability, sensor field of view, and clutter intensity. Since it was proposed, the δ-GLMB filter has been rapidly promoted and applied [83, 86, 94, 148–151], which shows that the GLMB filter is a general model with excellent performance.

In addition to using the above acceleration strategies, some scholars have sought cheaper approximations of the δ-GLMB filter to improve the performance, the most famous of which are the Mδ-GLMB [72] and LMB filters [71]. As mentioned earlier, the performance improvement of the δ-GLMB filter is obtained at the cost of higher computational complexity, which mainly comes from data association. For some applications, such as multi-sensor tracking or distributed estimation, it is not feasible to apply the δ-GLMB filter due to limited computational resources. Inspired by Mahler's independent and identically distributed cluster (IIDC) approximation in the CPHD filter, Ref. [152] derives a special tractable GLMB class: marginal δ-GLMB (Mδ-GLMB) density, which can be used to define a principle approximation of the δ-GLMB density representing the true posterior of the multi-target Bayesian filter. Since the δ-GLMB density can be used to optimally approximate any labeled multi-object density [152], the Mδ-GLMB density provides a tractable multi-target density approximation to a general labeled RFS density that captures statistical correlations between targets. Especially, it matches the cardinality distribution and the first order moment (probability hypothesis density, PHD) of labeled multi-target distribution of interest (such as the true δ-GLMB density), and minimizes the Kullback-Leibler divergence (KLD) on tractable GLMB densities (such as the Mδ-GLMB density). Based on this, Ref. [72] proposes the Mδ-GLMB filter, as it can be interpreted as performing marginalization on the data association histories generated by the δ-GLMB filter. Therefore, the Mδ-GLMB filter is computationally cheaper than the δ-GLMB filter, while preserving the key statistics of the multi-target posterior, in particular, it is easier to develop efficient tractable multi-sensor trackers based on Mδ-GLMB filter.

Another efficient approximation to the δ-GLMB filter is given in [71], namely, the labeled multi-Bernoulli (LMB) filter, which uses the δ-GLMB update step in each iteration. However, it approximates the δ-GLMB posterior from each update step using the LMB distribution, to reduce the computational complexity. The LMB filter, a generalization of the MB filter, inherits the advantages of the MB filter concerning particle implementation and state estimation, and also advantages of the δ-GLMB filter. It delivers a more accurate update approximation than the MB filter by calling the conjugate prior form of the labeled RFS, without the cardinality bias problem. In addition, it can also output the target tracks (labels), with performance significantly better than performances of the PHD, CPHD and MB filters [153], and comparable to that of the δ-GLMB filter. Besides, it gets rid of the limitation that the MB filter is only suitable for high signal-to-noise ratio (low clutter and high detection probability) conditions. In conclusion, the LMB filter can formally estimate the tracks with unbiased posterior cardinality distribution even in difficult scenarios such as low detection probability and high false alarm. This filter has been used in the environment perception system of the autonomous vehicle with multiple sensors (radar, lidar, and video sensor) [154], which demonstrates its real-time performance and robustness. Reference [155] improves the real-time performance of the LMB filter by further approximation.

To synthesize the advantages of the LMB and δ-GLMB filters: the low complexity of the LMB filter and the accuracy of the δ-GLMB filter, Ref. [156] proposes an adaptive LMB (ALMB) filter, which automatically switches between the LMB and δ-GLMB filters based on the KLD [157] and entropy [158]. Aiming at the problem that most methods usually discard partial or all of statistical correlations in order to reduce the amount of calculation, Ref. [159] proposes an improved labeled multi-object (LMO) density approximation by adaptively decomposing the LMO density into the densities of several independent subsets according to the analysis of the true statistical correlation between target states. Considering that the labeled Poisson RFS and labeled IIDC RFS are special cases of the GLMB RFS, Ref. [160] derives the labeled PHD (LPHD) and labeled CPHD (LCPHD) filters based on the GLMB filter. When the targets approach each other for a long time and then separate, a mixed label problem will occur [161]. In such case, the label-switching improvement (LSI) method that is not interested in label information [134] can be applied. The basic idea of this approach is as follows: the label can be regarded as an auxiliary variable when only the inference of the label-free target set is interested. At this time, it is equivalent to generate an additional degree of freedom, which helps improve the approximation of the posterior PDF, so that any labeled posterior PDF can be selected, as long as the corresponding unlabeled posterior PDF remains unchanged.