Mathematical Engineering

Yuri B. Zudin

Theory of Periodic Conjugate Heat Transfer

Fourth Edition

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Mathematical Engineering

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To my beloved wife Tatiana,
who has always been my source of
inspiration,
to my children Maxim and Nataliya,
and to my grandchildren Alexey and Darya

Preface

The material presented in this book crowns my long-term activity in the field of conjugate periodic heat transfer. Its first stage, which had passed under the scientific supervision of my teacher Professor Labuntsov (1929−1992), started with the publication (1977) of our first article and was completed in 1984 with the publication of our book in Russian: Labuntsov D. A., Zudin Y. B., “Processes of heat transfer with periodic intensity”. This stage was marked with the defense of my Candidate Thesis: Zudin Y. B., “Analysis of heat transfer processes with periodic intensity” (1980). The subsequent period of interpretation of the already gained results, and accumulation of new knowledge had taken seven years. In 1991, I started working on a new series of publications on this subject, which culminated in this book, the first edition of which appeared in 2007, the second one, in 2011, and the third one, in 2017. This stage was also marked with my habilitation script (Zudin Y. B., “Approximate theory of heat transfer processes with periodic intensity”, 1996), as well as with fruitful scientific collaborations with my respected German colleagues: Prof. U. Grigull, Prof. F. Mayinger, and Prof. J. Straub (TU München), Prof. W. Roetzel (University BW Hamburg), Prof. J. Mitrovic and Prof. D. Gorenflo (University Paderborn), Prof. K. Stephan, Prof. M. Groll, and Prof. B. Weigand (University Stuttgart).

The objective of the present monograph is to give an exhaustive answer to the question of how thermophysical and geometrical parameters of a body govern the heat transfer characteristics under conditions of thermohydraulic pulsations. An applied objective of this book is to develop a universal method for calculation of the average heat transfer coefficient for periodic conjugate processes of heat transfer.

As a rule, real “steady” processes of heat transfer can be looked upon as steady ones only on the average. In the actual fact, periodic, quasiperiodic, and various random fluctuations of parameters (velocities, pressure, temperatures, momentum and energy fluxes, vapor content, interface boundaries, etc.) around their average values always exist in any type of fluid flow, except for purely laminar flows. Owing to the conjugate nature of the “fluid flow–streamlined body” interface, both the fluctuation and the average values of temperatures and heat fluxes on the heat transfer surface generally depend on thermophysical and geometrical characteristics of the heat transferring wall.

This suggests the principle question about the possible effect of the material and the thickness of the wall on the key parameter of convective heat transfer, namely the heat transfer coefficient. Such an effect was earlier manifested in experimental investigations of heat transfer at nucleate boiling, dropwise condensation, and in some other cases. In these studies, the heat transfer coefficients, as defined as the ratio of the average heat flux on the surface and the average temperature difference "wall−fluid", could differ markedly for various materials of the wall (and also for different thicknesses of walls).

In 1977, a concept of a true heat transfer coefficient was first proposed by Labuntsov and Zudin. According to this concept, the actual values of the heat transfer coefficient (for each point of the heat transferring surface and at each moment of time) are determined solely by the hydrodynamic characteristics of the fluid flow; as a result, they are independent of the parameters of a body. Fluctuations of parameters occurring in the fluid flow will result in the respective fluctuations of the true heat transfer coefficient, which is also independent of the material and thickness of the wall. This being so, from the solution of the heat conduction equation with a boundary condition of third kind, it is possible to find the temperature field in the body (and, hence, on the heat transfer surface), and as a result, to calculate the required experimental heat transfer coefficient as the ratio of the average heat flux to that of the temperature difference. This value (as determined in traditional heat transfer experiments and employed in applied calculations) should in general case depend on the conjugation parameters.

The study of relations between the heat transfer coefficients averaged by different methods (the true and experimental ones) laid the basis for the first edition of the present book, in which the following fundamental result was obtained: the average experimental value of the heat transfer coefficient is always smaller than the average true value of this parameter.

Chapter 1 gives a qualitative description of the method for investigations of periodic conjugate convective–conductive "fluid flow–streamlined body" problems. An analysis of physical processes representing heat transfer phenomena with periodic fluctuations is also given.

In Chap. 2, a boundary problem for the two-dimensional unsteady heat conduction equation with a periodic boundary condition of third kind is examined. To characterize the thermal effects of a solid body on the average heat transfer, a concept of the factor of conjugation is introduced. The quantitative effect of the conjugation in the problem is shown to be rather significant.

Chapter 3 puts forward a construction of a general solution for the boundary value problem for the equation of heat conduction with periodic boundary condition of the third kind. Analytic solutions are obtained for the characteristic laws of variation of the true heat transfer coefficient, namely the harmonic, inverse harmonic, stepwise, and delta-like ones.

In Chap. 4, a universal algorithm of a general approximate solution of the problem is developed. On its basis, solutions are obtained for a series of problems with different laws of periodic fluctuations of the true heat transfer coefficient.

Chapter 5 deals with conjugate periodic heat transfer for involved cases of external heat supply: the heat transfer at a contact either with environment or with a second body. A generalized solution for the factor of conjugation for the bodies of the "standard form" is obtained. A problem of conjugate heat transfer for the case of bilateral periodic heat transfer is also investigated. The cases of asymmetric and nonperiodic fluctuations of the true heat transfer coefficient are examined.

Chapter 6 includes some applied problems of the periodic conjugate heat transfer theory such as jet impingement onto a surface, dropwise condensation, and nucleate boiling. We consider the conjugate heat transfer problem for turbulent flow of liquid in a pipe.

Chapter 7 studies the effects of thermophysical parameters and the channel wall thickness on the hydrodynamic instability of the so-called density waves. The boundary of stability of fluid flow in a channel at supercritical pressures is found analytically. As an application, the problem stability provision for a thermal regulation system for superconducting magnets is considered.

In Chap. 8, the Landau problem on the evaporation front stability is generalized to the case of finite thickness of the evaporating liquid layer. The analysis of the influence of additional factors, the impermeability condition of solid wall, and resulting pulsations of mass velocity is carried out. Parametric calculations of the stability boundary are performed when changing the liquid film thickness and the relationship between phase densities in the framework of the asymptotic Landau approach for large Reynolds numbers. The influence of liquid viscosity on the stability boundary is approximatively evaluated.

Chapter 9 deals with the hyperbolic heat conduction equation. An extension of the algorithm of computation of the factor of conjugation is given. The limiting case described by the telegraph equation is considered. The boundary between the Fourier and Cattaneo–Vernotte laws is found. The method of investigation of heat transfer processes with periodic intensity is extended to the case of finite heat propagation rate. For the case of a semi-infinite body, approximate solutions for characteristic types (harmonic, inverse harmonic, and stepwise) of heat transfer intensity pulsations are obtained. The phenomenon of spontaneous enhancement of the conjugation effect, which is analogous to principal properties of self-oscillating systems is revealed.

Chapter 10 is concerned with derivation of the generalized Rayleigh equation that describes the dynamics of a gas bubble. Its solution has spherical and cylindrical asymptotics. A periodic quantum-mechanical model is offered for the process of homogeneous bubble nucleation. A thermally controlled vapor bubble growth in a bulk of uniformly superheated liquid is studied. Analysis of asymptotics of the analytical Scriven solution is carried out. A refined approximation of the Scriven integral with error $<1.5\%$ for all rangers of parameters is provided.

Chapter 11 examines the periodic slug flow in a two-phase media. One of the important parameters of periodic two-phase flows (the rise velocity of the Taylor bubbles in round pipes) is determined.

Chapter 12 develops an analytic method for calculation of heat exchange for a turbulent flow in a channel of fluid in a region of supercritical pressures. This method is capable of taking into account the effect of variability of thermophysical properties of a fluid on the heat transfer coefficient, as averaged over the period of turbulent pulsations. A periodic model of mass transfer through the interface surface is considered. The mass transfer coefficient is evaluated for various realizations of the channel flow (a flow in pipes, a flat gap, an open channel, a bubble flow). The effect of heat sources (sinks) on the turbulent heat exchange in a tube is studied. The dependence of the relative heat exchange flow on the source function is evaluated.

In Chap. 13, we consider a boundary value problem for the heat conduction equation with a transient heat transfer coefficient. Analytical solutions for the above cases are given. A general approximation of the solution is provided, from which the temperature for several variants is evaluated. Within the class of quasiperiodic functions of true heat transfer coefficient, the temperature and the factor of conjugation are evaluated. It is shown that the solutions or the periodic and quasiperiodic проблем are practically equal in the entire range of variation of the Biot number. On this basis, a conjecture on universality of this method of analysis of the problem of transient conjugate heat transfer is put forward.

Chapter 14 is devoted to the problem of evaporating meniscus on the interface of three phases. An approximate method for solution of this problem is proposed. Expressions for the meniscus profile and the length-averaged heat transfer coefficient are given. A periodic stationary problem for nucleate boiling simulation is put forward. The limiting degree of thermal influence of the heated wall on the heat transfer characteristics is evaluated.

In Appendix A, proofs are given of some properties of the two-dimensional unsteady equation of heat conduction with a periodic boundary condition of the third kind. As a corollary, we find the limiting values of the factor of conjugation.

Appendix B examines the eigenfunctions of the solution to the two-dimensional unsteady equation of heat conduction, as obtained by the method of separation of variables.

In Appendix C, the problem of convergence of infinite continued fraction is considered. An extension of the proof of Khinchin's third theorem to the case where the terms in the fraction possess a negative sign is obtained.

In Appendix D, a proof of divergence of infinite series obtained in Chap. 3 for the particular solution of the heat conduction equation is given.

In Appendix E, the approximate solutions from Chap. 4 are corrected for various laws of THTC oscillation (the harmonic, inverse harmonic, and step laws).

In Appendix F, the heat balance integral method is studied. An approximate solution of the one-dimensional unsteady heat conduction equation for a semi-infinite body is given. Approximation of the complementary error function is used to deliver an expression for the dimensionless temperature on the surface. An approximate solution is shown to coincide with error at most 2.5% with the classical solution available in the literature.

In Appendix G, we briefly discuss the peculiarities of self-oscillating systems: dissipation of the oscillatory energy and its replenishment from instability, fast and slow motions, relaxation oscillations, limit cycles, and strange attractor.

I would like to deeply thank the Director of the ITLR, Series Editor Mathematical Engineering of Springer-Verlag, Prof. Dr.-Ing. habil. B. Weigand for his strong

support of my aspiration to successfully accomplish this work, as well as for his numerous valuable advices and fruitful discussions concerning all aspects of the analytical solution methods. Prof. B. Weigand repeatedly invited me to visit the Institute of Aerospace Thermodynamics to perform joint research. Our collaboration was of great help for me in the preparation of this book. I am deeply indebted to Dr. T. Ditzinger, Editor of Springer-Verlag, for his interest in the publication and very good cooperation during the preparation of this manuscript.

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I would like to thank my dear wife Tatiana for her invaluable moral support of my work, especially in these tough and challenging times.

I am also thankful to Dr. A. Alimov (Moscow State University) for his very useful comments, which contributed much toward considerable improvement of the English translation of this book.

In conclusion, I cannot but stress the most crucial role played in my academic career by the prominent Russian scientist Prof. Labuntsov who was my scientific advisor.

Stuttgart, Germany Yuri B. Zudin
November 2022

Contents

Abbreviations

APF	Almost Periodic Function
ATHTC	Averaged True Heat Transfer Coefficient
BC	Boundary Condition
BVP	Boundary Value Problem
CP	Cauchy Problem
DDF	Dimensionless Dissipative Function
EF	Evaporation Front
EHTC	Experimental Heat Transfer Coefficient
FC	Factor of Conjugation
FT	Fourier Transform
FT	Film Thickness
GM	Growth Modulus
HHCM	Hyperbolic Heat Conduction Model
HN	Homogeneous Nucleation
HTC	Heat Transfer Coefficient
LT	Laplace Transform
MSR	Molten Salt Reactor
MTP	Mass Transfer Parameter
PTE	Parameter of the Thermal Effect
RANS	Reynolds-Averaged Navier–Stokes Equations
SCP	Supercritical Pressures
SI	Scriven Integral
SP	Stagnation Point
SRM	Surface Rejuvenation Model
ST	Scriven Table
TB	Taylor Bubble
TBC	Thermal Boundary Condition
THTC	True Heat Transfer Coefficient
VC	Vapor Cluster
WN	Wave Number

Symbols

Set Theory Symbols

$\forall$	Universal quantifier
$\mathbb{R}$	Set of real numbers
$\in$	Is an element of
$\subseteq$	Subset
$:$	Such that

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