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# Control and Optimization Based on Network Communication



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### **Preface**

With the rapid development of science and technology, networks play an important role in control systems. Typically, in a control system, the plant and the controller may communicate via a network. Such systems are called networked control systems and they have received significant attention in the literature. The introduction of a network has many advantages such as low cost, easy installation, and maintenance. However, a network also introduces some imperfections such as varying sampling, transmission delays, transmission protocols, packet losses, and so on. Besides, the network may also suffer from network attacks such as Denial of Service attacks. These imperfections may degrade the system performance and even make the closed-loop systems unstable. Therefore, it is necessary to investigate the effect of these imperfections on the stability and performance of control systems in a quantitative manner.

The content of this book consists of two parts. The first part aims at proposing modeling and analyzing approaches for network-based control systems under different problem settings. Model-based event-triggered transmission strategies are proposed to reduce the data transmissions in the network. We also consider the stability problem for networked control systems in the presence of stochastic noise and stochastic detecting instants. For all of these cases, hybrid models are established and Lyapunov functions are constructed. The second part first investigates the distributed unconstrained optimization problem which is solved over a network communication topology. The network may be destroyed by persistent attackers that result in communication failures. A distributed switching algorithm is proposed and a hybrid model is established to analyze stability. Then, the proposed method is extended to constrained aggregative games which are subject to unknown timevarying disturbances and unmodeled terms, and the communication topology is also influenced by attacks. This book provides a unifying framework to understand the complex dynamics of network-based control systems. The content is appropriate

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for students, control engineers, and scientists who are interested in network-based control and optimization problems.

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### **Notations**

```
\mathbb{R}^n
                                     n-dimensional Euclidean space
|\cdot|
                                     2-norm of a matrix
\mathbb{B}
                                     unit ball
                                     ball with radius r
\mathbb{B}_r
\langle x, y \rangle
                                     inner product of two vectors
x^T
                                     transpose of vector x
                                     (x^T, y^T)^T
col(x,y)
\mathbb{Z}
                                     set of integers
\mathbb{R}_{>0}
                                     [0, \infty)
\mathbb{R}_{\leq 0}
                                     (-\infty, 0]
                                     set of non-negative integers
\mathbb{Z}_{>0}
\mathbb{Z}_{<0}
                                     set of non-positive integers
\mathbb{R}_{>0}
                                      (0, \infty)
\mathbb{Z}_{>0}
                                     set of positive integers
\mathbb{R}_{\geq 0}^{N}
\mathbb{Z}_{\geq 0}^{N}
                                      \left\{ \left( x_1, \cdots, x_N \right)^T \middle| x_i \in \mathbb{R}_{\geq 0} \right\}
                                      \left\{ (x_1, \cdots, x_N)^T \middle| x_i \in \mathbb{Z}_{\geq 0} \right\}
|x|_{\mathscr{W}}
                                     distance of a point x to a set W
                                     n-dimensional zero matrix
\mathbf{0}_{n}
I_n
                                     n-dimensional identity matrix
0
                                     zero vector with appropriate dimension
1,
                                     n-dimensional vector with all elements being 1
diag\{M_1, \cdots, M_N\}
                                     block diagonal matrix
\lambda_{max}(P)
                                     maximal eigenvalue of a positive definite matrix
\lambda_{min}(P)
                                     minimal eigenvalue of a positive definite matrix
A \otimes B
                                     Kronecker product of matrices A and B
\overline{\mathrm{con}}\Omega
                                     closed convex hull of a set
\overline{\Omega}
                                     closure of a set
x + \Omega
                                     \{y|y=x+z, z\in\Omega\}
x \cdot \Omega
                                     \{y|y=x\cdot z,z\in\Omega\}
int(\Omega)
                                     interior of a set \Omega
```

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$$\begin{array}{lll} S + \varepsilon \mathbb{B} & \{x : |x|_S \leq \varepsilon\} \\ S + \varepsilon \mathbb{B}^\circ & \{x : |x|_S < \varepsilon\} \\ S^p & \{(x_1, \cdots, x_p) | x_i \in S\} \\ sgn(x) & sgn(x) = 1, \text{ for } x > 0, 0, \text{ for } x = 0 \text{ and } -1 \text{ for } x < 0 \\ sgn\Big( \big(x_1^T, \cdots, x_n^T \big)^T \Big) & (sgn(x_1), sgn(x_2), \cdots, sgn(x_n))^T \\ SGN(x) & SGN(x) = 1, \text{ for } x > 0, [-1, 1], \text{ for } x = 0 \text{ and } -1 \text{ for } x < 0 \\ SGN\Big( \big(x_1^T, \cdots, x_n^T \big)^T \Big) & (SGN(x_1), SGN(x_2), \cdots, SGN(x_n))^T \end{array}$$

# Chapter 1 Introduction



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### 1.1 Network-Based Control

Networked control systems refer to control systems in which the plant and the controller communicate via a network channel. Introduction of the network brings some benefits such as low cost, easy installation and maintenance. On the other hand, the network also introduces some imperfections such as varying transmission intervals, varying transmission delays, communication constraints, quantization effects and packet losses. The network may also suffer from various kinds of attacks such as denial-of-service attacks which can block the transmission channels. Therefore, it is necessary to investigate the effects of various kinds of imperfections on stability and performance of the closed-loop systems in a quantitative manner. Besides, in networked control systems, another important problem is how to reduce the data transmissions because the network resources are limited. There are mainly two transmission strategies. One is the time-triggered transmission strategy, which triggers the data transmission periodically, and the other is the event-triggered transmission strategy, which triggers the data transmission based on a triggering condition involving the system state and error variable. The latter has the potential to reduce data transmissions because a transmission happens only when it is necessary.

In networked control systems, there are mainly two modeling approaches. One is the delay system approach, which models the sampled-data control system into an input delay system and then the Lyapunov functional approach is applied to analyze the stability and performance of the resulted closed-loop system. The delay system approach has the advantage of less conservativeness but is often restricted to linear systems. The other modeling approach is the hybrid system approach, which models the networked control system as a hybrid system. The hybrid system approach is more appropriate for nonlinear systems and is flexible enough to model various kinds of complex dynamical systems such as event-triggered control systems.