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Zeroing Neural Networks

Finite-time Convergence Design,
Analysis and Applications


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Zeroing Neural Networks

Finite-time Convergence Design, Analysis and Applications

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Library of Congress Cataloging-in-Publication Data is Applied for:

Hardback ISBN: 9781119985990

Cover design: Wiley

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To our parents and ancestors, as always

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Figure	Transient behavior of the residual error

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	with $\gamma = 1$, (c) noise $z_{ij}(t) = 0.1 \exp(0.2t)$ with $\gamma = 1$, and (d) noise $z_{ij}(t) = 0.4 \cos(2t)$ with $\gamma = 1$.
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Figure 6.2	<p>Simulative results generated by ZNN model (6.27) for solving the time-varying linear equation system with no noise. (a) The first element of the neural state $\mathbf{u}(t)$ and the theoretical solution $\mathbf{u}^*(t)$. (b) The second element of neural state $\mathbf{u}(t)$ and the theoretical solution $\mathbf{u}^*(t)$. (c) The residual error $\ A(t)\mathbf{u}(t) - \mathbf{b}(t)\ _2$ corresponding to the neural state $\mathbf{u}(t)$.</p>