Lin Xiao • Lei Jia

Zeroing Neural Networks

Finite-time Convergence Design, Analysis and Applications

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To our parents and ancestors, as always

Contents

List of Figures *xv*
List of Tables *xxxi*
Author Biographies *xxxiii*
Preface *xxxv*
Acknowledgments *xlv*

Part I Application to Matrix Square Root *1*

1 FTZNN for Time-varying Matrix Square Root *3*
1.1 Introduction *3*
1.2 Problem Formulation and ZNN Model *4*
1.3 FTZNN Model *4*
1.3.1 Model Design *5*
1.3.2 Theoretical Analysis *7*
1.4 Illustrative Verification *8*
1.5 Chapter Summary *11*
References *11*

2 FTZNN for Static Matrix Square Root *13*
2.1 Introduction *13*
2.2 Solution Models *14*
2.2.1 OZNN Model *14*
2.2.2 FTZNN Model *15*
2.3 Illustrative Verification *17*
2.3.1 Example 1 *18*
2.3.2 Example 2 *20*
2.4 Chapter Summary *21*
References *21*

Part II Application to Matrix Inversion *23*

3 Design Scheme I of FTZNN *25*
3.1 Introduction *25*
3.2 Problem Formulation and Preliminaries *25*
3.3 FTZNN Model *26*
3.3.1 Model Design *26*
3.3.2 Theoretical Analysis *29*
3.4 Illustrative Verification *30*
3.4.1 Example 1: Nonrandom Time-varying Coefficients *30*
3.4.2 Example 2: Random Time-varying Coefficients *34*
3.5 Chapter Summary *35*
References *36*

4 Design Scheme II of FT ZNN *39*
4.1 Introduction *39*
4.2 Preliminaries *40*
4.2.1 Mathematical Preparation *40*
4.2.2 Problem Formulation *41*
4.3 NT-FTZNN Model *41*
4.4 Theoretical Analysis *43*
4.4.1 NT-FTZNN in the Absence of Noises *43*
4.4.2 NT-FTZNN in the Presence of Noises *44*
4.5 Illustrative Verification *46*
4.5.1 Example 1: Two-dimensional Coefficient *47*
4.5.2 Example 2: Six-dimensional Coefficient *52*
4.5.3 Example 3: Application to Mobile Manipulator *54*
4.5.4 Example 4: Physical Comparative Experiments *54*
4.6 Chapter Summary *57*
References *57*

5 Design Scheme III of FTZNN *61*
5.1 Introduction *61*
5.2 Problem Formulation and Neural Solver *61*
5.2.1 FPZNN Model *62*
5.2.2 IVP-FTZNN Model *63*
5.3 Theoretical Analysis *64*
5.4 Illustrative Verification *70*
5.4.1 Example 1: Two-Dimensional Coefficient *70*
5.4.2 Example 2: Three-Dimensional Coefficient *73*
5.5 Chapter Summary *78*
References *78*

Part III Application to Linear Matrix Equation *81*

6 Design Scheme I of FTZNN *83*
6.1 Introduction *83*
6.2 Convergence Speed and Robustness Co-design *84*
6.3 R-FTZNN Model *90*
6.3.1 Design of R-FTZNN *90*
6.3.2 Analysis of R-FTZNN *91*
6.4 Illustrative Verification *93*
6.4.1 Numerical Example *93*
6.4.2 Applications: Robotic Motion Tracking *98*
6.5 Chapter Summary *101*
References *102*

7 Design Scheme II of FTZNN *105*
7.1 Introduction *105*
7.2 Problem Formulation *106*
7.3 FTZNN Model *106*
7.4 Theoretical Analysis *108*
7.4.1 Convergence *108*
7.4.2 Robustness *112*
7.5 Illustrative Verification *118*
7.5.1 Convergence *118*
7.5.2 Robustness *121*
7.6 Chapter Summary *122*
References *122*

Part IV Application to Optimization *125*

8 FTZNN for Constrained Quadratic Programming *127*
8.1 Introduction *127*
8.2 Preliminaries *128*
8.2.1 Problem Formulation *128*
8.2.2 Optimization Theory *128*
8.3 U-FTZNN Model *130*
8.4 Convergence Analysis *131*
8.5 Robustness Analysis *134*
8.6 Illustrative Verification *136*
8.6.1 Qualitative Experiments *136*
8.6.2 Quantitative Experiments *139*

8.7 Application to Image Fusion *143*
8.8 Application to Robot Control *146*
8.9 Chapter Summary *149*
References *149*

9 FTZNN for Nonlinear Minimization *151*
9.1 Introduction *151*
9.2 Problem Formulation and ZNN Models *151*
9.2.1 Problem Formulation *152*
9.2.2 ZNN Model *152*
9.2.3 RZNN Model *154*
9.3 Design and Analysis of R-FTZNN *154*
9.3.1 Second-Order Nonlinear Formula *155*
9.3.2 R-FTZNN Model *159*
9.4 Illustrative Verification *161*
9.4.1 Constant Noise *161*
9.4.2 Dynamic Noise *163*
9.5 Chapter Summary *165*
References *166*

10 FTZNN for Quadratic Optimization *169*
10.1 Introduction *169*
10.2 Problem Formulation *170*
10.3 Related Work: GNN and ZNN Models *172*
10.3.1 GNN Model *172*
10.3.2 ZNN Model *173*
10.4 N-FTZNN Model *174*
10.4.1 Models Comparison *175*
10.4.2 Finite-Time Convergence *176*
10.5 Illustrative Verification *178*
10.6 Chapter Summary *181*
References *181*

Part V Application to the Lyapunov Equation *183*

11 Design Scheme I of FTZNN *185*
11.1 Introduction *185*
11.2 Problem Formulation and Related Work *186*
11.2.1 GNN Model *186*
11.2.2 ZNN Model *187*

11.3 FTZNN Model *187*
11.4 Illustrative Verification *190*
11.5 Chapter Summary *193*
References *193*

12 Design Scheme II of FTZNN *197*
12.1 Introduction *197*
12.2 Problem Formulation and Preliminaries *197*
12.3 FTZNN Model *198*
12.3.1 Design of FTZNN *199*
12.3.2 Analysis of FTZNN *200*
12.4 Illustrative Verification *202*
12.5 Application to Tracking Control *205*
12.6 Chapter Summary *207*
References *207*

13 Design Scheme III of FTZNN *209*
13.1 Introduction *209*
13.2 N-FTZNN Model *210*
13.2.1 Design of N-FTZNN *210*
13.2.2 Re-Interpretation from Nonlinear PID Perspective *211*
13.3 Theoretical Analysis *212*
13.4 Illustrative Verification *219*
13.4.1 Numerical Comparison *219*
13.4.2 Application Comparison *224*
13.4.3 Experimental Verification *228*
13.5 Chapter Summary *229*
References *229*

Part VI Application to the Sylvester Equation *231*

14 Design Scheme I of FTZNN *233*
14.1 Introduction *233*
14.2 Problem Formulation and ZNN Model *233*
14.3 N-FTZNN Model *235*
14.3.1 Design of N-FTZNN *235*
14.3.2 Theoretical Analysis *237*
14.4 Illustrative Verification *243*
14.5 Robotic Application *248*
14.6 Chapter Summary *251*
References *251*

15 Design Scheme II of FTZNN *255*
15.1 Introduction *255*
15.2 ZNN Model and Activation Functions *256*
15.2.1 ZNN Model *256*
15.2.2 Commonly Used AFs *257*
15.2.3 Two Novel Nonlinear AFs *257*
15.3 NT-PTZNN Models and Theoretical Analysis *258*
15.3.1 NT-PTZNN1 Model *258*
15.3.2 NT-PTZNN2 Model *262*
15.4 Illustrative Verification *266*
15.4.1 Example 1 *266*
15.4.2 Example 2 *269*
15.4.3 Example 3 *273*
15.5 Chapter Summary *274*
References *274*

16 Design Scheme III of FTZNN *277*
16.1 Introduction *277*
16.2 ZNN Model and Activation Function *278*
16.2.1 ZNN Model *278*
16.2.2 Sign-bi-power Activation Function *279*
16.3 FTZNN Models with Adaptive Coefficients *282*
16.3.1 SA-FTZNN Model *282*
16.3.2 PA-FTZNN Model *284*
16.3.3 EA-FTZNN Model *286*
16.4 Illustrative Verification *289*
16.5 Chapter Summary *294*
References *294*

Part VII Application to Inequality *297*

17 Design Scheme I of FTZNN *299*
17.1 Introduction *299*
17.2 FTZNN Models Design *299*
17.2.1 Problem Formulation *300*
17.2.2 ZNN Model *300*
17.2.3 Vectorization *300*
17.2.4 Activation Functions *301*
17.2.5 FTZNN Models *302*
17.3 Theoretical Analysis *303*

17.3.1 Global Convergence *303*
17.3.2 Finite-Time Convergence *304*
17.4 Illustrative Verification *309*
17.5 Chapter Summary *314*
References *314*

18 Design Scheme II of FTZNN *317*
18.1 Introduction *317*
18.2 NT-FTZNN Model Deisgn *318*
18.2.1 Problem Formulation *318*
18.2.2 ZNN Model *318*
18.2.3 NT-FTZNN Model *319*
18.2.4 Activation Functions *319*
18.3 Theoretical Analysis *320*
18.3.1 Global Convergence *320*
18.3.2 Finite-Time Convergence *321*
18.3.3 Noise-Tolerant Convergence *326*
18.4 Illustrative Verification *327*
18.5 Chapter Summary *334*
References *335*

Part VIII Application to Nonlinear Equation *337*

19 Design Scheme I of FTZNN *339*
19.1 Introduction *339*
19.2 Model Formulation *339*
19.2.1 OZNN Model *340*
19.2.2 FTZNN Model *340*
19.2.3 Models Comparison *341*
19.3 Convergence Analysis *341*
19.4 Illustrative Verification *343*
19.4.1 Nonlinear Equation $f(u)$ with Simple Root *343*
19.4.2 Nonlinear Equation $f(u)$ with Multiple Root *346*
19.5 Chapter Summary *347*
References *347*

20 Design Scheme II of FTZNN *349*
20.1 Introduction *349*
20.2 Problem and Model Formulation *349*

20.2.1 GNN Model *350*
20.2.2 OZNN Model *350*
20.3 FTZNN Model and Finite-Time Convergence *351*
20.4 Illustrative Verification *354*
20.5 Chapter Summary *356*
References *356*

21 Design Scheme III of FTZNN *359*
21.1 Introduction *359*
21.2 Problem Formulation and ZNN Models *359*
21.2.1 Problem Formulation *360*
21.2.2 ZNN Model *360*
21.3 Robust and Fixed-Time ZNN Model *361*
21.4 Theoretical Analysis *362*
21.4.1 Case 1: No Noise *362*
21.4.2 Case 2: Under External Noises *363*
21.5 Illustrative Verification *367*
21.6 Chapter Summary *370*
References *371*

Index *375*

List of Figures

Figure 1.1 Transient behavior of $U(t)$ synthesized by the OZNN model (1.5) starting with 10 randomly generated initial states, where solid curves correspond to neural state $U(t)$, and dash curves correspond to theoretical matrix square root $U^*(t)$. *9*

Figure 1.2 Transient behavior of $U(t)$ synthesized by FTZNN model (1.13) starting with 10 randomly generated initial states, where solid curves correspond to neural state $U(t)$, and dash curves correspond to theoretical time-varying matrix square root $U^*(t)$. *10*

Figure 1.3 Transient behavior of the residual error $\|U^2(t) - B(t)\|_F$ corresponding to $U(t)$ synthesized by the OZNN model (3.2) and FTZNN model (3.3). (a) By the OZNN model (3.2) and (b) by FTZNN model (3.3). *10*

Figure 2.1 Simulative results of OZNN model (2.3) using linear activation functions under the condition of $\gamma = 1$ and a randomly generated $U(0) \in \mathbb{R}^{2\times2}$ close to $U^*(t)$. (a) Transient behavior of state matrix $U(t)$ and (b) transient behavior of residual error $\|Y(t)\|_F$. *18*

Figure 2.2 Simulative results of OZNN model (2.3) using power-sigmoid activation functions under the condition of $\gamma = 1$ and a randomly generated $U(0) \in \mathbb{R}^{2\times2}$ close to $U^*(t)$. (a) Transient behavior of state matrix $U(t)$ and (b) transient behavior of residual error $\|Y(t)\|_F$. *19*

Figure 2.3 Simulative results of FTZNN model (2.5) under the condition of $\gamma = 1$, $m = 0.25$ and a randomly generated $U(0) \in \mathbb{R}^{2\times2}$ close to $U^*(t)$. (a) Transient behavior of state matrix $U(t)$ and (b) transient behavior of residual error $\|Y(t)\|_F$. *19*

Figure 2.4 Transient behavior of residual error $\|Y(t)\|_F$ synthesized by FTZNN model (2.5) under the condition of $m = 0.25$ and a randomly generated $U(0) \in \mathbb{R}^{6\times 6}$ close to $U^*(t)$. (a) With $\gamma = 1$ and (b) with $\gamma = 10$. *20*

Figure 3.1 Transient behavior of $U(t)$ synthesized by GNN model (3.2) starting with 8 randomly generated initial states under the condition of $\gamma = 1$. *31*

Figure 3.2 Transient behavior of $U(t)$ synthesized by OZNN model (3.3) starting with 8 randomly generated initial states under the condition of $\gamma = 1$. *32*

Figure 3.3 Transient behavior of the residual error $\|A(t)U(t) - I\|_F$ corresponding to $U(t)$ synthesized by GNN model (3.2) and OZNN model (3.3). (a) By GNN model (3.3) and (b) by OZNN model (3.2). *33*

Figure 3.4 Transient behavior of $U(t)$ synthesized by FTZNN model (3.11) starting with 8 randomly generated initial states under the conditions of $\gamma = \kappa_1 = \kappa_2 = 1$. *33*

Figure 3.5 Transient behavior of the residual error $\|A(t)U(t) - I\|_F$ synthesized by FTZNN model (3.11) under the conditions of $\kappa_1 = \kappa_2 = 1$. (a) $\gamma = 1$ and (b) $\gamma = 100$. *34*

Figure 3.6 Transient behavior of the residual error $\|A(t)U(t) - I\|_F$ synthesized by FTZNN model (3.11) under the conditions of $\kappa_1 = \kappa_2 = 10$. (a) $\gamma = 100$ and (b) $\gamma = 10^6$. *34*

Figure 3.7 Transient behavior of the residual error $\|A(t)U(t) - I\|_F$ synthesized by FTZNN model (3.11) using random time-varying coefficients under the conditions of $\kappa_1 = \kappa_2 = 1$. (a) $\gamma = 1$ and (b) $\gamma = 10$. *35*

Figure 4.1 Transient behavior of NT-FTZNN model (4.6) activated by VAF for solving time-dependent matrix inversion (4.9) without noise. (a) State solutions and (b) residual error. *47*

Figure 4.2 Transient behavior of the ZNN model activated by LAF for solving time-dependent matrix inversion (4.9) with noise $z_{ij}(t) = 1$. (a) State solutions and (b) residual error. *48*

Figure 4.3 Transient behavior of the ZNN model activated by abbtextpower-sum activation function (PSAF) for solving time-dependent matrix inversion (4.9) with noise $z_{ij}(t) = 1$. (a) State solutions and (b) residual error. *49*

Figure 4.4 Transient behavior of the ZNN model activated by SBPAF for solving time-dependent matrix inversion (4.9) with noise $z_{ij}(t) = 1$. (a) State solutions and (b) residual error. *49*

Figure 4.5 Transient behavior of NT-FTZNN model (4.6) activated by VAF for solving time-dependent matrix inversion (4.9) with noise $z_{ij}(t) = 1$. (a) State solutions and (b) residual error. *50*

Figure 4.6 Transient behavior of residual errors $\|L(t)U(t) - I\|_F$ synthesized by NT-FTZNN model (4.6) activated by VAF and other ZNN models activated by different activation functions in different noise environments for solving time-dependent matrix inversion (4.9). (a) Noise $z_{ij}(t) = 0$ with $\gamma = 1$, (b) noise $z_{ij}(t) = 0.4|y_{ij}(t)|$ with $\gamma = 1$, (c) noise $z_{ij}(t) = 0.1\exp(0.2t)$ with $\gamma = 1$, and (d) noise $z_{ij}(t) = 0.4\cos(2t)$ with $\gamma = 1$. *50*

Figure 4.7 Transient behavior of residual errors $\|L(t)U(t) - I\|_F$ synthesized by NT-FTZNN model (4.6) activated by VAF and other ZNN models activated by different activation functions in different noise environments for solving time-dependent matrix inversion (4.9) with different values of parameter γ. (a) Noise $z_{ij}(t) = 2t$ with $\gamma = 10$ and (b) noise $z_{ij}(t) = 20$ with $\gamma = 20$. *51*

Figure 4.8 Transient behavior of residual errors $\|L(t)U(t) - I\|_F$ synthesized by NT-FTZNN model (4.6) activated by VAF, GNN model, IEZNN model, and other ZNN models activated by LAF, PSAF, and SBPAF under different noise environments for solving time-dependent matrix inversion (4.11). (a) $z_{ij}(t) = 1$, (b) $z_{ij}(t) = 0.5\sin(2.2t)$, and (c) $z_{ij}(t) = 0.15\exp(0.2t)$. *53*

Figure 4.9 Circular task tracking synthesized by the original ZNN model activated by the SBP activation function in the presence of additive noise $z_{ij}(t) = 0.35$. (a) Whole tracking process, (b) task comparison, and (c) position error. *55*

Figure 4.10 Circular task tracking synthesized by NT-FTZNN model (4.6) in the presence of additive noise $z_{ij} = 0.35$. (a) Whole tracking process, (b) task comparison, and (c) position error. *56*

Figure 4.11 Physical comparative experiments of a butterfly-path tracking task generated by different ZNN models and performed on the Kinova JACO^2 robot manipulator when disturbed by external noise. (a) Failure by SBPAF activated ZNN and (b) success by NT-FTZNN. *56*

Figure 5.1 Simulative results using FPZNN model (5.4) with SBPAF when solving TVMI (5.1) of Example 1 with $k = 1.5$. (a) State solution $U(t)$ and (b) residual error $\|Y(t)\|_F$. *71*

Figure 5.2 Simulative results using EVPZNN model (5.7) with SBPAF when solving TVMI (5.1) of Example 1 with $k = 1.5$. (a) State solution $U(t)$ and (b) residual error $\|Y(t)\|_F$. *71*

Figure 5.3 Simulative results using VPZNN model (5.9) with SBPAF when solving TVMI (5.1) of Example 1 with $k = 1.5$. (a) State solution $U(t)$ and (b) residual error $\|Y(t)\|_F$. *72*

Figure 5.4 Simulative results using IVP-FTZNN model (5.11) with SBPAF when solving TVMI (5.1) of Example 1 with $k = 1.5$. (a) State solution $U(t)$ and (b) residual error $\|Y(t)\|_F$. *72*

Figure 5.5 Residual errors $\|Y(t)\|_F$ of FPZNN (5.4), EVPZNN (5.4), VPZNN (5.7), and IVP-FTZNN (5.9) with SBPAF when solving TVMI (5.11) of Example 1 with $\gamma = k = 1.5$. *73*

Figure 5.6 Residual errors $\|Y(t)\|_F$ of IVP-FTZNN model (5.11) with different activation functions when solving TVMI (5.1) of Example 1 with $k = 1.5$. *73*

Figure 5.7 Simulative results using IVP-FTZNN model (5.11) with SBPAF when solving TVMI (5.1) of Example 2 with $k = 1.5$. (a) State solution $U(t)$ and (b) residual error $\|Y(t)\|_F$. *74*

Figure 5.8 Simulative residual errors using FPZNN (5.4), EVPZNN (5.7), VPZNN (5.9), and IVP-FTZNN (5.11) with SBPAF when solving TVMI (5.1) of Example 2 with different γ and k. (a) With $\gamma = k = 0.5$, (b) with $\gamma = k = 2$, (c) with $\gamma = k = 5$, and (d) with $\gamma = k = 10$. *75*

Figure 5.9 Simulative residual errors using FPZNN (5.4), EVPZNN (5.7), VPZNN (5.9), and IVP-FTZNN (5.11) with SBPAF when solving TVMI (5.1) of Example 2 with different γ, k and noises $n_{ij}(t)$. (a) With $\gamma = k = 0.5$ and $n_{ij}(t) = 0.2\|Y(t)\|_F$, (b) with $\gamma = k = 1.2$ and $n_{ij}(t) = 0.5$, (c) with $\gamma = k = 2$ and $n_{ij}(t) = 0.5\sin(2t)$, and (d) With $\gamma = k = 2$ and $-1 \leq n_{ij}(t) \leq 1$. *76*

Figure 5.10 Simulative residual errors using FPZNN (5.4), EVPZNN (5.7), VPZNN (5.9), and IVP-FTZNN (5.11) with SBPAF when solving TVMI (5.1) of Example 2 with $\gamma = k = 5$ and different noises $n_{ij}(t)$. (a) With $n_{ij}(t) = 1.2t$ and with $n_{ij}(t) = 0.8\exp(0.2t)$. *77*

Figure 6.1 Simulative results generated by the R-FTZNN model (6.26) for solving the time-varying linear equation system with no noise. (a) The first element of the neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (b) The second element of neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (c) The residual error $\|A(t)\boldsymbol{u}(t) - \boldsymbol{b}(t)\|_2$ corresponding to the neural state $\boldsymbol{u}(t)$. *94*

Figure 6.2 Simulative results generated by ZNN model (6.27) for solving the time-varying linear equation system with no noise. (a) The first element of the neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (b) The second element of neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (c) The residual error $\|A(t)\boldsymbol{u}(t) - \boldsymbol{b}(t)\|_2$ corresponding to the neural state $\boldsymbol{u}(t)$. *95*

Figure 6.3 Simulative results generated by the R-FTZNN model (6.26) for solving the time-varying linear equation system with the constant noise $\eta_j = 0.5$. (a) The first element of the neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (b) The second element of neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (c) The residual error $\|A(t)\boldsymbol{u}(t) - \boldsymbol{b}(t)\|_2$ corresponding to the neural state $\boldsymbol{u}(t)$. *96*

Figure 6.4 Simulative results generated by ZNN model (6.27) for solving the time-varying linear equation system with the constant noise $\eta_j = 0.5$. (a) The first element of the neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (b) The second element of neural state $\boldsymbol{u}(t)$ and the theoretical solution $\boldsymbol{u}(t)$. (c) The residual error $\|A(t)\boldsymbol{u}(t) - \boldsymbol{b}(t)\|_2$ corresponding to the neural state $\boldsymbol{u}(t)$. *97*

Figure 6.5 Residual errors $||A(t)\boldsymbol{u}(t) - \boldsymbol{b}(t)||_2$ generated by the R-FTZNN model (6.35) and ZNN model (6.36) for solving the time-varying linear equation system with the different noises and parameters. (a) The bounded noise $\eta_j = 20$ with $\gamma_1 = \gamma_2 = \gamma = 10$. (b) The time-varying noise $\eta_j(t) = 5\boldsymbol{u}(t)$ with $\gamma_1 = \gamma_2 = \gamma = 10$. (c) The bounded noise $\eta_j = 20$ with $\gamma_1 = \gamma_2 = \gamma = 100$. *98*

Figure 6.6 Actual ellipse-tracking results synthesized by ZNN model (6.27) in the presence of the additive noise $\eta_j = 2t$. (a) Actual ellipse-tracking process. (b) Desired ellipse path and actual ellipse-tracking trajectory. (c) Position error between desired ellipse path and actual ellipse-tracking trajectory. *99*

Figure 6.7 Actual ellipse-tracking results synthesized by R-FTZNN model (6.26) in the presence of the additive noise $\eta_j = 2t$. (a) Actual ellipse-tracking process. (b) Desired ellipse path and actual ellipse-tracking trajectory. (c) Position error between desired ellipse path and actual ellipse-tracking trajectory. *100*

Figure 6.8 Actual circle-tracking results of the thee-dimensional (3D) manipulator synthesized by R-FTZNN model (6.26) in the presence of the additive noise $\eta_j = 1$. (a) Desired circle path and actual circle-tracking trajectory. (b) Position error between desired circle path and actual circle-tracking trajectory. (c) Dynamic behavior of joint angle. *101*

Figure 7.1 Solved by the FTZNN1 model with $\tau = 0.2$ and $\gamma = 1$, where dash curves represent theoretical solution of (7.1) and solid curves represent state solution of (7.5). *119*

Figure 7.2 Solved by the FTZNN2 model with $\tau = 0.2$ and $\gamma = 1$, where dash curves represent theoretical solution of (7.1) and solid curves represent state solution of (7.5). *120*

Figure 7.3 Steady-state error $||Y(t)||_{\mathrm{F}}$ produced by proposed FTZNN models with $\tau = 0.2$ and $\gamma = 1$. (a) By FTZNN1. (b) By FTZNN2. *120*

Figure 7.4 Steady-state error $||Y(t)||_{\mathrm{F}}$. (a) By ZNN model (7.5) activated by different activation functions with $\tau = 0.2$ and $\gamma = 1$. (b) By the perturbed ZNN model (7.11) activated by different activation functions with $\tau = 0.2$ and $\gamma = 3$. *121*

Figure 7.5 Steady-state error $||Y(t)||_{\mathrm{F}}$ produced by perturbed ZNN model (7.11) activated by NFTAF activation functions with $\tau = 0.2$. (a) By NFTAF1 activation function with different γ. (b) By NFTAF2 activation function with different γ. *122*

Figure 8.1 The state solutions computed by U-FTZNN model (8.12) on solving different TVQP problems. (a) TVQPEI (8.27)–(8.29). (b) TVQPE (8.30)–(8.31). *137*

Figure 8.2 The residual error $\|\boldsymbol{y}(t)\|_2$ generated by U-FTZNN model (8.9) on solving different TVQP problems. (a) TVQPEI (8.27)–(8.29). (b) TVQPE (8.30)–(8.31). *138*

Figure 8.3 Comparisons of residual error $\|\boldsymbol{y}(t)\|_2$ produced by different models with different noises. (a) by perturbed ZNN (8.21) and perturbed U-FTZNN (8.19) with $0.1\boldsymbol{\xi}_1(t)$. (b) by perturbed U-FTZNN with $\boldsymbol{\xi}_{\mathrm{c}}$, $\boldsymbol{\xi}_1(t)$ and $\boldsymbol{\xi}_{\mathrm{s}}(t)$. *139*

Figure 8.4 The convergence time of the U-FTZNN model (8.10) solving the TVQPEI-C2 problem starting from different initial condition $q(0)$ with different model variable settings. T_{p} is the predefined convergence time bound by Eq. (8.18). (a) $r_2 = 0.5, \gamma_1 = \gamma_2 = 1$ while changing r_1. (b) $r_1 = 3, \gamma_1 = \gamma_2 = 1$ while changing r_2. (c) $r_1 = 2, r_2 = 0.6, \gamma_1 = \gamma_2$ while changing γ_1 and γ_2. *141*

Figure 8.5 The maximum steady-state residual error (MSSRE) of $\boldsymbol{\xi}_c(t, a_n)$ noise perturbed U-FTZNN model (8.21) solving the TVQPEI-C2 problem with different model variable settings, where $\mathrm{MSSRE}_{\mathrm{pred}}$

is the predicted MSSRE bound by inequality (8.26) when $\phi = 4 \times 10^3$. (a) $r_1 = 4, r_2 = 0.6, \gamma_1 = \gamma_2 = 1$ while changing a_n and $q(0)$. (b) $r_1 = 2, r_2 = 0.6, q(0) = q_1(0), a_n = 10$ while changing γ_1 and γ_2. (c) $\gamma_1 = \gamma_2 = 1, a_n = 10$ while changing r_1 and r_2. *142*

Figure 8.6 Results of image fusion on a 512×512 gray scale picture using U-FTZNN model (8.10). (a) Original image. (b) Noisy image with SNR = 1 dB. (c) Fused image when $S = 20$. (d) Fused image when $S = 50$. *144*

Figure 8.7 Computation error $\|\boldsymbol{y}(t)\|_2$ synthesized by U-FTZNN model (8.9) during image fusion with different S. (a) $S = 20$. (b) $S = 50$. *145*

Figure 8.8 Results of image fusion on T1-MR images using U-FTZNN model (8.10). (a) Slice 50 of T1-MR. (b) Slice 50 polluted by Gaussian noise. (c) Fused image of noise-polluted slice 50 when $S = 100$. (d) Slice 80 of T1-MR. (e) Slice 80 polluted by the Rician noise. (f) Fused image of noise-polluted slice 80 when $S = 100$. *145*

Figure 8.9 Residual error $\|\boldsymbol{y}(t)\|_2$ synthesized by U-FTZNN model (8.10) during image fusion when using different T1-MR images. (a) Using Gaussian noise polluted Slice 50. (b) Using Rician noise polluted Slice 80. *146*

Figure 8.10 Simulation results of using U-FTZNN model (8.10) to control PUM560 robot manipulator to track the Lissajous curve path. (a) Relationship between θ_1 and η_1. (b) Trajectory of end effector and desired path. (c) Full view of path tracking with robot arm in Cartesian space. (d) Traces of joint angles. (e) Traces of joint velocities. (f) Position errors of the end effector in three dimensions. *148*

Figure 9.1 Computing nonlinear minimization problem by ZNN model (9.6) using the sign-bi-power activation function with $\varpi = 5$ in front of constant noise $\upsilon = 0.5$. (a) Neural output $U(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *162*

Figure 9.2 Computing nonlinear minimization problem by RZNN model (9.8) with $\gamma_1 = \gamma_2 = 5$ in front of constant noise $\upsilon = 0.5$. (a) Neural output $U(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *162*

Figure 9.3 Computing nonlinear minimization problem by R-FTZNN model (9.27) using the sign-bi-power activation function with $\gamma_1 = \gamma_2 = 5$ in front of constant noise $\upsilon = 0.5$. (a) Neural output $U(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *163*

Figure 9.4 Computing nonlinear minimization problem by ZNN model (9.6) using the sign-bi-power activation function with $\varpi = 10$ under constant noise $\upsilon = 2\sin(t)$. (a) Neural output $\boldsymbol{u}(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *164*

Figure 9.5 Computing nonlinear minimization problem by RZNN model (9.8) with $\gamma_1 = \gamma_2 = 10$ under dynamic noise $\upsilon = 2\sin(t)$. (a) Neural output $\boldsymbol{u}(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *164*

Figure 9.6 Computing nonlinear minimization problem by R-FTZNN model (9.27) using the sign-bi-power activation function with $\gamma_1 = \gamma_2 = 10$ in front of dynamic noise $\upsilon = 2\sin(t)$. (a) Neural output $\boldsymbol{u}(t)$. (b) Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$. *165*

Figure 9.7 Residual error $\|\boldsymbol{z}(\boldsymbol{u}(t), t)\|_2$ generated by R-FTZNN model (9.8), ZNN model (9.6), and RZNN model (9.8) with $\varpi = \gamma_1 = \gamma_2 = 10$ in front of different types of noises. (a) Bounded additive noise $\upsilon = 15$. (b) Linearly increasing noise $\upsilon = 3t$. *165*

Figure 10.1 "Moving" nonlinear constraint, "Moving" objective function, and "Moving" optimal solution of nonstationary quadratic optimization (10.1) and (10.2). (a) Snapshot at $t = 0.15$ second. (b) Snapshot at $t = 1.50$ seconds. (c) Snapshot at $t = 3.25$ seconds. (d) Snapshot at $t = 5.80$ seconds. *171*

Figure 10.2 Simulative results of real-time solution to nonstationary quadratic optimization (10.4) and (10.5) synthesized by GNN model (10.12) with $\gamma = 1$. (a) Transient behavior of neural state $g(t)$. (b) Transient behavior of $\|C(t)\mathbf{g}(t) - \mathbf{d}(t)\|_2$. *179*

Figure 10.3 Simulative results of real-time solution to nonstationary quadratic optimization (10.4) and (10.5) synthesized by ZNN model (10.15) with $\gamma = 1$. (a) Transient behavior of neural state $g(t)$. (b) Transient behavior of $\|C(t)\mathbf{g}(t) - \mathbf{d}(t)\|_2$. *180*

Figure 10.4 Simulative results of real-time solution to nonstationary quadratic optimization (10.4) and (10.5) synthesized by N-FTZNN model (10.18) with $\gamma = 1$. (a) Transient behavior of neural state $g(t)$. (b) Transient behavior of $\|C(t)\mathbf{g}(t) - \mathbf{d}(t)\|_2$. *180*

Figure 11.1 Transient behavior of $U(t)$ synthesized by GNN model (11.3) and ZNN model (11.5) starting from randomly generated initial state matrices $U(0) \in \mathbb{R}^{2\times2}$. (a) By GNN model (11.3) and (b) by ZNN model (11.5). *191*

Figure 11.2 Transient behavior of $\|A^{\mathrm{T}}U(t) + U(t)A + C\|_{\mathrm{F}}$ synthesized by GNN model (11.3) and ZNN model (11.5) starting from randomly generated initial state matrices $U(0) \in \mathbb{R}^{2\times2}$. (a) By GNN model (11.3) and (b) by ZNN model (11.5). *192*

Figure 11.3 Simulative results synthesized by FTZNN model (11.8) starting from randomly generated initial state matrices $U(0) \in \mathbb{R}^{2\times 2}$. (a) Transient behavior of state matrices and (b) transient behavior of residual errors. *192*

Figure 11.4 Transient behavior of $\|A^{\mathrm{T}}U(t) + U(t)A + C\|_{\mathrm{F}}$ synthesized by FTZNN model (11.8) starting with a randomly generated initial state, where dash-dotted curves denote the time-varying theoretical solution of (11.1) and solid curves denote the neural-state solution. (a) With $\gamma = 100$ and (b) With $\gamma = 1000$. *193*

Figure 12.1 Transient behavior of neural state $U(t)$ synthesized by original ZNN model (12.4). *203*

Figure 12.2 Transient behavior of neural state $U(t)$ synthesized by FTZNN model (12.11). *204*

Figure 12.3 Transient behavior of residual error $\|A^{\mathrm{T}}(t)U(t) + U(t)A(t) + C(t)\|_{\mathrm{F}}$ corresponding to the neural state $U(t)$ synthesized by original ZNN model (12.4) and FTZNN model (12.11). (a) By the original ZNN model (12.4) and (b) by FTZNN model (12.11). *205*

Figure 12.4 Tracking ellipse-path results of the mobile manipulator synthesized by the proposed model (12.15). (a) Whole tracking motion trajectories and (b) top graph of tracking motion trajectories. *206*

Figure 12.5 Tracking ellipse-path results of the mobile manipulator synthesized by the proposed model (12.15). (a) Desired path and actual trajectory and (b) tracking errors at the joint position level. *206*

Figure 13.1 Block diagram of the control architecture of N-FTZNN model (13.4) for solving dynamic Lyapunov from the nonlinear PID perspective. *212*

Figure 13.2 Convergence property of each element of state output $U(t)$ corresponding to the one of theoretical solution $U^*(t)$ synthesized by N-FTZNN model (13.5) with $\gamma_1 = \gamma_2 = 1$ in the presence of additive constant noise $\varsigma = 1$. *220*

Figure 13.3 Convergence property of each element of state output $U(t)$ corresponding to the one of theoretical solution $U^*(t)$ synthesized by ZNN model (13.6) with $\gamma_1 = \gamma_2 = 1$ in the presence of additive constant noise $\varsigma = 1$. *221*

Figure 13.4 Convergence of residual error $\|C^{\mathrm{T}}(t)U(t) + U(t)C(t) + G(t)\|_{\mathrm{F}}$ produced by two different models with $\gamma_1 = \gamma_2 = 1$ in the presence of additive constant noise $\varsigma = 1$. (a) By N-FTZNN model (13.5). (b) By ZNN model (13.6). *221*

Figure 13.5 Convergence of residual error $\|C^{\mathrm{T}}(t)U(t) + U(t)C(t) + G(t)\|_{\mathrm{F}}$ produced by N-FTZNN model (13.5) with different values of parameters γ_1 and γ_2 in the presence of additive constant noise $\varsigma = 1$. (a) $\gamma_1 = \gamma_2 = 10$. (b) $\gamma_1 = \gamma_2 = 10^6$. *222*

Figure 13.6 Convergence of residual error $\|C^{\mathrm{T}}(t)U(t) + U(t)C(t) + G(t)\|_{\mathrm{F}}$ produced by N-FTZNN model (13.5) and ZNN model (13.6) with $\gamma_1 = \gamma_2 = 10$ in the presence of different kinds of additive noises. (a) Bounded random additive $\varsigma = 10$. (b) Time-varying additive $\varsigma = 5U(t)$. *223*

Figure 13.7 Circular tracking results of the planar six-link manipulator synthesized by the N-FTZNN-based control law with $\gamma_1 = \gamma_2 = 100$ in the presence of additive constant noise $\varsigma = 5$. (a) Joint motion trajectories of planar six-link manipulator. (b) Comparison between the desired path and the actual trajectory. (c) Position error between the desired path and the actual trajectory. (d) Dynamic behavior of joint angle. *225*

Figure 13.8 Circular tracking results of the planar six-link manipulator synthesized by the ZNN-based control law with $\gamma_1 = \gamma_2 = 100$ in the presence of additive constant noise $\varsigma = 5$. (a) Joint motion trajectories of planar six-link manipulator. (b) Comparison between the desired path and the actual trajectory. (c) Position error between the desired path and the actual trajectory. (d) Dynamic behavior of joint angle. *226*

Figure 13.9 Circular tracking experiment results synthesized by the N-FTZNN-based control law with $\gamma_1 = \gamma_2 = 100$ in the presence of additive constant noise $\varsigma = 1$. *227*

Figure 13.10 Data profiles during the circular tracking experiment of the 3-axis physical manipulator. (a) Comparison between the desired path and the actual trajectory. (b) Position error between the desired path and the actual trajectory. (c) Dynamic behavior of joint angle. (d) Dynamic behavior of joint velocity. *228*

Figure 14.1 The circuit topology of design formula (14.6) for hardware implementation. *236*

Figure 14.2 Trajectories of theoretical solution $U^*(t)$ and state output $U(t)$ generated by N-FTZNN model (14.9) with $\gamma_1 = \gamma_2 = 1$ in front of no noise, where solid curves denote the elements of state output $U(t)$, and dash curves denotes the elements of $U^*(t)$. *244*

Figure 14.3 Trajectories of residual error $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ generated by N-FTZNN model (14.9) with different values of γ_1 and γ_2 in front of no noise. (a) With $\gamma_1 = \gamma_2 = 1$. (b) With $\gamma_1 = \gamma_2 = 1000$. *245*

Figure 14.4 Trajectories of theoretical solution $U^*(t)$ and state output $U(t)$ generated by ZNN model (14.4) with $\gamma_1 = \gamma_2 = 1$ in front of the additive constant noise $\varepsilon = 3$, where solid curves denote the elements of state output $U(t)$, and dash curves denotes the elements of $U^*(t)$. *246*

Figure 14.5 Trajectories of residual error $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ generated by two different models with $\gamma = \gamma_1 = \gamma_2 = 1$ in front of additive constant noise $\varepsilon = 3$. (a) By ZNN model (14.5). (b) By N-FTZNN model (14.9). *246*

Figure 14.6 Trajectories of residual error $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ generated by two different models with $\gamma = \gamma_1 = \gamma_2 = 10$ in front of additive constant noise $\varepsilon = 3$. (a) By ZNN model (14.5). (b) By N-FTZNN model (14.9). *247*

Figure 14.7 Trajectories of residual error $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ generated by two different models with $\gamma = \gamma_1 = \gamma_2 = 10$ in front of additive dynamic noise $\varepsilon = 4t$. (a) By ZNN model (14.5). (b) By N-FTZNN model (14.9). *247*

Figure 14.8 Motion trajectories of a two-link planar manipulator synthesized by the N-FTZNN-based model (14.43) with $\gamma_1 = \gamma_2 = 50$ in front of the additive constant noise $\varepsilon = 2.5$. (a) The actual circle trajectory and the desired path. (b) The whole motion process. (c) The control law $u = \dot{\theta}$. (d) The position tracking error $Y(t) = r(t) - r_d(t)$. *249*

Figure 14.9 Motion trajectories of a two-link planar manipulator synthesized by ZNN-based model (14.31) with $\gamma = 50$ in front of the additive constant noise $\varepsilon = 2.5$. (a) The actual circle trajectory and the desired path. (b) The whole motion process. (c) The control law $u = \dot{\theta}$. (d) The position tracking error $Y(t) = r(t) - r_d(t)$. *250*

Figure 15.1 Transient behavior of state solutions $U(t)$ generated by NT-PTZNN1 model (15.9) and NT-PTZNN2 model (15.13) when solving time-variant Sylvester equation of Example 1 with noise $Y(t) = 0$. (a) By NT-PTZNN1 model (15.9). (b) By NT-PTZNN2 model (15.13). *267*

Figure 15.2 Transient behavior of residual errors $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ generated by NT-PTZNN1 model (15.9) and NT-PTZNN2 model (15.13) when solving time-variant Sylvester equation of Example 1 with noise $d_{ij}(t) = 0$. (a) By NT-PTZNN1 model (15.9). (b) By NT-PTZNN2 model (15.13). *268*

Figure 15.3 Transient behavior of residual errors $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ synthesized by NT-PTZNN1 model (15.9) activated by AF (15.5), NT-PTZNN2 model (15.13) activated by AF (15.6) and ZNN model (15.4) activated by LAF, PSAF, and SBPAF under different kinds of noises $D(t)$. (a) With noise $d_{ij} = 0$. (b) With noise $d_{ij} = 0.45|y_{ij}(t)|$. (c) With noise $d_{ij} = 1$. (d) With noise $d_{ij} = 0.45\cos(2t)$. *268*

Figure 15.4 Transient behavior of state solution $U(t)$ generated by NT-PTZNN1 model (15.9) and NT-PTZNN2 model (15.13) when solving time-variant Sylvester equation of Example 2 with noise $D(t) = 1$. (a) By NT-PTZNN1 model (15.9). (b) By NT-PTZNN2 model (15.13). *270*

Figure 15.5 Transient behavior of residual errors $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ synthesized by NT-PTZNN1 model (15.9), NT-PTZNN2 model (15.13), and ZNN model (15.4) activated by LAF, PSAF, and SBPAF under different kinds of noises $D(t)$ with $\gamma = 1$. (a) With noise $d_{ij}(t) = 1$. (b) With noise $d_{ij}(t) = 0.6|y_{ij}(t)|$. (c) With noise $d_{ij}(t) = 0.6\cos(2.5t)$. (d) With noise $d_{ij}(t) = 0.125\exp(0.2t)$. *271*

Figure 15.6 Transient behavior of residual errors $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ synthesized by NT-PTZNN1 model (15.9), NT-PTZNN2 model (15.13), and ZNN model (15.4) activated by LAF, PSAF, and SBPAF under different kinds of noises $D(t)$ with $\gamma = 10$ and $\gamma = 20$. (a) With noise $d_{ij}(t) = 2.1t$ and $\gamma = 10$. (b) With noise $d_{ij}(t) = 18.5$ and $\gamma = 20$. *272*

Figure 15.7 Transient behavior of residual errors $\|A(t)U(t) - U(t)B(t) + C(t)\|_F$ synthesized by NT-PTZNN1 model (15.9), NT-PTZNN2 model (15.13), and ZNN model (15.4) activated by LAF, PSAF, and SBPAF under different kinds of noises $D(t)$ with $\gamma = 1$. (a) With noise $d_{ij}(t) = 1$. (b) With noise $d_{ij}(t) = 0.5|y_{ij}(t)|$. (c) With noise $d_{ij}(t) = 0.5\sin(1.6t)$. (d) With noise $d_{ij}(t) = 0.1\exp(0.1t)$. *273*

Figure 16.1 Trajectories of theoretical solutions (black solid lines) of the time-varying Sylvester equation and state solutions (dotted curves) generated by SA-FTZNN model (16.22), PA-FTZNN model (16.29), and EA-FTZNN model (16.37). *290*

Figure 16.2 Comparisons of the computational errors generated by SA-FTZNN model (16.22), PA-FTZNN model (16.29), EA-FTZNN model (16.37), and the ZNN activated by sign-bi-power function (16.5). *290*

Figure 16.3 Comparisons of the SA-FTZNN model (16.22) with different values of k. *292*

Figure 16.4 Comparisons of SA-FTZNN model (16.22), PA-FTZNN model (16.29), EA-FTZNN model (16.37) and the ZNN activated by SP-1 function (16.7) and SP-2 function (16.8) while setting a large and small initial state. (a) Large initial state. (b) Small initial state. *292*

Figure 17.1 Trajectories of state vector $U(t)$ by applying FTZNN-1 model (17.17) to solve LMI (17.1) when $U(0)$ is outside $\Omega(0)$ with $\gamma = 1$ and $\alpha = 0.3$. (a) $u_{11}(t)$, (b) $u_{12}(t)$, (c) $u_{21}(t)$, (d) $u_{22}(t)$, (e) $u_{31}(t)$, and (f) $u_{32}(t)$. *310*

Figure 17.2 Comparisons of three FTZNN models with the conventional ZNN models activated by other AFs with $\gamma = 1$ and $\alpha = 0.3$ when $p(0) > 1$. (a) By FTZNN-1 model (17.17) and FTZNN-2 model (17.19). (b) By FTZNN-1 model (17.17) and FTZNN-3 model (17.21). *311*

Figure 17.3 Comparisons of three FTZNN models with the conventional ZNN models activated by other AFs with $\gamma = 1$ and $\alpha = 0.3$ when $p(0) < 1$. (a) By FTZNN-1 model (17.17) and FTZNN-2 model (17.19). (b) By FTZNN-1 model (17.17) and FTZNN-3 model (17.21). *312*

Figure 17.4 Comparisons among three cases of FTZNN-3 model (17.21) with different tunable parameters. (a) $p(0) > 1$. (b) $p(0) < 1$. *313*

Figure 17.5 Transient behaviors of the error function $\|Y(t)\|_2$ synthesized by FTZNN-3 model (17.21) with $k_2 = 1$, $k_3 = 1$ and different values of k_1. *313*

Figure 18.1 Trajectories of state matrix $U(t)$ by applying NT-FTZNN model (18.6) to solve linear inequalities (18.1) when $U(0)$ is outside $\Omega(0)$ with $\gamma = 1$ and $\lambda = 10$. (a) $u_{11}(t)$, (b) $u_{12}(t)$, (c) $u_{21}(t)$, and (d) $u_{22}(t)$. *328*

Figure 18.2 Trajectories of five random state matrix $U(t)$ by applying NT-FTZNN model (18.6) with $\gamma = 1$ and $\lambda = 10$ to solve linear inequalities (18.1) when $U(0)$ is outside $\Omega(0)$. *328*

Figure 18.3 Comparisons of the error function generated by NT-FTZNN model (18.6) with $\gamma = 1$ and $\lambda = 1$ and the error function created by the CZNN model with $\gamma = 1$ which are both activated by linear AF. *329*

Figure 18.4 Comparisons of the error functions generated by NT-FTZNN model (18.6) with $\gamma = 1$ and $\lambda = 1$ which are activated by several of AFs under the same conditions. *329*

Figure 18.5 Trajectories of the error function generated by NT-FTZNN model (18.6) activated by two sign-bi-power AFs with $\gamma = 1$, $\lambda = 1$ and $r = 0.3$ to solve linear inequalities (18.1). (a) When the initial state $p(0) < 1$. (b) When the initial state $p(0) > 1$. *330*

Figure 18.6 Comparisons among three cases of NT-FTZNN model (18.6) with different α to solve linear time-varying inequalities (18.1). *332*

Figure 18.7 Comparisons among three cases of NT-FTZNN model (18.6) with $\alpha = 0.3$ and different λ to solve linear time-varying inequalities (18.1). *332*

Figure 18.8 Trajectories of theoretical solutions of the higher-order time-varying inequalities and actual solutions starting with random initial state using NT-FTZNN model (18.6) with $\gamma = 1$, $\lambda = 1$, and $\alpha = 0.3$. *333*

Figure 18.9 Comparisons of the error functions generated by NT-FTZNN model (18.6) with $\gamma = 1$ and $\lambda = 10$ which are activated by different AFs for solving higher-order time-varying inequalities. *334*

Figure 19.1 Transient behavior of neural states $u(t)$ synthesized by different dynamical models with $\gamma = 1$ and 50 randomly generated initial states within $[-10, 10]$. (a) By OZNN model (19.4). (b) By FTZNN model (19.5). *344*

Figure 19.2 Transient behavior of residual functions $|y(t)|$ synthesized by different dynamical models with $\gamma = 1$ and 50 randomly generated initial states within $[-10, 10]$. (a) By OZNN model (19.4). (b) By FTZNN model (19.5). *345*

Figure 19.3 Transient behavior of residual functions $|y(t)|$ synthesized by FTZNN model (19.5) with different values of γ and 50 randomly generated initial states within $[-10, 10]$. (a) $\gamma = 10$. (b) $\gamma = 10^6$. *345*

Figure 19.4 Transient behavior of residual functions $|y(t)|$ synthesized by different dynamical models with $\gamma = 1$ and 5 randomly generated initial states within $[-4, 4]$. (a) By OZNN model (19.4). (b) By FTZNN model (19.5). *346*

Figure 20.1 Transient behavior of neural states $u(t)$ solved by GNN model (20.2) and OZNN model (20.4) starting with 20 randomly generated initial states and with $\gamma = 1$, where solid curves