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VIRTUAL EXPERIMENTS in MECHANICAL VIBRATIONS

Structural Dynamics and Signal Processing

with website

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Structural Dynamics and Signal Processing

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To Our Wives Laura and Xiudan, and Our Children Emma, Jingde, and Jinghui

“All models are wrong, but some are useful”

George Box

Preface

The idea to write this book came about from many years of interacting with students, both undergraduate and postgraduate. There seemed to be a disconnect between the theoretical treatment of mechanical vibrations and the signal processing procedures needed to measure vibration in the laboratory. They are often treated as separate subjects, sometimes taught in different departments by different lecturers. When the first author of the book came to UNESP Ilha Solteira in Brazil at the end of 2010, he decided to teach a course that combined the two approaches. The notes developed for that course form the basis of this book.

At the beginning of 2010 Bin Tang came as an academic visitor, supported from the China Scholarship Council (Grant No. 2009821053), to the Institute of Sound and Vibration Research (ISVR) in Southampton, UK, where Mike Brennan had a position as professor of engineering dynamics. They worked together for about one year on research related to nonlinear vibrations. Bin Tang then returned to his position as an assistant professor at Dalian University of Technology (DUT), and Mike departed for Brazil. The following year Mike visited Bin Tang in DUT, and about two years later, Bin Tang came to Brazil as an academic visitor, supported by the Brazilian National Council for Scientific and Technological Development (CNPq). He stayed for two years, and during this time they had many discussions about the topics in this book, honing the ideas and concepts. A decision was made to write the book, but this never really began in earnest until the COVID 19 pandemic struck in 2020. This curtailed the much-enjoyed academic activity of travelling and meeting colleagues around the world, and freed up some time to work on the book.

The authors are extremely grateful for the many discussions with both colleagues and students over the years that have helped to form the perspective from which the book is written. The authors would like to acknowledge the financial support of the Brazilian National Council for Scientific and Technological Development (CNPq), (Grant No. 401360/2012-1) and the National Natural Science Foundation of China (Grant No. 11672058). It is hoped that students who are new to the topic, or those who are more experienced in some areas of either vibration or signal processing will find the book useful.

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List of Abbreviations

CPSD
: cross power spectral density

DFT
: discrete Fourier transform

DOF
: degrees-of-freedom

DTFT
: discrete time Fourier transform

Env
: envelope

ESD
: energy spectral density

FEM
: finite element method

FFT
: fast Fourier transform

FRF
: frequency response function

FS
: Fourier series

FT
: Fourier transform

IDFT
: inverse discrete Fourier transform

IDTFT
: inverse discrete time Fourier transform

IFT
: inverse Fourier transform

Im
: imaginary part

IRF
: impulse response function

ln
: natural logarithm

LTI
: linear time-invariant

MDOF
: multi-degree-of-freedom

ODE
: ordinary differential equation

PDE
: partial differential equation

PSD
: power spectral density

Re
: real part

rms
: root mean square

SDOF
: single-degree-of-freedom

SIMO
: single input multiple outputs

SISO
: single input single output

SNR
: signal-to-noise ratio

List of Symbols

Symbol	Description	Units
$a(t)$	Analytic signal (displacement)	[m]
$(A_{n,\,l})_p$	Modal constant for the p-th mode	[1/kg]
c	Viscous damping coefficient	[Ns/m]
c_{B}	Phase speed in a beam	[m/s]
c_{R}	Phase speed in a rod	[m/s]
$\mathbf{C}$	Damping matrix	[Ns/m]
$\tilde{\mathbf{C}}$	Modal damping matrix	[Ns/m]
E	Young's modulus	[N/m^2]
E	Expectation operator	
f	Frequency	[Hz]
$f_c(t)$	Damping force	[N]
$f_e(t)$	Excitation force	[N]
$\hat{f}_e$	Force impulse	[Ns]
$f_i(t)$	Force applied at point i	[N]
$f_k(t)$	Stiffness force	[N]
$f_m(t)$	Inertia force	[N]
f_n	Natural frequency	[Hz]
f_s	Sampling frequency	[Hz]
$\mathbf{f}(t)$	Vector of forces	[N]

Symbol	Description	Units
$\bar{\mathbf{f}}(\mathrm{j}\omega)$	Vector of complex force amplitudes	[N]
$\mathcal{F}$	Fourier transform operator	
$\mathcal{F}^{-1}$	Inverse Fourier transform operator	
$\bar{F}$	Complex force amplitude	[N]
$\lvert\bar{F}\rvert$	Force amplitude	[N]
$\bar{F}_c$	Complex damping force	[N]
$\bar{F}_k$	Complex stiffness force	[N]
$\bar{F}_m$	Complex inertia force	[N]
$F(\mathrm{j}\omega)$	FT of $f_e(t)$	[N/Hz]
$\mathbf{g}$	Modal force vector	[N]
$g_p(t)$	Modal force for the p-th mode	[N]
$\tilde{G}_{xx}(f)$	Estimate of the single-sided PSD of $x(t)$	[m^2/Hz]
$h(t)$	Displacement impulse response function	[m/Ns]
$\dot{h}(t)$	Velocity impulse response function	[m/Ns^2]
$\ddot{h}(t)$	Acceleration impulse response function	[m/Ns^3]
$H(\mathrm{j}\omega)$, $H(f)$	Receptance FRF	[m/N]
$\mathbf{H}(\mathrm{j}\omega)$	Receptance matrix	[m/N]
$H_{\mathrm{vel}}(\mathrm{j}\omega)$	Mobility FRF	[m/Ns]
$H_{\mathrm{acc}}(\mathrm{j}\omega)$	Accelerance FRF	[m/Ns^2]
$H_1(\mathrm{j}\omega)$	H_1 estimator	[m/N]
$H_2(\mathrm{j}\omega)$	H_2 estimator	[m/N]
$i(t)$	Train of delta functions	[1/s]

$i_s(t)$	Current supplied to the shaker	[A]
I	Second moment of area for the cross-section of a beam	[m^4]
j	$\sqrt{-1}$	
k	Stiffness	[N/m]
$\tilde{k}_p$	Modal stiffness of the p-th mode	[N/m]
$K(\mathrm{j}\omega)$	Dynamic stiffness	[N/m]
$\mathbf{K}$	Stiffness matrix	[N/m]
$\tilde{\mathbf{K}}$	Modal stiffness matrix	[N/m]
m	Mass	[kg]
$\tilde{m}_p$	Modal mass of the p-th mode	[kg]
$M(\mathrm{j}\omega)$	Apparent mass	[kg]
$\overline{M}(\mathrm{j}\omega)$	Complex moment amplitude	[Nm]
$\mathbf{M}$	Mass matrix	[kg]
$\tilde{\mathbf{M}}$	Modal mass matrix	[kg]
$\mathbf{q}$	Vector of modal displacements	[m]
$q_p(t)$	Modal participation factor of the p-th mode	[m]
R_{ij}	Residual for the modal model	[m/N]
S	Cross-sectional area of a rod or a beam	[m^2]
$S_{ff}(\omega)$	PSD of $f_e(t)$	[N^2/Hz]
$\tilde{S}_{ff}(\omega)$	Estimate of the PSD of $f_e(t)$	[N^2/Hz]
$S_{fx}(\mathrm{j}\omega)$	CPSD between $f_e(t)$ and $x(t)$	[Nm/Hz]
$\tilde{S}_{fx}(\mathrm{j}\omega)$	Estimate of the CPSD between $f_e(t)$ and $x(t)$	[Nm/Hz]
$S_{xx}(f)$	PSD of $x(t)$	[m^2/Hz]

$\tilde{S}_{xx}(f)$	Estimate of the PSD of $x(t)$	[m^2/Hz]
$S_{xf}(j\omega)$	CPSD between $x(t)$ and $f_e(t)$	[Nm/Hz]
$\tilde{S}_{xf}(j\omega)$	Estimate of the CPSD between $x(t)$ and $f_e(t)$	[Nm/Hz]
t	Time	[s]
T	Time duration	[s]
T_d	Damped natural period	[s]
T_n	Undamped natural period	[s]
T_p	Fundamental period of a periodic signal	[s]
$u(t)$	Heaviside function	
$u(x, t)$	Axial displacement of a rod	[m]
$\overline{U}(j\omega)$	Complex axial displacement amplitudes for a rod	[m]
$w(x, t)$	Lateral displacement of a beam	
$w(t)$, $W(f)$	windows in the time domain and its FT	[m]
$\overline{W}(x)$	Complex displacement amplitude of a beam	[m]
$\overline{W}(j\omega)$	Complex lateral displacement amplitude for a beam	[m]
$x(t)$	Displacement	[m]
$\mathbf{x}(t)$	Vector of displacements	[m]
$\dot{x}(t)$	Velocity	[m/s]
$\dot{\mathbf{x}}(t)$	Vector of velocities	[m/s]
$\ddot{x}(t)$	Acceleration	[m/s^2]
$\ddot{\mathbf{x}}(t)$	Vector of accelerations	[m/s^2]
	Vector of complex displacement	