MICHAEL J. BRENNAN
BIN TANG

# VIRTUAL EXPERIMENTS in MECHANICAL VIBRATIONS

## Structural Dynamics and Signal Processing

with website

WILEY

**Virtual Experiments in Mechanical Vibrations**

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## Structural Dynamics and Signal Processing

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*To Our Wives Laura and Xiudan, and Our Children Emma, Jingde, and Jinghui*

*“All models are wrong, but some are useful”*

George Box

# Contents

# Preface

The idea to write this book came about from many years of interacting with students, both undergraduate and postgraduate. There seemed to be a disconnect between the theoretical treatment of mechanical vibrations and the signal processing procedures needed to measure vibration in the laboratory. They are often treated as separate subjects, sometimes taught in different departments by different lecturers. When the first author of the book came to UNESP Ilha Solteira in Brazil at the end of 2010, he decided to teach a course that combined the two approaches. The notes developed for that course form the basis of this book.

At the beginning of 2010 Bin Tang came as an academic visitor, supported from the China Scholarship Council (Grant No. 2009821053), to the Institute of Sound and Vibration Research (ISVR) in Southampton, UK, where Mike Brennan had a position as professor of engineering dynamics. They worked together for about one year on research related to nonlinear vibrations. Bin Tang then returned to his position as an assistant professor at Dalian University of Technology (DUT), and Mike departed for Brazil. The following year Mike visited Bin Tang in DUT, and about two years later, Bin Tang came to Brazil as an academic visitor, supported by the Brazilian National Council for Scientific and Technological Development (CNPq). He stayed for two years, and during this time they had many discussions about the topics in this book, honing the ideas and concepts. A decision was made to write the book, but this never really began in earnest until the COVID 19 pandemic struck in 2020. This curtailed the much-enjoyed academic activity of travelling and meeting colleagues around the world, and freed up some time to work on the book.

The authors are extremely grateful for the many discussions with both colleagues and students over the years that have helped to form the perspective from which the book is written. The authors would like to acknowledge the financial support of the Brazilian National Council for Scientific and Technological Development (CNPq), (Grant No. 401360/2012-1) and the National Natural Science Foundation of China (Grant No. 11672058). It is hoped that students who are new to the topic, or those who are more experienced in some areas of either vibration or signal processing will find the book useful.

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January 2022

# List of Abbreviations

| | |
|---|---|
| CPSD | cross power spectral density |
| DFT | discrete Fourier transform |
| DOF | degrees-of-freedom |
| DTFT | discrete time Fourier transform |
| Env | envelope |
| ESD | energy spectral density |
| FEM | finite element method |
| FFT | fast Fourier transform |
| FRF | frequency response function |
| FS | Fourier series |
| FT | Fourier transform |
| IDFT | inverse discrete Fourier transform |
| IDTFT | inverse discrete time Fourier transform |
| IFT | inverse Fourier transform |
| Im | imaginary part |
| IRF | impulse response function |
| ln | natural logarithm |
| LTI | linear time-invariant |
| MDOF | multi-degree-of-freedom |
| ODE | ordinary differential equation |
| PDE | partial differential equation |
| PSD | power spectral density |
| Re | real part |
| rms | root mean square |
| SDOF | single-degree-of-freedom |
| SIMO | single input multiple outputs |
| SISO | single input single output |
| SNR | signal-to-noise ratio |

# List of Symbols

| Symbol | Description | Units |
|---|---|---|
| $a(t)$ | Analytic signal (displacement) | [m] |
| $(A_{n,l})_p$ | Modal constant for the $p$-th mode | [1/kg] |
| $c$ | Viscous damping coefficient | [Ns/m] |
| $c_{\mathrm{B}}$ | Phase speed in a beam | [m/s] |
| $c_{\mathrm{R}}$ | Phase speed in a rod | [m/s] |
| $\mathbf{C}$ | Damping matrix | [Ns/m] |
| $\tilde{\mathbf{C}}$ | Modal damping matrix | [Ns/m] |
| $E$ | Young's modulus | [N/m$^2$] |
| $\mathrm{E}$ | Expectation operator | |
| $f$ | Frequency | [Hz] |
| $f_c(t)$ | Damping force | [N] |
| $f_e(t)$ | Excitation force | [N] |
| $\hat{f}_e$ | Force impulse | [Ns] |
| $f_i(t)$ | Force applied at point $i$ | [N] |
| $f_k(t)$ | Stiffness force | [N] |
| $f_m(t)$ | Inertia force | [N] |
| $f_n$ | Natural frequency | [Hz] |
| $f_s$ | Sampling frequency | [Hz] |
| $\mathbf{f}(t)$ | Vector of forces | [N] |
| $\overline{\mathbf{f}}(\mathrm{j}\omega)$ | Vector of complex force amplitudes | [N] |
| $\mathcal{F}$ | Fourier transform operator | |
| $\mathcal{F}^{-1}$ | Inverse Fourier transform operator | |
| $\overline{F}$ | Complex force amplitude | [N] |
| $\lvert\overline{F}\rvert$ | Force amplitude | [N] |
| $\overline{F}_c$ | Complex damping force | [N] |
| $\overline{F}_k$ | Complex stiffness force | [N] |
| $\overline{F}_m$ | Complex inertia force | [N] |

| Symbol | Description | Units |
|---|---|---|
| $F(\mathrm{j}\omega)$ | FT of $f_e(t)$ | [N/Hz] |
| $\mathbf{g}$ | Modal force vector | [N] |
| $g_p(t)$ | Modal force for the $p$-th mode | [N] |
| $\tilde{G}_{xx}(f)$ | Estimate of the single-sided PSD of $x(t)$ | [m$^2$/Hz] |
| $h(t)$ | Displacement impulse response function | [m/Ns] |
| $\dot{h}(t)$ | Velocity impulse response function | [m/Ns$^2$] |
| $\ddot{h}(t)$ | Acceleration impulse response function | [m/Ns$^3$] |
| $H(\mathrm{j}\omega)$, $H(f)$ | Receptance FRF | [m/N] |
| $\mathbf{H}(\mathrm{j}\omega)$ | Receptance matrix | [m/N] |
| $H_{\mathrm{vel}}(\mathrm{j}\omega)$ | Mobility FRF | [m/Ns] |
| $H_{\mathrm{acc}}(\mathrm{j}\omega)$ | Accelerance FRF | [m/Ns$^2$] |
| $H_1(\mathrm{j}\omega)$ | $H_1$ estimator | [m/N] |
| $H_2(\mathrm{j}\omega)$ | $H_2$ estimator | [m/N] |
| $i(t)$ | Train of delta functions | [1/s] |
| $i_s(t)$ | Current supplied to the shaker | [A] |
| $I$ | Second moment of area for the cross-section of a beam | [m$^4$] |
| j | $\sqrt{-1}$ | |
| $k$ | Stiffness | [N/m] |
| $\tilde{k}_p$ | Modal stiffness of the $p$-th mode | [N/m] |
| $K(\mathrm{j}\omega)$ | Dynamic stiffness | [N/m] |
| $\mathbf{K}$ | Stiffness matrix | [N/m] |
| $\tilde{\mathbf{K}}$ | Modal stiffness matrix | [N/m] |
| $m$ | Mass | [kg] |
| $\tilde{m}_p$ | Modal mass of the $p$-th mode | [kg] |
| $M(\mathrm{j}\omega)$ | Apparent mass | [kg] |
| $\overline{M}(\mathrm{j}\omega)$ | Complex moment amplitude | [Nm] |
| $\mathbf{M}$ | Mass matrix | [kg] |
| $\tilde{\mathbf{M}}$ | Modal mass matrix | [kg] |
| $\mathbf{q}$ | Vector of modal displacements | [m] |
| $q_p(t)$ | Modal participation factor of the $p$-th mode | [m] |
| $R_{ij}$ | Residual for the modal model | [m/N] |
| $S$ | Cross-sectional area of a rod or a beam | [m$^2$] |
| $S_{ff}(\omega)$ | PSD of $f_e(t)$ | [N$^2$/Hz] |
| $\tilde{S}_{ff}(\omega)$ | Estimate of the PSD of $f_e(t)$ | [N$^2$/Hz] |
| $S_{fx}(\mathrm{j}\omega)$ | CPSD between $f_e(t)$ and $x(t)$ | [Nm/Hz] |
| $\tilde{S}_{fx}(\mathrm{j}\omega)$ | Estimate of the CPSD between $f_e(t)$ and $x(t)$ | [Nm/Hz] |
| $S_{xx}(f)$ | PSD of $x(t)$ | [m$^2$/Hz] |
| $\tilde{S}_{xx}(f)$ | Estimate of the PSD of $x(t)$ | [m$^2$/Hz] |
| $S_{xf}(\mathrm{j}\omega)$ | CPSD between $x(t)$ and $f_e(t)$ | [Nm/Hz] |
| $\tilde{S}_{xf}(\mathrm{j}\omega)$ | Estimate of the CPSD between $x(t)$ and $f_e(t)$ | [Nm/Hz] |

| Symbol | Description | Units |
|---|---|---|
| $t$ | Time | [s] |
| $T$ | Time duration | [s] |
| $T_d$ | Damped natural period | [s] |
| $T_n$ | Undamped natural period | [s] |
| $T_p$ | Fundamental period of a periodic signal | [s] |
| $u(t)$ | Heaviside function | |
| $u(x, t)$ | Axial displacement of a rod | [m] |
| $\overline{U}(\mathrm{j}\omega)$ | Complex axial displacement amplitudes for a rod | [m] |
| $w(x, t)$ | Lateral displacement of a beam | |
| $w(t)$, $W(f)$ | windows in the time domain and its FT | [m] |
| $\overline{W}(x)$ | Complex displacement amplitude of a beam | [m] |
| $\overline{W}(\mathrm{j}\omega)$ | Complex lateral displacement amplitude for a beam | [m] |
| $x(t)$ | Displacement | [m] |
| $\mathbf{x}(t)$ | Vector of displacements | [m] |
| $\dot{x}(t)$ | Velocity | [m/s] |
| $\dot{\mathbf{x}}(t)$ | Vector of velocities | [m/s] |
| $\ddot{x}(t)$ | Acceleration | [m/s$^2$] |
| $\ddot{\mathbf{x}}(t)$ | Vector of accelerations | [m/s$^2$] |
| $\overline{\mathbf{x}}(\mathrm{j}\omega)$ | Vector of complex displacement amplitudes | [m] |
| $\overline{X}$ | Complex displacement amplitude | [m] |
| $\lvert\overline{X}\rvert$ | Displacement amplitude | [m] |
| $\lvert\overline{X}_n\rvert$ | Amplitude of the $n$-th harmonic of the Fourier series | [m] |
| $\tilde{X}_n$ | Complex amplitude of the $n$-th harmonic of the complex | [m/Hz] |
| | Fourier series | [m] |
| $X(\mathrm{j}\omega)$, $X(f)$ | FT of $x(t)$ | |
| $X_s(f)$ | DTFT of $x(t)$ | [m] |
| $X(k\Delta f)$ | DFT of $x(n\Delta t)$ | [m] |
| $Z(\mathrm{j}\omega)$ | Impedance | [Ns/m] |

## Greek Symbols

| | | |
|---|---|---|
| $\alpha$ | Time delay | [s] |
| $\beta_\mathrm{R}$ | Wavenumber for a rod | [1/m] |
| $\beta_\mathrm{B}$ | Wavenumber for a beam | [1/m] |
| $\delta(t)$ | Delta function | [1/s] |
| $\delta(x)$ | Delta function | [1/m] |
| $\Delta f$ | Frequency resolution | [Hz] |
| $\Delta t$ | Time resolution | [s] |
| $\varepsilon$ | Time duration | [s] |
| $\phi, \theta, \psi$ | Phase angle | [rad] |

| | | |
|---|---|---|
| $\phi_p(x)$ | Mode shape for the $p$-th mode of a rod or a beam | |
| $\mathbf{\phi}_p$ | Mode shape vector for the $p$-th mode of a lumped parameter system | |
| $\mathbf{\Phi}$ | Matrix of mode-shape vectors for a lumped parameter system | |
| $\gamma_{fx}^2(\omega)$ | Coherence function between $f$ and $x$ | |
| $\gamma$ | Ratio of absorber nat. freq. to host structure nat. freq. | |
| $\eta$ | Structural loss factor | |
| $\mu$ | Mass ratio | |
| $\rho$ | Density | [kg/m$^3$] |
| $\sigma$ | Standard deviation | |
| $\zeta$ | Damping ratio | |
| $\zeta_p$ | Damping ratio of the $p$-th mode | |
| $\omega$ | Circular excitation frequency | [rad/s] |
| $\omega_a$ | Undamped natural frequency of a vibration absorber | [rad/s] |
| $\omega_d$ | Damped natural frequency | [rad/s] |
| $\omega_n$ | Undamped natural frequency or the $n$-th harmonic | [rad/s] |
| $\omega_p$ | Undamped natural frequency of the $p$-th mode | [rad/s] |
| $\Omega$ | Non-dimensional frequency | |

# About the Companion Website

This book is accompanied by a companion website which has MATLAB files.

www.wiley.com/go/brennan/virtualexperimentsinmechanicalvibrations

# 1

# Introduction

## 1.1 Introduction

Knowledge of the dynamic behaviour of systems and structures becomes increasingly important as organisations and companies strive to produce devices and products that outperform the competition. This means that engineers from a wide range of disciplines covering, for example, automotive, acoustical, aeronautical, aerospace, civil, mechanical, and marine engineering, are required to have knowledge of vibration engineering. Of course, some will need to be experts in this discipline, but others will simply need to be aware of some basic issues. This means that university engineering programmes for all the disciplines mentioned above generally have a course in mechanical vibrations. These courses tackle the subject in different ways, depending on the particular discipline. For example, civil engineers start from the study of the static behaviour of structures. Once this has been mastered, they move to the dynamic behaviour of structures, i.e. they start at a frequency of 0 Hz, and then investigate the behaviour as frequency increases. This sequence of study is similar for many disciplines, with the exception, perhaps, of physicists and acoustical engineers, who may tackle the subject using a wave description of the structural dynamics. Acoustical engineers generally restrict their frequency range of interest to that of human hearing, which is from about 20 Hz to 20 kHz. Thus, the way in which mechanical vibration is taught may vary enormously from course to course. To illustrate the diversity of the topic, Michael Brennan, the first author of this book, started his career in vibration engineering by investigating high-frequency (>500 Hz) structure-borne noise through a helicopter gear box support strut, whereas Bin Tang, the second author of this book, started his career by investigating the relatively low-frequency torsional vibration (<30 Hz) of a ship's propellor shaft.

The terms 'mechanical vibration' and 'engineering vibration' are used interchangeably in this book. To master this topic from a theoretical and a practical point of view, the student is required to have some knowledge of physics, mathematics, and engineering. This is illustrated schematically in the Venn diagram shown in Figure 1.1. It is acknowledged that not all vibration engineers have the same profile. For example, some have a much more mathematical bias, focusing on theoretical aspects of the subject, perhaps working as researchers in universities, and others follow a much more practical career, working on the implementation of vibration control strategies in consulting or engineering companies. Notwithstanding this, it is the firm belief of the authors, that engineers/researchers will only gain mastery of the topic, if their knowledge base is in the area of the overlapping circles shown in Figure 1.1.

It is not the aim of this book to provide basic knowledge in mechanical vibrations, although it is expected that the reader will gain some insight into the dynamical behaviour of a simple vibrating

*Virtual Experiments in Mechanical Vibrations: Structural Dynamics and Signal Processing,*
First Edition. Michael J. Brennan and Bin Tang.

Companion website: www.wiley.com/go/brennan/virtualexperimentsinmechanicalvibrations

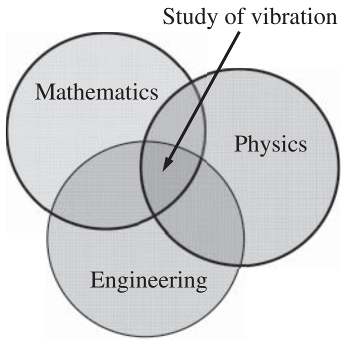


**Figure 1.1** The subject of vibration engineering.

system. There are several textbooks devoted to basic vibration theory, for example, Tse et al. (1978), Thompson (2002), Clough and Penzien (2003), de Silva (2006), Inman (2007), and Rao (2016). There is also the classic book (Den Hartog, 1956) that offers some excellent physical descriptions of vibrating systems. The aim of this book is to provide a text that will help to bridge the gap between vibration theory and laboratory-based experimental work. Many students study vibration from a purely theoretical point of view. In some institutions, the lecturers may not even be experts in vibration engineering, and so they teach by closely following a textbook. Inevitably, this is often a mathematical exposition, with the underlying physics being frequently masked by mathematical complexity. Accordingly, many students do not gain the necessary physical insight, which would be helpful in their future careers. One way to overcome this problem is to formulate vibration problems in a more physical way in terms of variables that are measurable in a laboratory setting. In many situations, these are forces applied to the system or structure and the resulting accelerations/velocities/displacements. This means that before the theory is taught, some thought should be given to an accompanying experiment, to ensure that the output from the theoretical model involves measurable variables. It is, of course, desirable that any course has a practical component to complement and support the theory.

Much of the physical insight gained in vibration engineering, whether it be theoretical or experimental, occurs by viewing data in the frequency domain. However, all vibration signals are measured in the time domain, so these signals must be transformed to the frequency domain using signal processing techniques. This, of course, means that the vibration engineer should have some knowledge of the way in which this is done, and the mathematical basis behind the techniques. The way data are processed in practice is to first sample the data and then to work on them in digitised form using a computer. Processing sampled data brings further complications, which are discussed in Chapter 4. Many students of vibration engineering may have studied some signal processing techniques, such as Fourier analysis, but often this is done in a mathematics department, and therefore is often not related directly to the vibration theory taught in the engineering departments. There can thus be a chasm between the taught vibration theory and the way in which corresponding experimental data are captured and processed to enable comparisons between predictions and reality. It is the intention of this book to bring together these two disciplines and to give the reader some experience in applying the required signal processing techniques on simulated vibration data. There is one book on signal processing, which is specifically tailored for sound and vibration engineers (Shin and Hammond 2008), and there are other more general textbooks on the subject, which may help the reader with some of the more theoretical aspects, for example Papoulis (1962, 1977), Oppenheim and Schafer (1975), Oppenheim et al. (1997), and Bendat and Piersol (1980, 2000).

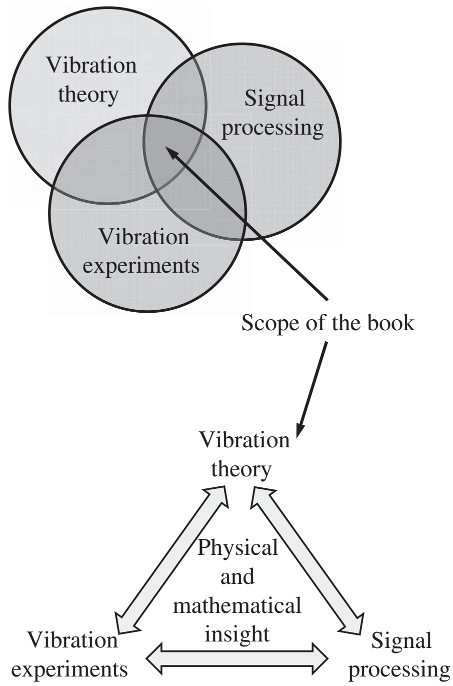


**Figure 1.2** Schematic diagram showing the scope of the book.

The scope of the book is encapsulated in the Venn diagram shown in the top part of Figure 1.2. It can be seen that it contains three elements, vibration theory, vibration experiments, and signal processing. At the end of the book the reader will have been exposed to elements of these three topics and will have carried out some 'virtual' experiments using simulated data. Through the theoretical development and exercises in the book, some proficiency should be gained, which hopefully will result in improved physical insight into both vibration theory and the rationale between the choices to be made in the signal processing procedures. At the end of the book, the reader should be in a position to carry out an experiment in the laboratory and process the measured signals, provided that the experimenter has been given some additional tuition on the practical aspects of how to set up an experiment and how to handle the transducers correctly.

## 1.2 Typical Laboratory-Based Vibration Tests

Two typical vibration tests are shown in Figure 1.3. In the top part of the figure, an electrodynamic shaker is used to excite the structure under test, and in the lower part of the figure an instrumented impact hammer is used to excite the structure. In both cases, the resulting vibration response is measured using an accelerometer, as shown in the figure. Details of some typical signals, which are used to drive the shaker and the type of force signal generated by the impact hammer, are discussed in Chapter 5. For the shaker excitation, a signal is provided by a signal generator, which is then passed through a power amplifier, before supplying the shaker. The signal then has enough power to drive the shaker. In many cases the signal generator forms part of a software package in a computer. The force is measured using a force gauge attached to the structure, and this signal together

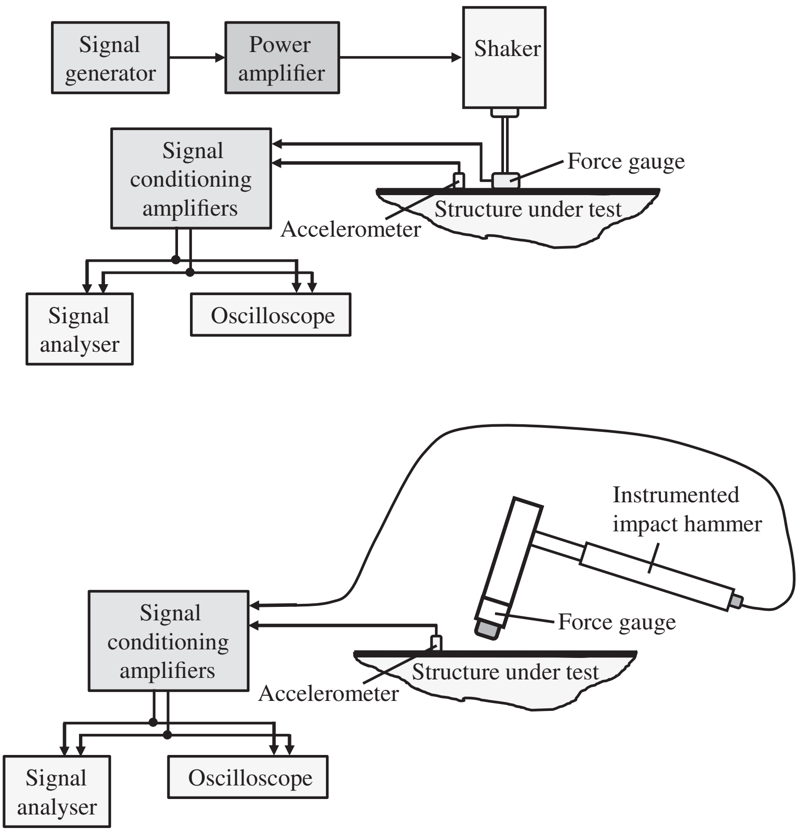


**Figure 1.3** Typical experimental set-ups to measure a frequency response function (FRF). Source: Modified from Waters (2013) / Taylor & Francis.

with the signal from the accelerometer are passed through conditioning amplifiers before entering the signal analyser, and being viewed in analogue form using the oscilloscope. For hammer excitation, the force gauge is in the tip of the hammer and measures the force applied to the structure during the impact. The signals from the force gauge and the accelerometer are processed in a similar way for both shaker and force excitation. Further details on how to set up a vibration experiment similar to that shown in Figure 1.3 are given in Waters (2013). General textbooks on vibration testing have been written by Ewins (2000), McConnell and Varoto (2008), Brandt (2011), and Avitabile (2017).

The test set-ups shown in Figure 1.3. are designed to measure a single input and single output (SISO). More accelerometers can be added at different points on the structure to form a single input multi-output (SIMO) system, and an example of this type of measurement is described in Chapter 9. As mentioned above, more insight is gained by examining the data in the frequency domain – specifically the output for a given input at each frequency of excitation. This is achieved by studying this relationship which is called the *frequency response function* (FRF). The FRF is the backbone of this book, both theoretically and experimentally. It is derived analytically for a simple vibrating system in Chapter 2, and the way in which it is estimated from measurements or simulations using time domain force and acceleration data is described in Chapter 8.

## 1.3 Relationship Between the Input and Output for a SISO System

The relationships between the signals from a vibration measurement are shown in Figure 1.4. However, note that in this figure, displacement rather than acceleration is the response variable. This has been chosen for convenience, but also note that acceleration signals can easily be converted to velocity or displacement, by time-domain integration as discussed in Appendix A. The engineering units are shown for all the variables in Figure 1.4, as this is considered to be important in the context of this book and is rarely provided in books on signal processing. The input to the system is a force $f_e(t)$ which has the SI unit of N, and the displacement response $x(t)$ which has the unit of m. The vibrating system connecting the input to the output has a time domain description $h(t)$, which is the *impulse response function* (IRF) and has units of m/Ns. The displacement output can be determined by convolving $f_e(t)$ with $h(t)$, which is discussed further in Chapter 2, and is used extensively throughout the book.

As mentioned above, it is necessary to transform the data to the frequency domain. This is achieved by using the Fourier transform. The Fourier transform of the force time history is given by $\mathcal{F}\{f_e(t)\}$ and results in $F(\mathrm{j}\omega)$, where $\mathrm{j} = \sqrt{-1}$ and $\omega$ is angular frequency, which has units of rad/s; $F(\mathrm{j}\omega)$ has units of N/Hz. Note that in this book time domain quantities are denoted by lower-case italic symbols and frequency domain quantities are denoted by upper-case italic symbols. The Fourier transform of the displacement time history is given by $X(\mathrm{j}\omega) = \mathcal{F}\{x(t)\}$, which has units of m/Hz. Chapter 3 is devoted to the Fourier transform applied to continuous and sampled time histories. Note that frequency domain data can be transformed to the time domain, and this is achieved using the inverse Fourier transform, which is also discussed in Chapter 3. The frequency domain description of the system is the FRF, denoted by $H(\mathrm{j}\omega)$. This is related to the IRF by the Fourier transform, i.e. $H(\mathrm{j}\omega) = \mathcal{F}\{h(t)\}$ and has units of m/N. The output in the frequency domain $X(\mathrm{j}\omega)$ can be determined by multiplying $F(\mathrm{j}\omega)$ with $H(\mathrm{j}\omega)$, and this is discussed in Chapter 2.

You will become aware as you read this book that most of the analysis is conducted using FRFs. The theoretical FRFs shown are analytical because the systems discussed are relatively simple. However, if modelling is carried out using numerical tools such as finite element analysis (FEA), Petyt (2010), which is used extensively in industry, it is also important that structures are modelled so that FRFs can be easily extracted for analysis and comparison with measurements.

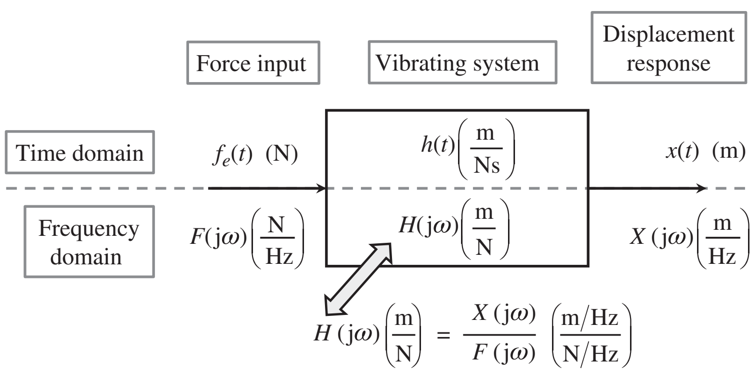


**Figure 1.4** Block diagram representing of a simple single-input, single-output vibration test. Note that the response in this case is displacement for convenience, which can be obtained by integrating acceleration twice as described in Appendix A.

## 1.4 A Virtual Vibration Test

As mentioned previously, the aim of this book is to bridge the gap between vibration theory and vibration experiments. The book can also be used by students who do not have access to a laboratory to conduct experiments. They can carry out 'virtual' experiments. In a real experiment both force input and displacement output are measured, but in a virtual experiment the output data are generated using a model of the system. The concept is shown in Figure 1.5. The virtual experiment has a major advantage as a learning tool, in that the processed data in terms of an IRF or FRF, can be compared with the original model, which was used to generate the output time series. In this way, any artefacts in the data due to the processing can be clearly identified, which is not always possible in a real experiment.

Several methods can be used to determine the displacement output data, three of which are used in this book, and are described in Chapter 6. They are:

1. If the differential equation(s) of the vibrating system are known, then the response can be calculated by numerical integration of the equation(s) of motion. Generally, this is a straightforward procedure using a computer and is described in Appendix D.
2. If the IRF of the vibrating system is known, the response can be determined using convolution. Again, this is a relatively straightforward procedure and is described in Appendix G.
3. If the FRF of the vibrating system is known, the input force time history can be transformed to the frequency domain using the Fourier transform. The frequency domain response can then be calculated by multiplying this by the FRF, which can then be transformed to the time domain using the inverse Fourier transform to give the time history of the response. Alternatively, the FRF can be transformed to give the IRF and then the method in 2 can be used.

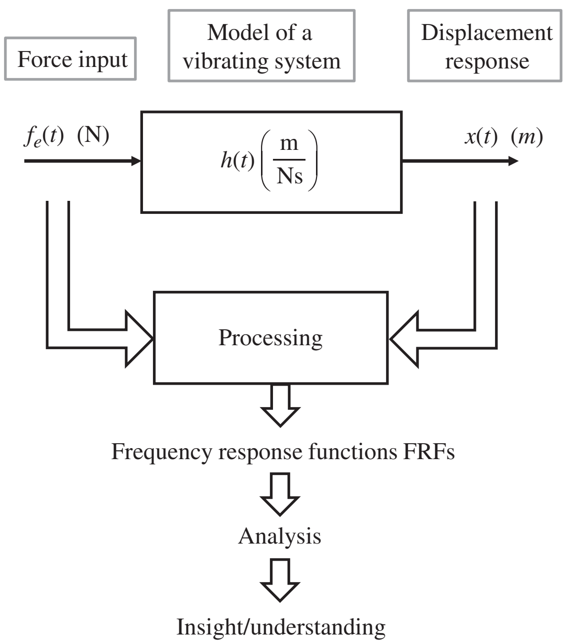


**Figure 1.5** The process and rationale for a virtual vibration experiment.