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Mathematical Modeling of Physical Systems

Applications of Fields, Circuits and Signal Processing

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Dr. Adhir Baran Chattopadhyay
Children (Rimpu, Riki), Wife (Pratima) and
Sons-in-law (Anand and Aritra)

Dr. Shazia Hasan
Syed Aquil Manzar Hasan (Father)
You have been a constant source of my
inspiration
&
Sarfraj, Iqra, Sidra and Inaya
for your unconditional love and support

Dr. Snehaunshu Chowdhury
Late Smt. Rina Chowdhury (mother)
You had always been a source of constant
inspiration for me
and
Dolon and Samadarshi
for your unconditional love and support

Preface

Generally, it is felt that a new research book on classical foundation concepts should focus on a crystal-clear and reasonable point of view not highlighted by research-based books already published and used by the readers. Based on this philosophy, this book has been written with an anticipation that the experience and vision of the authors are reflected. The authors of this book also expect that the readers who are not intellectually convinced by the above-said view point of thoughts can search elsewhere the suitable academic or research materials. The mode of presentations and contents of the book express our personal opinions in different ways, as follows.

The Place of Magnetic Levitation and Propulsion as a Major Application Area of Electromagnetic Field Theory

Electromagnetic Field Theory is basically based on the concept of the interaction of a source quantity with the concerned field quantity. Again, based on the concepts of electromagnetic field theory, analytical discussions of some major applications may be presented in any research book. However, seeing the limitations of inclusion of the application-based topics, this book partially presents the mathematical modelling of a linear induction motor (with long stator and short rotor), which is the most important component of a commonly used magnetic levitation and propulsion system.

Applications in Fluid Mechanics

The theory of irrotational flows (potential flows) plays a vital role in the study of fluid mechanics. Not only are such flows of historical importance but they are still used to understand a lot of complex flows by superposition of simple elementary flows. An effort is made to introduce the students to fluid flow and include the governing equations describing their flow. Care and emphasis have been made to limit the discussion to irrotational flows as the governing equations for inviscid, steady, irrotational flows are the Laplace equations. But due to the extremely limited nature of complex geometry in real systems, the Laplace equations are not always solved analytically. Therefore, the readers are introduced to various numerical procedures such as finite difference, finite volume and finite element techniques. The mathematical nature of the governing equations and their physical significance are discussed. Finally, several cases of the Laplace equations are solved with various boundary conditions using the finite element method demonstrating the different fields in which such solutions could play an important role.

Mathematical Modelling Using the Generalized Theory of Electrical Machines

The authors believe that till date, the mathematical modelling of any composite engineering system (or, problem) involves the mathematical modelling of the electrical drives part, or electrical power generation part (in general), as a major share. Such necessity basically leads to a unified type of mathematical modelling of all types of electrical machines. This particular terminology, "unified type of mathematical modelling" is basically known as "Generalized Theory of Electrical Machines". Chapter 7 is dedicated to the sharing of the rigorous analytical aspects of electrical machines using the generalized theory of electrical machines.

Mathematical Modelling of a Non-linear System Using Circuit Theory Approach and Linearization Technique

In contrast to the "Field Theory Approach", the "Circuit Theory Approach" bears some advantages, and these advantages are encashed for modelling a power electronic D.C drive system involving some non-linearities. Moreover, a non-linear state variable modelling approach has been implemented based on linearization techniques. Chapter 8 reflects these aspects.

Applications of Signal Processing to Certain Classes of Electrical Power System Problems

It is well known that a major part of modelling a power system involves mathematical handling of different types of data. Such work, in the long run, merges to be a major responsibility in the area of signal processing. The power system data generally are analysed using Kalman Filter (KF), Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) which are being considered as the very effective methods in discrete domains. Chapters 9 and 10 of this book basically reflects the above-said modelling philosophy.

Organization

This book may be conceived as divided into four conceptual (also chronologically placed) area as follows:

- Mathematical modelling of Linear Induction Motors having different constructional features, based on "Electromagnetic Field Theory" (Chaps. 1–3).
- Mathematical modelling of the selected systems using the concept of "Fluid Fields" (Chaps. 4–6).
- Mathematical modelling of the selected systems using "Circuit Theory Approach" (Chaps. 7 and 8).
- Mathematical modelling of the selected power system phenomena using "Signal Processing Approach" (Chaps. 9 and 10).

Suggestions for Using This Book

(i) This book can be partially tailored as a course work for the research programmes (leading to the Ph.D. degree) in the universities.
(ii) This book also may be used as a helping tool for the researchers working in the field of mathematical modelling of physical systems.
(iii) A book chapter may be separately written based on the concept of comparing the "Electromagnetic Field Theory" with the "Fluid Field Theory".

Notable Features

(i) **Chapter 1: Sect. 1.2**: This section actually gives a feeling to the researchers about formulating and solving a partial differential equation (PDE) problem in two dimensions.
(ii) **Chapter 1: Sects. 1.2** and **1.5**: The difference in the philosophical aspects of these two sections works as an eye opener before any researcher from the view point of giving extension to one existing class of PDE formulations. In other words, such sections show how the number of PDEs can be increased while pursuing the mathematical modelling of a practical problem.

(iii) **Chapter 2: Sect. 2.2.1.1 and Appendix B**: This particular direction of thinking clearly shows how the deep knowledge of "Theory of Complex Variable" or "Contour Integration" helps a researcher to solve the so-called difficult type of improper integrals involving an integrand whose convergence generally cannot be assured.

(iv) **Chapter 4**: The continuity in this chapter is based on elementary fluid mechanics taught in undergraduate courses. After a brief introduction on how flow fields are described, the governing equations of fluid flow are presented. Subsequently, the potential flow equation, both in terms of velocity potential and stream function, is developed. The concept of elementary flows is introduced along with the idea of linear superposition of elementary flows to reproduce certain physical flow cases. Methods for solving the Laplace equation are also provided in this chapter.

(v) **Chapter 5**: Introduction to the various numerical techniques to solve the Laplace equation is presented. Finite difference schemes, finite volume as well as finite element techniques are introduced. It should be noted that where analytical solutions are not possible either due to complex boundary conditions or geometry, numerical solutions could be used. These techniques, therefore, refer to a generalized way of solving PDEs by eventually converting them into a system of algebraic equations. These techniques are extremely powerful and both academia and industry have been extremely diligent in pushing the frontiers of these techniques.

(vi) **Chapter 6**: This chapter demonstrates the use of finite element techniques to solve the Laplace equation numerically. The Laplace equation forms the governing equation in multiple disciplines spanning fluid mechanics, groundwater seepage, electrostatics, steady-state heat transfer, etc. The readers should grasp the beauty of the interdisciplinary nature of this technique and its application.

(vii) **Chapter 7: Sect. 7.2.3 and Reference literature (5)**: The authors feel that these directions will clear the ideas of a researcher about modelling any transient or sub-transient equivalent circuits.

(viii) **Chapter 9: Sects. 9.2.4 and 9.2.5**: The in-depth studies of these two sections will throw light on the concept of the basic difference between "Extended Kalman Filter" and "Unscented Kalman Filter". This study also will help in identifying the drawbacks of the "Linearization Technique" being used during the problem formulation in the area of signal processing.

(ix) **Chapter 10: Sect. 10.2.2**: In power systems, due to sudden load change or short circuit, there may happen a drastic reduction in system frequency, and in turn, such reduction may affect the overall performance of the particular power system in a most negative manner. That is why "Frequency Estimation" becomes very much necessary as a part of the mathematical modelling work. Moreover, this particular section has another feature that a detailed application of "Z-Transform" is involved and such exercise may accelerate the mathematical modelling skill of a researcher.

(x) **Appendix A**: Appendix A puts light on the actual use of Fourier Series for dealing with a practical problem. Such analysis dictates that the finiteness of rotor width leads to the deviation in the conductivity or resistivity from the original value.

(xi) **Appendix C**: The content of this chapter is not actually needed from the view point of mathematical modelling but it is very much needed for prototype development. The authors believe that a perfect combination of knowledge and skill of prototype development and the knowledge of mathematical modelling helps any researcher to create a very high level of research environment.

Kolkata, India — Adhir Baran Chattopadhyay
Dubai, United Arab Emirates — Shazia Hasan
Dubai, United Arab Emirates — Snehaunshu Chowdhury

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(i) **By Dr. Adhir Baran Chattopadhyay**:

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Contents

About the Editors

Dr. Adhir Baran Chattopadhyay completed his Ph.D. (Electrical Engineering) from Indian Institute of Technology (IIT), Kharagpur, India, in 1999. He has worked as an Assistant Engineer (Quality Control) in M/S Crompton Greaves Limited, Ahmednagar, Maharashtra, India, from 1980 to 1985; as a faculty-member (Associate Professor) in National Institute of Technology, Jamshedpur, India (from 1985 to 2006), and as a Professor (EEE Department), BITS Pilani, Dubai, UAE (from 2006 to 2020). He is presently working as a Professor (Electrical Engineering Department), Budge Budge Institute of Technology (B.B.I.T), Kolkata-700137, since March, 2021. His research interests include electrical machines and drives, control systems, power electronics, power systems, electromagnetic fields and waves and applied mathematics. He has published more than 45 research papers in reputed international journals. He serves as a reviewer in many popular journals. He has been awarded (jointly) **"Cogent Engineering Best Paper 2020" award, by Cogent Engineering Journal, Publisher—Taylor and Francis, U.K—Award declared on 7th May, 2021. He has published, as a first author, the book, "Advanced Electrical and Power System Design" in July, 2021 (Publisher: New Age International Publishers, New Delhi, London).**

Dr. Shazia Hasan has more than 15 years of teaching and research experience in reputed institutes in India and currently working as an Assistant Professor at BITS Pilani Dubai Campus. She received her Ph.D. in Engineering from Biju Patnaik University of Technology, India, in the year 2012. She won the 'Young Scientist Award' from VIFRA in 2015. She has published more than 35 research papers in peer-reviewed international journals/conferences. She has served as a reviewer for various reputed Journals like as IEEE, Elsevier, Springer and IET. She is a Senior Member of IEEE. She has served as convenor/co-convenor for several international conferences. She is the recipient of "Academic achievement for university professor award" Leadership Excellence for Women Awards (LEWAS) 2020. Her research interest includes Digital Signal Processing, signal processing application to Power systems, renewable energy integration, and statistical filter design.

Dr. Snehaunshu Chowdhury received his B.E. degree in 2001 from Regional Engineering College (currently National Institute of Technology), Trichy, India; M.S.M.E. degree from Indiana University Purdue University Indianapolis (IUPUI), Indiana, USA, in 2004; M.E. degree from Colorado State University, Fort Collins, USA, in 2005; and a Ph.D. degree from the University of Maryland College Park, USA, in 2012, all in Mechanical Engineering. He then worked as a postdoctoral fellow in the "Clean Combustion Research Center" at King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia, from 2013 to 2016. Currently, Dr. Chowdhury is working as an assistant professor in the department of BITS Pilani, Dubai Campus, Dubai, UAE, since 2017 after a short stint at B.C. Roy Engineering College, Durgapur, India. Dr. Chowdhury's research interests include nanoparticle characterization and combustion, soot particle and flame speed measurements and computational fluid dynamics. He has authored many papers and serves as a reviewer of internationally reputed journals.

Chapter 1
Applications of Field Theory: Analysis of a Single Sided Linear Induction Motor with Stator and Rotor of Infinite Length and Width

1.1 Introduction

Linear Induction motors (LIMs) have several practical applications [1–13] and they have been analysed by various researchers [14–23]. However, the bulk of literature on LIMs deals with the double-sided, short primary and long secondary LIM which is considered as one of the most suitable means of propulsion for high-speed ground vehicles [15, 24–26]. There is an interesting work on “High-Temperature Superconducting(HTS) LIM” [27] which gives a good view on the coordination between HTS and LIM, leading to a realistic practical application of Single-sided Linear Induction Motor (supported with back iron) to the rail system.

However, in extra high-speed transportation, it is needed that the vehicle is to be free from wheel-rail friction, and therefore, it is necessary to provide both lift and propulsion to the vehicle, and in such context, preferably an optimum design may be needed [28]. Also, it is desirable to have a completely contactless and lightweight vehicle. Such requirements can be met by using a single-sided linear induction motor (SLIM) with long primary and short secondary (without back iron support for the secondary) and its analysis and performance (in different stages) form the subject matter of the first three chapters of the Part-A of this book.

A single-sided LIM has an open-sided, infinitely long stator in which the flux is forced through long air paths and the rotor consists of simply a thin sheet of aluminium of finite length and width placed over the stator. When the stator winding is excited by three-phase currents, the induced currents in the rotor produce propulsion and levitation forces. However, it may be noted that the magnetic circuit of such a machine is not efficient and the construction is highly expensive compared to double-sided LIM.

The design of a single-sided linear induction motor calls for an analysis of the flux distribution on the open-side of a long primary supported by back iron. Such an analysis has been first carried out by West and Hesmondhalgh [29]. Representing the stator current distribution by a sinusoidal current sheet and assuming the stator to be

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infinitely long and wide, Laplace's equation in two dimensions was solved by them to find tangential and normal components of flux density. They also analysed the current distribution in a slab of conductor placed over the stator winding assuming that the slab was infinitely thick and extending to infinity both along length and width. These assumptions are too idealistic and unrealistic for practical applications. Therefore, an analysis based on more realistic assumptions is needed. The analysis in this particular direction is taken up in two stages and those stages constitute the next two chapters of Part-A of this book.

The analysis of a single-sided linear induction motor(SLIM) is based on the analysis of electromagnetic fields in the open air gap of the machine subject to the condition that a thin rotor slab of finite length and finite width is placed in the air-gap zone, in the proximity of the stator iron block. The finiteness in the rotor dimensions gives rise to so-called "Longitudinal End Effects" and "Transverse Edge Effects", respectively. In order to simplify our analysis, a SLIM without any end effect, either in a longitudinal or in a transverse direction is being considered. In other words, a SLIM with an infinitely long and wide stator and rotor is considered for preliminary analysis [30]. Figure 1.1 shows such an idealized model with a coordinate system fixed relative to the rotor and the origin lying just above the stator surface. The rotor sheet is of infinitely small thickness and is made of non-magnetic material. The sheet is placed over the stator surface with its longitudinal edges running parallel to the stator. The length of the gap between the surface of the stator and the rotor sheet along the y-axis direction is "h". As per the construction of the SLIM in Fig. 1.1, conceptually there exist three regions in the y-direction where the field quantities are to be mathematically derived. Those regions include one particular region which is the rotor sheet itself. But practically, as the thickness of the rotor sheet is very small and as also the rotor is magnetically equivalent to air, only two regions (region I and region II) are considered for field calculations. Region I consists of the air gap between the stator surface and the rotor sheet and region II consists of all the space from the rotor surface to infinity in the y-direction. Thus, the problem becomes a two-region problem.

With the configuration of the SLIM shown in Fig. 1.1, calculations of the following are hereby proposed:

(i) *Fields in the two regions and the current in the rotor sheet*
(ii) *Electromagnetic forces exerted on the rotor sheet.*

1.2 Formulation for Fields and Currents

Three-phase distributed windings are laid on laminated stator iron. With reference to Fig. 1.1, the stator windings are approximated by a surface current sheet having sinusoidal current distribution along the length and time. This is similar to a travelling wave or travelling field. It is well known that for the practical purpose of developing a distributed winding, there should exist slots and teeth on the stator iron surface. It is also very interesting to notice that if a plane sheet (made of aluminium or

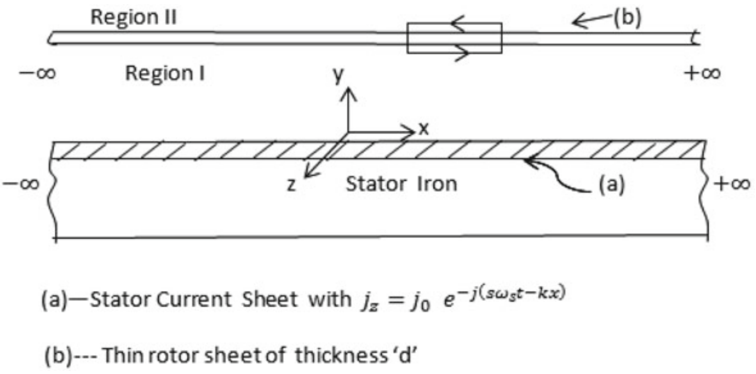

Fig. 1.1 A SLIM with thin rotor sheet. *Note* x, y, z-coordinate system is fixed relative to the rotor for purpose of analysis and the origin is just above the stator surface

iron) is placed above such a slotted structure of the iron sheet, the air-gap length between the stator and rotor will have a non-uniform variation along the length of the stator structure (or, along the distribution of the winding). However, in practice, such variation may not affect the performance of the SLIM, much. Ignoring such variation of the air gap and also the effect of unbalances in three-phase windings (if any), with the *coordinate system fixed to the rotor,* the equivalent stator current sheet can be represented by

$$j_z = j_0\left[e^{j(s\omega_s t - kx)}\right], \tag{1.1}$$

where

j_0	peak value of the linear current density of the stator winding
s	slip of the rotor
k	wave number $= \frac{\pi}{\tau}$, where $\tau =$ pole pitch
ω_s	supply frequency in electrical radians per second
t	time instant in seconds
j	imaginary number $= \sqrt{-1}$.

As the stator winding system is considered to be infinitely wide, only one component of the stator linear current density (j_z) exists. Similarly, because of the assumption of an infinitely wide rotor, only one component of the induced current (which flows in the z-direction) exists. Since the primary and the secondary currents have only z-components, the magnetic vector potential at any field point will also become an one-dimensional vector, $A_z \overrightarrow{1_z}$, where $\overrightarrow{1_z}$ stands for unit vector in the z-axis direction. The field is uniform in the z-direction so that all the field quantities are independent of "z".

As regions I and II are current-free zones, Laplace's equation is satisfied by A_z in both the zones. The relevant equation is

$$\frac{\partial^2}{\partial x^2}(A_z) + \frac{\partial^2}{\partial y^2}(A_z) = 0 \tag{1.2}$$

Within the gap between the stator surface and rotor sheet (region I) and in the region beyond the rotor sheet (region II), the solution of Eq. (1.2) can be expressed as

$$A_{z1} = \left(C_1 e^{ky} + C_2 e^{-ky}\right)\left[e^{j(s\omega_s t - kx)}\right] \tag{1.3}$$

and

$$A_{z2} = \left(C_3 e^{ky} + C_4 e^{-ky}\right)\left[e^{j(s\omega_s t - kx)}\right] \tag{1.4}$$

respectively, where C_1, C_2, C_3 and C_4 are unknown coefficients. The usual method of finding the unknown coefficients using the associated boundary conditions of the differential equation follows in the next section.

1.2.1 Boundary Conditions and Determination of the Coefficients, C_1, C_2, C_3 and C_4

Region II extends up to infinity in the positive y-direction, A_{z2} must vanish at $y = \infty$. Therefore, from Eq. (1.4), we obtain

$$C_3 = 0 \tag{1.5}$$

Hence,

$$A_{z2} = \left(C_4 e^{-ky}\right)\left[e^{j(s\omega_s t - kx)}\right] \tag{1.6}$$

In connection with Eq. (1.2), the boundary conditions to be satisfied are as follows:

(i) On the surface of the stator current sheet (at $y = 0$),

$$\frac{\partial A_{z1}}{\partial y} = -\mu_0 j_z. \tag{1.7}$$

In Eq. (1.7), μ_0 ($= 4\pi \times 10^{-7}$) is the permeability of air. The stator surface is taken as a $y = 0$ plane.

(ii) At the interface of regions I and II:

(a) Normal component of flux density is continuous, i.e.
at $y = h$,

$$\left(-\frac{\partial A_{z_1}}{\partial x}\right)_{y=h} = \left(-\frac{\partial A_{z2}}{\partial x}\right)_{y=h} \tag{1.8}$$

(b) Considering a rectangular contour (in the *x*–*y* plane), enclosing a section of the rotor sheet as shown in Fig. 1.1 and taking the line integral of *H*, we get, for a thin conducting rotor sheet,

$$(H_x)_{\text{bottom}} - (H_x)_{\text{top}} = j_{\text{zr}}. \tag{1.9}$$

In Eq. (1.9), $(H_x)_{\text{bottom}}$ *and* $(H_x)_{\text{top}}$ are the tangential components of magnetic field intensity at the bottom and top surfaces of the rotor sheet (respectively) and j_{zr} is the linear rotor current density (in the *z*-direction).

From Eqs. (1.3), (1.6) and (1.8), it yields

$$C_1 e^{kh} + C_2 e^{-kh} = C_4 e^{-kh} \tag{1.10}$$

The rotor linear current density, j_{zr} can be expressed as

$$j_{\text{zr}} = J_{\text{zr}}(d) = \sigma(E_z). \tag{1.11}$$

In Eq. (1.11), 'j_{zr}' is the rotor current density in A/m^2 and '*d*' is the thickness of the rotor sheet and 'σ' and 'E_z' are the conductivity and electric field intensity in the rotor, respectively. From the relation, $\text{Curl}(\vec{E}) = \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$, we can write, *for an infinitely wide rotor*,

$$E_z = -\frac{d}{dt}\left[(A_{z2})_{y=h}\right] \tag{1.12}$$

In Eq. (1.12), $(A_{z2})_{y=h}$ represents the *z*-axis component of the magnetic vector potential in region II, at a height (*y*-value) of "*h*".

Equations (1.11), (1.12) and (1.6) give the final expression for j_{zr} as

$$j_{\text{zr}} = -j\,\sigma s(\omega_s)(d) C_4 e^{-ky}\left[e^{j(s\omega_s t - kx)}\right] \tag{1.13}$$

From Eqs. (1.3) and (1.6), we obtain $(H_x)_{\text{bottom}}$ and $(H_x)_{\text{top}}$ as

$$\begin{aligned}(H_x)_{\text{bottom}} &= \left(\frac{1}{\mu_0}\right)\left[\left(\frac{\partial A_{z1}}{\partial y}\right)_{y=h}\right]\\ &= \left(\frac{k}{\mu_0}\right)\left(C_1 e^{kh} - C_2 e^{-kh}\right)\left[e^{j(s\omega_s t - kx)}\right]\end{aligned} \tag{1.14}$$

and

$$\begin{aligned}(H_x)_{\text{top}} &= \left(\frac{1}{\mu_0}\right)\left[\left(\frac{\partial A_{z2}}{\partial y}\right)_{y=h}\right]\\ &= -\left(\frac{k}{\mu_0}\right)\left(C_4 e^{-kh}\right)\left[e^{j(s\omega_s t - kx)}\right]\end{aligned} \tag{1.15}$$

Substituting Eqs. (1.14), (1.15) and (1.13) in Eq. (1.9), it yields

$$\left(\frac{k}{\mu_0}\right)\left(C_1 e^{kh} - C_2 e^{-kh}\right) = -\left[\frac{k}{\mu_0} + j\sigma s(\omega_s)(d)\right]\left(C_4 e^{-kh}\right) \tag{1.16}$$

From Eqs. (1.10) and (1.16), we obtain (after C_4 being eliminated)

$$\left[\frac{\left(C_1 e^{kh} - C_2 e^{-kh}\right)}{\left(C_1 e^{kh} + C_2 e^{-kh}\right)}\right] = -\left[1 + j\left(\frac{\sigma s(\omega_s)(d)\mu_0}{k}\right)\right] \tag{1.17}$$

The quantity "$\frac{\sigma s(\omega_s)(d)\mu_0}{k}$" is non-dimensional and may be designated as Magnetic Reynold's number(R). Thus, we can write

$$\begin{aligned}R &= \frac{\sigma s(\omega_s)(d)\mu_0}{k}\\ &= \mu_0 \sigma d(s v_s)\\ &= \mu_0 \sigma d v_{\text{rel}}\end{aligned} \tag{1.18}$$

In Eq. (1.18), 'v_{rel}' is the relative velocity of the travelling field with respect to the rotor. Equations (1.17) and (1.18) give

$$\frac{\left(C_1 e^{kh} - C_2 e^{-kh}\right)}{\left(C_1 e^{kh} + C_2 e^{-kh}\right)} = -[1 + jR] \tag{1.19}$$

The boundary condition in Eq. (1.7), with the help of Eqs. (1.1) and (1.3), gives

$$-(\mu_0)(j_0) = k(C_1 - C_2) \tag{1.20}$$

From Eqs. (1.19), (1.20) and (1.10), it yields

$$C_1 = \frac{\left(\frac{(\mu_0)(j_0)}{k}\right)\left(-jRe^{-2kh}\right)}{(2 + jR) + jRe^{-2kh}} \tag{1.21}$$

$$C_2 = \frac{\left(\frac{(\mu_0)(j_0)}{k}\right)(2 + jR)}{(2 + jR) + jRe^{-2kh}} \tag{1.22}$$

and

$$C_4 = \frac{2\frac{(\mu_0)(j_0)}{k}}{(2 + jR) + jRe^{-2kh}} \tag{1.23}$$

Based on the values of C_1, C_2 and C_4, the magnetic vector potential (A_z) in regions I and II can be computed.

1.2.2 Calculation of Flux Density Components in the Rotor Sheet

The longitudinal component of the flux density, B_{xr}, in the rotor sheet is not uniform along its thickness (in the y-direction) because of the rotor current. As seen from Eq. (1.9), the values of H_x at the bottom and top surfaces of the rotor sheet differ by a finite quantity equal to the rotor linear current density as decided by the current enclosed in the same sheet. Therefore, B_{xr} can be considered as the average of its values at the bottom and top surfaces of the sheet. Based on this consideration, B_{xr} can be expressed as

$$B_{\mathrm{xr}} = \frac{1}{2}(\mu_0)\left[(H_x)_{\mathrm{bottom}} + (H_x)_{\mathrm{top}}\right]$$

The above-said expression can be simplified to

$$B_{\mathrm{xr}} = \left(\frac{k}{2}\right)\left(-2C_2e^{-kh}\right)\left[e^{j(s\omega_s t - kx)}\right] \tag{1.24}$$

The normal component of flux density in the rotor, B_{yr}, being continuous at the interface of regions I and II, can be expressed as

$$\begin{aligned} B_{\mathrm{yr}} &= \left(-\frac{\partial A_{z1}}{\partial x}\right)_{y=h} \\ &= jk\left(C_1e^{kh} + C_2e^{-kh}\right)\left[e^{j(s\omega_s t - kx)}\right] \end{aligned} \tag{1.25}$$

In the equation the unknowns C_1 *and* C_2 are known from Eqs. (1.21) and (1.22).

1.2.3 Calculation of Current Density in the Rotor Sheet

From Eqs. (1.11) and (1.13), we get

$$J_{\mathrm{zr}} = -j\,\sigma s(\omega_s)C_4e^{-kh}\left[e^{j(s\omega_s t - kx)}\right] \tag{1.26}$$

In the equation the unknown C_4 is known from Eq. (1.23). From J_{zr}, the propulsion and levitation forces on the rotor can be calculated using the relation, $\vec{F} = \vec{J} \times \vec{B}$ as explained below.

1.3 Calculation of Forces

In any system, mechanical forces can be produced by so many ways. Such forces also can be produced by using the ever-known principle of magnetic flux–current interaction and they are known as electromagnetic forces. In the present problem, two-dimensional electromagnetic forces, namely “levitation force” and “propulsion force” are produced. The computation methods for the calculation of these forces are explained in the subsequent sections.

1.3.1 Calculation of Levitation Force

The time average of the levitation force on the rotor sheet, per unit width and unit length, F_y is expressed as

$$F_y = \frac{1}{2}\left\{\mathrm{Real}\left(\mathrm{J_{zr}B^{*}_{xr}}\right)\right\}\{(1)(1)(d)\} \tag{1.27}$$

In Eq. (1.27), B^{*}_{xr} indicates the complex conjugate of B_{xr}. The expression for F_y finally simplifies to

$$F_y = \frac{1}{2}\left[\mathrm{Real}\left\{(\sigma\,\mathrm{sd}\omega_{\mathrm{s}}\mathrm{k})\mathrm{jC_1C^{*}_2}\right\}\right] \tag{1.28}$$

Substituting for C_1 and C^{*}_2, it finally yields

$$F_y = \frac{1}{2}\mu_0(j_0)^2\left[\frac{\frac{R^2}{2}e^{-2kh}}{1+\frac{R^2}{4}\left(1+e^{-2kh}\right)^2}\right] \tag{1.29}$$

1.3.2 Calculation of Propulsion Force

The time average of the propulsion force on the rotor per unit width and length, F_x is given by