Logic, Language, and the World

Volume 2

Time and Space in Formal Logic

Richard L. Epstein

Advanced Reasoning Forum

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Preface

What is time? What is space? I will not try to answer these questions, if they even make sense. What we can do is try to understand what role time and space play in our talk and reasoning, hoping to come to some better understanding of what it is we believe and perhaps a better understanding of the world.

In this book I present two very different approaches. Both start with the basics of propositions and inferences set out in Chapter 1 and the relation of formal logic to ordinary language set out in Chapter 2. Each part can then be read independently.

Relative Times

We say "Spot barked before Dick yelled"; we say "Tom met Suzy after Tom broke his foot". We talk of before and after. But before and after what? We can and do pick out times with true descriptions: "Spot barked", "Dick yelled", "Tom met Suzy", "Tom broke his foot". We order them as describing before and after. This is all we need to take account of time in our reasoning: a minimal metaphysics of before and after, codified with temporal propositional connectives.

Times and Locations as Things

A different approach assumes instead that we can talk about times and locations as if they were things, quantifying over them in an extension of classical predicate logic. This is closer to what has been done by others in trying to include talk of time and space in the scope of formal logic and mathematics. But paying attention to the metaphysical assumptions on which to proceed, there are many hard questions to investigate, even before setting up formal systems, and then many more arise in formalizing ordinary language propositions and inferences.

Throughout, I have tried to be clear about what metaphysical assumptions are assumed in establishing the formal systems. Those assumptions and the mechanics of the formal systems are tested by many examples of formalizing ordinary language propositions and inferences. To facilitate comparisons of those there is an Index of Examples. The last chapter, "Metaphysical Bases of Logics of Time and Space" reviews all of the book, setting out what metaphysical assumptions were introduced, why they were needed, and how they were used. That chapter could be read as an extended introduction.

There is so much more we could do, so much begun but not done. And surely there will be more questions, doubts, and interests that will arise. I hope that you can go farther.

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This is the third volume in the series *Logic, Language, and the World*. Volume 0, *An Introduction to Formal Logic*, gives the background for all the work here and can be referred to as needed. The material in Volume 1, *The Internal Structure of Predicates and Names*, is discussed in separate sections and used in examples of formalizing, but those can be skipped without loss of continuity.

Works cited in the text not attributed to anyone are by me.

Supplements to this text which examine extensions of the logics here can be found at <www.AdvancedReasoningForum.org> .

1 Propositions and Inferences

Propositions
We would like to know how to reason well. We would like rules to guide us in finding truths and when that is not straightforward to determine what are the consequences of what we believe. To do so, we must first agree about what it is that is true or false.

Proposition A *proposition* is a written or uttered part of speech used in such a way that it is true or false, but not both.

By "uttered" I include silent uttering to oneself, what we might call thinking of the speech. The propositions we'll focus on in this text are sentences.

Some say that propositions are abstract objects or thoughts shared by all people and that what I have defined are physical linguistic representatives of propositions. But those who hold such views reason using linguistic propositions, which can serve as a common basis on which to begin our work.

Others say that there are not two truth-values but many or that some propositions have no truth-value. But all who reason and all who devise formal models for reasoning divide propositions into those we can proceed on to arrive at what we are justified in believing or acting on and those we cannot. We are right to label any such dichotomy a true-false division, as I explain in "Truth and Reasoning".

A proposition is meant as a description of the world, or at least some part of it. For a proposition to be true it must in some way be an accurate description of the world, relative to whatever you reckon the world is.[1]

Propositions are types
When I say that "dog" is a word, you understand that I do not mean just that particular inscription but any inscription like it or any utterance that we deem the same, for example "*DOG*", or "DoG" or "`dog`". To treat a word or sentence as a *type* in this way is to draw equivalences: the inscriptions and utterances are not the same but only the same for the uses we plan to make of them. We say that they are *equiform*. We identify equiform words for our reasoning. We might or might not say that the English "dog" and the French "chien" are the same word. Similarly, we identify equiform propositions, though for those it is more difficult to be clear about what is or is not important for reasoning. We can say that our

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[1] Or an accurate description of how we do or should use words, as I discuss in *Reasoning about the World as the Flow of All*. The coherence theory of truth invokes consistency rather than description as the basis of truth, using only a syntactic analysis. But unexamined metaphysical assumptions about how language connects to the world are needed to justify whatever notion of consistency is used.

identifications are provisional, allowing that we may later uncover differences that are important for reasoning. Explicitly, we make the following assumption.

Words and propositions are types Throughout any particular discussion, equiform words can be treated as the same for our reasoning. We identify them and treat them as if they were the same word. Equiform propositions, too, will be identified and treated as the same for our reasoning. Briefly, *a word or a proposition is a type*.[2]

Inferences

What does it mean to say that a proposition follows from one or more propositions?

Inference An *inference* is a collection of two or more propositions—one of which is designated the *conclusion* and the others the *premises*—that is intended by the person who sets it out either to show that the conclusion follows from the premises or to investigate whether that is the case.[3]

When does an inference show that the conclusion follows from the premises? That depends in part on what kind of reasoning we are analyzing. Different conditions apply depending on whether we are concerned with evaluating arguments, explanations, reasoning in mathematics, reasoning about cause and effect, reasoning with prescriptive claims, or conditionals.[4] However, for all kinds of reasoning a fundamental criterion for whether the conclusion follows is that the inference is valid or strong.

Valid and strong inferences An inference is *valid* means that there is no way the premises could be true and conclusion false in the same way and at the same time.

An inference is *strong* means that there is a way for the premises to be true and conclusion false, but any such way is unlikely.

For example, the following is valid:

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[2] Some say that types are abstract objects in accord with their belief that propositions are abstract. In that case the assumption that words and propositions are types concerns which inscriptions and utterances (which are what we use in reasoning together) represent or express or point to the same abstract thing.

[3] Note that intent is crucial in determining whether what has been uttered is an inference, as can be seen in hundreds of examples in *Critical Thinking*. The examples of inferences in this book should be understood as prefaced by "imagine that someone has put forward the following inference".

Some say that inferences, too, are abstract things. But all who reason use linguistic inferences, and it is those we can study, whether or not we consider them to be representatives of abstract entities.

[4] See my *Reasoning in Science and Mathematics*, *The Fundamentals of Argument Analysis*, *Prescriptive Reasoning*, and *Cause and Effect, Conditionals, Explanations*.

(1) Ralph is a dog.
All dogs bark.
Therefore, Ralph barks.

I can't prove that to you. At best I can rephrase it in other words. If you understand English, it's clear that it's valid. That's not to say it's a good inference. More is required for an inference to be good, but what that more is depends on what kind of reasoning we are analyzing.

Similar inferences are also valid:

Dick is a student.
All students study hard.
Therefore, Dick studies hard.

Suzy is a cheerleader.
All cheerleaders have a liver.
Therefore, Suzy has a liver.

It would facilitate our reasoning if we could clarify in what way these inferences are similar, for it seems that we don't need to know anything about cheerleaders, or students, or dogs to see that they are valid. Somehow, it is the forms of the propositions in these inferences that matter.

Formal logic *Formal logic* is the analysis of inferences for validity in terms of the structure of the propositions appearing in the inference as well as the analysis of propositions for truth in terms of their structure.

Formal logic does not in itself give us rules for how to reason well. It is a tool we can use in the analysis of inferences and propositions in our reasoning.

Propositions and schemes of propositions

Consider another inference that looks a lot like (1):

(2) Ralph barks.
All dogs bark.
Therefore, Ralph is a dog.

Is it valid? We consider ways the premises could be true and conclusion false. One way is that Ralph could be a fox or seal that barks and that all dogs bark; then the premises would be true and conclusion false. So the inference is not valid.

If (2) is an inference, then the premises and conclusion are propositions, true or false. But in analyzing whether the conclusion follows from the premises, we take each of those sentences to be a scheme of propositions, needing a specification of how the world is in order for it to be true or false.

In formal logic, we establish rules for what sentences we'll consider to be propositions in terms of form relative to some assumptions about the nature of

the world. Then given a collection of propositions, a way the world could be is a description in which each of those propositions is true or is false, what we call *a model*. We use schemes of propositions to establish relations among propositions in terms of not whether they are true or false but how they could be true or false.

Given a sentence such as "Ralph barks", for us to agree that it has a truth-value we need to know what thing "Ralph" is meant to name and what "barks" means. Often it's obvious, and we don't bother to make the agreements explicit. Sometimes it's not. But agreements there are that turn the sentence into a proposition; those are what determine that the sentence is used in such a way that it is true or false.

Thus "Spot is a dog" is a proposition relative to the choice of what "Spot" refers to and what "is a dog" means. If the choice is our usual understanding of "is a dog" and "Spot" refers to my burro, then it's false. If by "Spot" we mean Dick's dog, and "is a dog" means it's a purebred show dog, then it's false. If by "Spot" we mean Dick's dog and have our usual understanding of "is a dog", then it's true. These are three different propositions. They are not one proposition that has different truth-values, but a scheme that is made into three different propositions by choosing how we understand the parts of the scheme, that is, relative to a way the world could be. We reason with propositions. We reason about propositions using schemes of propositions.

Schemes of propositions A (type of) utterance is a *scheme of propositions* means that for each choice relative to a specified range of choices of semantic values for its parts it is true or false, that is, it is a proposition.

Classical logic

In Volume 0 (*An Introduction to Formal Logic*), we saw how to formulate and use classical propositional logic and classical predicate logic. In Volume 1 (*The Internal Structure of Predicates and Names*) we saw how to extend the scope of that work by looking at the internal structure of what we had taken to be atomic in classical predicate logic. Now consider:

> I never hit an old man.
> My brother is an old man.
> Therefore, I never hit my brother.

Even in that richer system, the formalization of this is valid. But it is clearly invalid: I hit my brother more than a few times when he and I were children. To evaluate this inference, we need to take account of time in our reasoning. How shall we proceed?

2 Logic and Language

Developing logic from the point of view of formalizing reasoning in ordinary language is essential for us to see why we should accept its norms and how we can apply it.

In *An Introduction to Formal Logic* (Volume 0) and in *The Internal Structure of Predicates and Names* (Volume 1), I have motivated formal logics based on formalizing reasoning using hundreds of examples from English. This was suitable because the metaphysical assumptions used in establishing classical propositional logic and classical predicate logic are compatible with the usually unstated metaphysical assumptions in speaking English. In particular, classical propositional logic is based on the assumptions of the previous chapter; for classical predicate logic we further assume that the world is made up at least in part of individual things.

In order to take account of time and space in our formalizations of ordinary reasoning using the tools of formal logic, we will have to assume more. But time is treated differently in different languages. In English we treat time in two ways. One is with the use of tenses along with temporal connectives such as "before" to indicate time relationships. The other is by talking of times, using names such as "May 31, 2009" and informal quantifiers like "sometimes" and "always". Modalities, such as duration, are treated mainly through the use of specific words. In contrast, in American Sign Language modalities such as duration are treated as forms of verbs, and in Chinese time indications are established with specific words.

How, then, shall we proceed? Shall we continue to formalize reasoning in English, knowing that to do so will be to build into our formal models particular conceptions of time? Or shall we look for what is common to the treatment of time in all languages, some more universal assumptions?

If we do not base our formal systems on abstracting from reasoning in some ordinary language we will not be able to test our work with examples. We can do no experiments to see if what we have done is apt even for the metaphysics we have assumed. We must begin somewhere. But how we proceed will depend on how we understand our project in terms of the relation of formal logic to reasoning in ordinary language. Consider what Benjamin Lee Whorf says in "Grammatical Categories":

> English adjectives form two main cryptotypes with subclasses. A group referring to "inherent" qualities—including color, material, physical state (solid, liquid, porous, hard, etc.), provenience, breed, nationality, function, use—has the reactance of being placed nearer the noun than the other group, which we may call one of noninherent qualities, though it is rather the residuum outside the first group—including adjectives of size, shape, position, evaluation (ethical, esthetic, or economic). These come before the inherent group, e.g. "large red house" (not

> "red large house"), "steep rocky hill, nice smooth floor." The order may be reversed to make a balanced contrast, but only by changing the normal stress pattern, and the form is at once sensed as being reversed and peculiar. p. 179

Whorf is describing the grammar of English: what is correct and incorrect to say in our language. From that perspective, "Anubis is a wild big dog" is just a bad or odd way to say "Anubis is a big wild dog". But those two sentences can have distinct truth-values and distinct consequences: wild dogs are generally small so that a big wild dog may not be a big dog. Since our formalizations must respect consequences, we formalize the two differently (Chapter 14 of Volume 1). To follow grammar strictly as a guide would leave us unable to say much less to note the logical role of sentences that use an "odd" order of adjectives. Grammar can suggest but cannot be a strict guide to our logical investigations. As Whorf says in "Languages and Logics":

> I can sympathize with those who say, "Put it into plain, simple English," especially when they protest against the empty formalism of loading discourse with pseudolearned words. But to restrict thinking to the patterns merely of English, and especially to those patterns which represent the acme of plainness in English, is to lose a power of thought which, once lost, can never be regained. It is the "plainest" English which contains the greatest number of unconscious assumptions about nature. . . . Western culture has made, through language, a provisional analysis of reality and, without correctives, holds resolutely to that analysis as final. The only correctives lie in all those other tongues which by aeons of independent evolution have arrived at different, but equally logical, provisional analyses. pp. 235–236

I will start with English as the source of the reasoning we'll formalize. But at each point I'll try to make clear what metaphysical assumptions we are building into our formal system so we can go back and adjust or abandon those as needed in developing models of how to reason well for other languages or simply to investigate other assumptions. We shall see as we begin with English that many issues of how to deal with time and later space will lead us to consider ways of conceiving of experience and reasoning that are far from what we usually accept as speakers of English. Being aware of those assumptions, we can begin to have a more general view of formalizing reasoning about time and space, and a more ample understanding of different ways of encountering the world.

3 Times and Propositions

We cannot point to a time as we point to a cow or a table. We cannot point to a time as some say we point through our intellects to a number. We pick out a part of time we are paying attention to with a description: "the time when Spot barked", "the time when George Washington was inaugurated as the first President of the U.S.A.", "the time when dinosaurs lived". Perhaps you or I can pick out times by some sense, some feeling or intellectual apprehension. But reasoning together, we have only linguistic descriptions. And those descriptions are propositions. Loosely speaking, we might say that we pick out a time by what happened at it. But that's too loose. It assumes that the part of time we are discussing existed as an individual unit before we made the description, which is no more reasonable than to say that the water that fills the glass I submerged in the bathtub was a separate and distinct thing before I put the glass in the tub. It is also too loose in assuming that "what happened" is not just a description but some doing or configuration. Yet that doing or configuration would have to be all that happened at that time. So June 6, 1970 is the time picked out by all that "occurred" in the universe at that moment. But that is impossible for us to use in our talk and reasoning. We have no idea of "all that happened". We don't even have a single clock we can use to pick out "what happened" at distant places, physicists tell us. Simply, we pick out or describe times using linguistic propositions. "Spot barked" picks out a part of time which, if we know enough more about the world, like who Spot is, is sufficiently clear that we can treat it as an individual unit of time.

Often we need and can use several temporal propositions together to pick out a time: "Spot barked", "Dick yelled", "Suzy closed the gate". For example, Suzy and Tom were talking last week:

Suzy: Spot barked.
Tom: When?
Suzy: Before Dick yelled.

If what Suzy said is true, Spot barked. When did he bark? He barked when he barked. That's very unsatisfying. Tom wants to know more. We orient ourselves in time by relating one time to another, each established with a proposition. We relate the "when" of one proposition by comparing:

Spot barked before Dick yelled.

If that's true, then "Spot barked" picks out a time that is before the time that "Dick yelled" picks out. Suzy has given Tom a better idea of the "when" of "Spot barked".

If "Spot barked" is true, we can use it along with "Dick yelled" and other true propositions to establish time, relative time. The "when" of this, the time it describes, is before the "when" of that.

What if "Spot barked" is false? Then it does not describe the world, it establishes no time, just as "Spot jumped three meters high" picks out no time. A false proposition is not false of a time; it marks no time relative to any other time.

Yet Zoe said:

Dick yelled at 3 p.m.

Surely if this is true, it's true about 3 p.m.; and if it's false, it's false about 3 p.m.

We have a standard measure for time, one no less arbitrary than saying that the distance on a rod in Paris is one meter. We have a standard clock. We think that this measures time quite independently of us. Perhaps it does. But the times of the standard measure are no less picked out with propositions:

Dick yelled *at the same time as* the standard clock read 3 p.m.

or

Dick yelled *at the same time as* Zoe looked at her watch and saw "3 p.m.".

We do not locate times. We locate ourselves in time by comparing the "when" of true propositions. We use the words and phrases "before", "after", "at the same time as", "during", "while", and others to connect propositions in order to make assertions about relative times.

Suzy: Spot barked.
Tom When?
Suzy: Before Dick yelled.

Zoe: Suzy is going to the grocery store.
Dick: When?
Zoe: After I go to her apartment to take care of Puff.

Tom: Puff scratched Zoe.
Dick: When?
Tom: While Suzy was at the grocery store.

It's not that one proposition becomes true before another. No, if "Spot barked" is true, it was true when first uttered, it is true now, and it is, was, and will be true whenever those words are meant to make the same description of the world.

We can make further comparisons:

(1) Spot barked. Then Dick yelled. And then Spot barked.

If these are true, then there are three times that are established here. But the first and the last are both described as when "Spot barked" is true. They are distinguished from each other by one being before when Dick yelled and one being after when Dick yelled. There might be no other words we can use to pick out those times: we might not have fuller descriptions of "what happened" then. In (1), there are two inscriptions that we classify as the same linguistic type, a sentence-type, yet we cannot identify them for our reasoning. They are (or represent) distinct propositions.

We need to distinguish them, as in:

(2) (Spot barked)$_1$. Then Dick yelled. Then (Spot barked)$_2$.

"(Spot barked)$_1$" is a linguistic type that is also a type of a proposition: that is, we can identify distinct inscriptions and utterances of this linguistic type as being (or representing) the same proposition. Generally, any sentence-type that we previously treated as an atomic proposition we will now want to index. Let's use "1", "2", . . . for that . To make the discussion easier to read when there's only one occurrence of a sentence-type in an example I'll suppress the index, understanding it to be "1". So in (2), "Dick yelled" is shorthand for "(Dick yelled)$_1$".

A true proposition establishes a time but only in relation to other true propositions. A false proposition is not about any time. Perhaps "Dick yelled" picks out a time that Suzy remembers; perhaps it picks out a "real time". But together in our reasoning we have only relative times marked by propositions. So when I use the phrase "the time of" I mean it as shorthand for something like "the time that this proposition establishes relative to the times established by other true propositions we are considering". I'm not assuming—though you may if you wish—a reality of time "out there", parts of which are picked out by propositions. Perhaps all we have are propositions we classify as true that together in their temporal relations create our shared web of time. Whether that ordering is objective or intersubjective doesn't matter here—if it even makes sense to ask which it is.

That "Dick yelled" establishes a time relative to other true propositions is primitive in our work here. We leave open to many interpretations what we mean by the truth of a proposition in time, just as we leave open to many interpretations what we mean by the truth of an atomic proposition in classical propositional logic. If we say that "Spot barked" is true of some time (that we establish with the help of other propositions), does that mean he never stopped barking for even a second to take a breath or look around during that time? Did sound have to keep coming out of his mouth the entire time? These are not questions we need to resolve if we want to make our work open to many conceptions of truth in time.

To summarize, here's what we've assumed in this chapter.

Propositions and times

- A *temporal proposition* is a proposition meant as a description of the world in time. Being a proposition, it is true or false; it is not a scheme of propositions, true of some times and false of others.
- A true temporal proposition establishes a time of which it is a correct description of "the world", however you conceive that, relative to the times established by other true temporal propositions.
- A false temporal proposition establishes no time. There is no "when" of a false proposition.

- Times as established by propositions are ordered with some notion of before and after.
- We cannot take sentence types as (representing) temporal propositions because distinct instances of a single type can be used for distinct propositions. We use instead indexed versions of sentence types as propositions.
- The truth-value of a proposition in time is taken as primitive.

TIME in PREDICATE LOGIC

14 The Timelessness of Classical Predicate Logic

The following is often taken as an archetype of a valid inference:

> All men are mortal.
> Socrates is a man.
> Therefore, Socrates is mortal.

Moreover, it's said, the premises are true. So the conclusion is true. Yet Socrates does not exist. That doesn't matter. We use the simple present tense to indicate that we're not talking about any time in particular nor about all times, for this inference would be valid even if there were no men anymore.

We don't often reason like this. Mostly we talk like this when we're doing science, or mathematics, or metaphysics, trying to find out what's true of things without any consideration of time: electrons have spin; dogs bark; 2 + 2 = 4; God is omniscient; the world is made up of individual things.

Predicate logic is designed to formalize such reasoning. No account is taken of the times at which the things in a universe of a model are meant to exist. They all exist in a timeless status, their coming into existence and going out of existence is of no concern. The only existence we reason about in classical predicate logic is that which is suitable to allow for a thing to be the value given to a variable by an assignment of references.[10] We say that we can assign Socrates to x, though Socrates does not exist now. This is not the issue of whether we have the ability to pick out a thing, for we think that we can pick out Socrates among all other things to be the reference of a variable.

So identity is timeless in predicate logic. Though Norma Jeane Mortensen adopted the name "Marilyn Monroe" only when she was an adult, "Marilyn Monroe ≡ Norma Jeane Mortensen" is true: the names pick out the same thing.

Quantification is timeless in predicate logic. For an existential quantification to be true there must be something in the universe that can be assigned to the variable that makes the resulting proposition true, and that assignment is timeless. For a universal quantification to be true, each thing in the universe, regardless of any considerations of time, must satisfy the predicate.

So consider:

(1) All dogs bark.
Spot is a dog.
Therefore Spot barks.

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[10] This is the way to understand W. V. O. Quine's memorable phrase "To be is to be the value of a variable" in "Designation and Existence", p. 708. Quine didn't mean that mud doesn't exist since it can't be the value of a variable, and he amended that dictum to read on p. 13 of "On What There Is": "To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable". In *Predicate Logic* I suggest that predicate logic as a whole characterizes our notion of thing.

In classical predicate logic we formalize this as:

(2) $\forall x$ ((— is a dog) (x) $\rightarrow$ (— barks) (x))
(— is a dog) (Spot)
Therefore, (— barks) (Spot)

The formal inference is valid. We justify that by saying that if the collection of all things that are dogs is within the collection of all things that bark, then if Spot is a dog, he barks. It is remarkably unclear what we mean by this. The reading of the predicates isn't atemporal nor omnitemporal. It's more like ascribing essential attributes or permanent capabilities or dispositions to things. If Spot is a dog, that is, if he has that attribute without any consideration of time, then Spot barks, without any consideration of time. But that Spot barks is not an essential attribute of Spot. Or perhaps we need to think that it is in order to use classical predicate logic to formalize (1). And perhaps that is indeed what we mean by (1). This is how we must understand the wffs at (2): they are true because being a dog and barking are attributes we ascribe to an object independent of time.

But suppose we wish to use both "— is a puppy" and "— is a dog" in a semi-formal language. These are related in meaning: a puppy is an immature dog. So we should adopt a meaning axiom to codify that:

(3) $\forall x$ ((— is a puppy) (x) $\leftrightarrow$ ((— is a dog) (x) $\wedge \neg$ (— is mature) (x)))

What does this mean? It's said to be atemporal, but nothing is a puppy atemporally. The truth-conditions for (3) are: something is in the collection of things that are puppies if and only if it is in the collection of things that are not mature dogs. Suppose, then, that my dog Birta is in the universe of a model. Is she in the collection of mature dogs or is she in the collection of puppies? To use (3), we must choose, but that means we cannot formalize a true proposition about Birta when she is in the other.

Whatever properties we ascribe to an object in a model must be unchanging. That does not preclude, however, having all of the following true in a model :

(— was a puppy) (Birta)
(— is a dog) (Birta)
(— is mature) (Birta)

Such a model amounts to setting out what is true of certain objects at one particular time. In 2009 both "(— is a dog) (Birta)" and "(— was a puppy) (Birta)" are true. Yet if that's how we interpret our models, then (3) does not ensure that the following is true in the model.

(— was a dog) (Birta) $\wedge \neg$ (— was mature) (Birta)

We would have to add as well:

$\forall x$ ((— was a puppy) (x) $\leftrightarrow$ ((— was a dog) $\wedge \neg$ (— was mature) (x)))

But that is false, for Birta was both a puppy and was a mature dog, just not at the same time. We'd need to find a better way to relate "— was a dog" to "— is a dog", and "— was a woman" to "— is a woman", and . . . . To reason about things in time in classical predicate logic, about how objects have different properties at different times, we would have to adopt *ad hoc* meaning axioms governing every atomic predicate.

Classical predicate logic is useful for formalizing reasoning about things outside of time or about essential attributes or permanent attributes of things that are in time. What will occupy us now is how or whether we can modify classical predicate logic to reason about things in time, for it is not clear that the semantic assumptions of classical predicate logic are compatible with taking account of when things exist.

15 Time and Reference in Predicate Logic

If we are to take account of time in classical predicate logic, then the objects we talk about, the things that can be values of variables, will be things in time. Those objects have to have sufficient stability that we can identify and re-identify them, at least in theory. For example, "Birta" refers to a dog—*that*. Yes "she" changed: she was a puppy, now she's a mature dog; she shed hair; she's grown hair. Yet we say that there is one object we pick out with the name "Birta" that persisted through all those "changes". The name "Birta" picks out an object that is not atemporal but supratemporal: not of one time but across many times. It is a difficult question how and when we are justified in making such identifications, which we'll consider more in Chapter 19. To start, it is enough that in practice we can make judgments that are more or less generally accepted.

So when I say that Birta is the value of the variable x, that is a supratemporal reference; not timeless, but ignoring any particular times at which she existed. That is, reference to an object is independent of any time or times. And it is also independent of any other object. Birta was adopted by me, and perhaps the only way we can pick her out is with a description that talks of other objects and times, such as "the brown dog that Arf adopted at the animal pound on August 23, 2001". But once the reference is established, we need not consider those objects and times. It's enough to say that Birta is the value of x.[11]

But when we as speakers of English divide time into past, present, and future, it seems that when a thing exists is important for how we refer to it.

There are some future-tense propositions we have good reason to believe. I have good reason to believe "No dog lives more than 30 years" and "Chocolate is a dog", so I can conclude "Chocolate will be dead 31 years from now". Every scientific law is meant as true of all times, including the future. For instance, "Electrons have spin" is meant not as a proposition that is true of the present or timelessly, but true for all times, including the future where it serves as a prediction: any electron in the future will have spin.

On the other hand, now as I'm writing we have no reason to believe that "Eduardo Ribeiro will have a cold two years from now" is true nor to believe it is false. When we do not have good reason to believe that a future-tense sentence is true or that it is false, we can treat it as we treat any other proposition whose truth-value we do not know: we consider a model in which it is true and a model in which it is false in order to investigate its consequences and perhaps learn whether it is true or whether it is false. But that doesn't help us with the problem of how to use variables in such reasoning. We need a notion of pointing and naming.

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[11] As I write this now in 2017, sadly Birta no longer is alive. But she continues to exist in the sense needed to reason about her in predicate logic.

Suppose we have a realization and wish to take for its universe all creatures that are living, have lived, or will live. What can it mean to have in the universe some thing that has never lived but will live three years from now? If we wish to use predicate logic, we are under the obligation to explain what we mean when we say that we can pick out and re-identify any object in the universe to serve as a reference for a variable.

If everything in the universe is picked out with a name from our ordinary speech, there is no problem; we can assume that we know how to use ordinary names well enough. But for a universe of all mountain lions in New Mexico it would be too difficult to name each one of them. Yet we understand well enough what it means to say "that mountain lion" when pointing at one, and we take that as our explanation of how we'll use variables.

For a universe of all creatures that have ever lived, it is more difficult to say what we mean by pointing to a donkey that lived in Julius Caesar's time. But unless we have some notion of what we mean by that, the semantics and inductive definition of truth in a model for predicate logic are an empty formalism. We can pick out such an animal by linguistic means by saying "the donkey that is referred to by — in —". But of all the creatures that have lived, the existence of only a few were ever noted in writing. Yet we want to say that there must have been a donkey alive when the first humans lived in Africa. How do we know that such a donkey lived? We infer its existence from evidence we now have.

To pick out things by evidence is to use an informal logic to establish the semantics of our formal logic. It is an informal logic that must remain informal on pain of an infinite regress on what we mean by pointing and naming. Yet we can say that we know well enough what it means to point and name for donkeys, or for any creature that ever lived. The problem is that we can't be there to do the pointing and naming, just as we can't point and name every mountain lion now. Our notion is clear enough; it's just that we're not able to do it. And, we could argue, it is only the method not the execution that is needed for the use of the semantics of predicate logic. We need not be complete empiricists to use predicate logic.

For a universe that includes all creatures that will live any time up to 30 years from now the problem of pointing and naming seems quite different. To say that the baby that will be born to Tom and Suzy three years from now is the reference of x seems to invoke a determinism, an assumption that such a thing will exist, though it doesn't exist now and we have no evidence that it will exist. We can't invoke evidence to use a variable in that way.

The problem would be simpler if we could confine our reasoning to only things in the future that are named. We could treat "Humbert", "Lucilinda", "Meribel" as names for things that are not alive now but will be alive within 30 years from now in the same way we treat fictional names, as we did in Volume 1. A proposition such as "Meribel will be a girl" is assigned a truth-value, and we reason with that without

assuming there is an object that "Meribel" refers to. But we want to analyze inferences about the future that are not confined to talk about things we provide names for now. It seems clear that the following is valid:

> If Tom and Suzy have a baby, it will be born in a hospital.
> If a baby is born in a hospital, the parents will have to pay a doctor.
> Therefore, if Tom and Suzy have a baby, they will have to pay a doctor.

If we wish to use predicate logic to reason about future-tense sentences, it is not enough to say that such sentences are propositions because we can treat them as if they are true or false. We need to explain what we mean by pointing and naming. To do that I see no way except to invoke the general method we use for the kind of objects we are talking about that do exist. We talk about all creatures that will live 30 years from now by invoking the method of pointing and naming we use for creatures that are alive now. We survey different universes to determine what is valid: one in which there is a baby born to Tom and Suzy three years from now, one in which there is a baby born to Tom and Suzy eight years from now, one in which there is no baby born to Tom and Suzy, ... For each of those we claim that the notion of pointing and naming is clear (enough).[12]

If you agree that these comments are adequate justification for our use of predicate logic to analyze reasoning with future-tense sentences as propositions, perhaps our work here will be useful to you. If you disagree, then you can excise all such applications to allow for formalizations of how to reason only about the present and the past. If you disagree further that we can't use the semantics of predicate logic for past-tense sentences as propositions, then the applications that follow will be at best curious, an attempt to convert a logic of timeless propositions into a logic of temporal propositions.

Aside: *Waismann on future tense propositions and determinism*

We have made no judgment here about whether the future is determined.[13] Given a future-tense proposition, such as "Richard L. Epstein will die in 2029", we consider one model in which it is true and one in which it is false. We see the consequences of each choice, but we do not assume or know which is the "correct" model. This is consonant with the analysis that Friedrich Waismann gives in "How I See Philosophy", pp. 8–10:

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[12] Nicholas Rescher and Alasdair Urquhart in *Temporal Logic*, p. 236, take a similar approach to resolving issues of temporal reference:

> Let us, for reasons such as this, eschew a rigorous nominalism, in the context of the present discussion, and be prepared to accept collections that are given, not by an extensionalistic display, but in terms of an abstract criterion of membership, and so be prepared to accept future individuals of a given type (e.g. liars) which, *ex hypothesi* lies beyond the reach of any ostensive procedure such as an enumeration or labeling.

[13] See Richard Taylor, "The Problem of Future Contingencies" for a discussion that touches on many issues in what follows.

This doubt has taken many different forms, one of which I shall single out for discussion—the question, namely, whether the law of excluded middle, when it refers to statements in the future tense, forces us into a sort of logical Predestination. A typical argument is this. If it is true now that I shall do a certain thing tomorrow, say, jump into the Thames, then no matter how fiercely I resist, strike out with hands and feet like a madman, when the day comes I cannot help jumping into the water; whereas, if this prediction is false now, then whatever efforts I may make, however many times I may nerve and brace myself, look down at the water and say to myself, "One, two, three—", it is impossible for me to spring. Yet that the prediction is either true or false is itself a necessary truth, asserted by the law of excluded middle. From this the startling consequence seems to follow that it is already now decided what I shall do tomorrow, that indeed the entire future is somehow fixed, logically preordained. Whatever I do and whichever way I decide, I am merely moving along lines clearly marked in advance which lead me towards my appointed lot. We are all, in fact, marionettes. If we are not prepared to swallow *that*, then—and there is a glimmer of hope in the "then"—there is an alternative open to us. We need only renounce the law of excluded middle for statements of this kind, and with it the validity of ordinary logic, and all will be well. Descriptions of what will happen are, at present, neither true nor false. (This sort of argument was actually propounded by Łukasiewicz in favour of a three-valued logic with "possible" as a third truth-value alongside "true" and "false".)

The way out is clear enough. The asker of the question has fallen into the error of so many philosophers: of giving an answer before stopping to ask the question. For is he clear what he is asking? He seems to suppose that a statement referring to an event in the future is at present undecided, neither true nor false, but that when the event happens the proposition enters into a sort of new state, that of being true. But how are we to figure the change from "undecided" to "true"? Is it sudden or gradual? At what moment does "it will rain tomorrow" begin to be true? When the first drop falls to the ground? And supposing that it will not rain, when will the statement begin to be false? Just at the end of the day, at 12 p.m. sharp? Supposing that the event *has* happened, then the statement *is* true, will it remain so for ever? If so, in what way? Does it remain uninterruptedly true, at every moment of day and night? Even if there were no one about to give it any thought? Or is it true only at the moments when it is being thought of? In that case, how long does it remain true? For the duration of the thought? We wouldn't know how to answer these questions; this is due not to any particular ignorance or stupidity on our part but to the fact that something has gone wrong with the words "true" and "false" applied here.

If I say, "It is true that I was in America", I am saying that I was in America and no more. That in uttering the words "It is true that—" I take responsibility upon myself is a different matter that does not concern the present argument. The point is that in making a statement prefaced by the words "It is true that" I do not *add* anything to the factual information I give you. *Saying* that something is true is not *making* it true: cf. the criminal lying in court, yet every time he is telling a lie protesting, his hand on his heart, that he is telling the truth.

What is characteristic of the use of the words "true" and "false" and what the pleader of logical determinism has failed to notice is this. "It is true" and "It is false", while they certainly have the force of asserting or denying, are not descriptive. Suppose that someone says, "It is true that the sun will rise tomorrow" all it means

is that the sun will rise tomorrow: he is not regaling us with an extra-description of the trueness of what he says. But supposing that he were to say instead, "It is true *now* that the sun will rise tomorrow", this would boil down to something like "The sun will rise tomorrow now"; which is nonsense. To ask, as the puzzle-poser does, "Is it true or false *now* that such-and-such will happen in the future?" is not the sort of question to which an answer can be given: which *is* the answer.

16 Times as Things

One way we can take account of time in predicate logic is suggested by how we talk.

> Some time before Spot barked, Suzy opened the gate.
> Dick never pets Puff.
> Flo spilled a drink three times yesterday.
> Harry always calls before he visits.

We talk about how many times, counting them: some, none, three, all.

If we're going to use the methods of classical predicate logic to quantify over times, we'll have to treat times as things, for it is things we quantify over in classical predicate logic. That seems to be a big assumption.

Time, in our experience, is not a collection of times. Time is a mass. Every part of time is time, just as every part of mud is mud. There are, in our experience, no smallest parts of time, just as there are no smallest parts of mud. The time when Spot barked contains the time when Suzy was startled. The time when Suzy was startled contains the time when Suzy flinched. The time when Suzy flinched contains the time when Suzy blinked. We have no experience of smallest times, so how can times be things?

Water is a mass, too, pervading the world, not coming in bits that are things.[14] Yet we can talk of this cup of water, of that lake of water, of that bathtub of water, treating them, conceiving of them, experiencing them as things. So, too, with time. We can use containing descriptions: the time when Birta barked at Buddy, the time when I slipped getting out of the shower. I can focus my attention on a part of the mass of time, treating that part as a thing, and hope to direct your attention to that part of time by using a description. Time does not come packaged as things as dogs come to us as individual things. The parts of time, the things that are times are what we pay attention to by containing descriptions.

Some disagree. Time, they say, is composed of instants, indivisible, like points on a line. Each instant has no duration, as each point on a line has no breadth. The time that Birta barked is a collection of those instants. Instants, like points on a line, are real, independent of us and our descriptions. Times are things: instants and collections of instants.

This is to make into things the result of our abstracting. A point is a part of space we mark or describe that is sufficiently small for our purposes at hand to treat as having no dimension: the point we make with a pencil when we wish to draw a line to saw a piece of wood; a distant star as a "point mass" in calculations. An instant is a part of time we describe that is sufficiently small for our purposes at hand to treat as having no duration, such as the time that Suzy blinked.

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[14] To assert that H_2O molecules are the smallest bits of water is to mistake an abstraction for our experience, as I explain in "Models and Theories" in *Reasoning in Science and Mathematics*.

If time were the totality of durationless instants, we would have to explain how it is that many of those together can make an interval of time that does have duration. Some say it's the same as with points of space: uncountably many together can give an interval that has length. But that would be to postulate a metaphysics that is ungrounded in experience, mistaking our abstracting—paying attention to some times as if they were indivisible for our purpose—for a reality. It would leave us in a mathematical wonderland with no justification for how to analyze simple temporal inferences.

Let us base our work here on experience. We can treat times as things, each of which can contain other times until we no longer wish to consider smaller parts of time. We pick them out, as we must in order to use them in the semantics of predicate logic, by descriptions.

But what of times we name, like "September 20, 2009", "1985", "5:02 p.m. March 4th, 1831"? Surely those pick out times without any description: the times are there, independent of us and any way we may choose to talk of them. When I say, "I fed the sheep at 7:09 p.m. Saturday, April 7, 2018", the time name is shorthand for a description: when Arf looked at his watch and it said "7:09 p.m. Saturday, April 7, 2018". A scientist who uses the time name "April 8, 2018 at 6:04.0002113 p.m." is using a description: when the standard clock indicated "April 8, 2018 at 6:04.0002113 p.m.". Our clocks are used to describe parts of time.[15]

To treat times as things in the semantics of predicate logic, we will need to assume that we have some way of picking out times, of specifying a reference, a value, for a variable. The specific method of picking out times, whether with verbal descriptions of "what happened" or pointing to a clock, are part of how we specify a model, as the ways we pick out dogs or numbers are part of how we specify a model.

As with our talk of individual things, we can treat a reference to a time as independent of other times and things. That's not to say that a particular description of an object or a time does not and need not involve talk of any other object or other time—that would be to invoke some kind of abstract pure reference, as discussed in Chapter 5.H of Volume 0. It's that we can focus on the individual thing-in-time or time-as-a-thing in our reasoning independently of what things or times are used in a particular description.

Things in Time, the World, and Propositions The world is made up at least in part of individual things that exist in time. Times are also (can be conceived of as) individual things. Reference to each of those things-in-time is independent of any time or any other individual thing. Reference to each of those times-as-things is independent of any individual thing or other time.

The only propositions in which we are interested in are those that are about things-in-time and/or times-as-things.

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[15] Time names have to specify a particular place on earth for which the times are marked. Here let's assume that's the center of the universe, Dogshine, the home of the Advanced Reasoning Forum.

A time can be a value of a variable. But we don't want "$\forall x_1$ (— is a dog) (x_1)" to be false because yesterday isn't a dog. So we'll adopt new variables $t_0, t_1, t_2, \ldots$ to stand for times. These new variables are meant to take values from a universe of times T in a model that is distinct from the universe U of things from which the usual variables $x_0, x_1, x_2, \ldots$ take values.

As noted above, we also use names for times, such as "5:02 p.m. March 4th, 1831", which we'll call *fixed time markers*. We needn't limit ourselves to just those names, however. I might want to use "BB" for the time that my dog Birta was born, since I don't know the date. So let's adopt the symbols $\mathsf{b}_0, \mathsf{b}_1, \mathsf{b}_2, \ldots$ that can be realized as names of times from T, different from the name symbols $\mathsf{c}_0, \mathsf{c}_1, \mathsf{c}_2, \ldots$ that can be realized as names of things from U.

I'll use $t, t', w, w', w_1, w_2, \ldots$ to stand for time variables or time name symbols.

Aside: *Time as things and time as later than*

In English it seems natural to speak of times as things. But there are languages in which there is no or only a quite secondary notion of individual thing and which do not talk of times. Speakers of those "reckon time" using a relation of later than, as we do with "Dick yelled after Spot barked". Vera da Silva Sinha in *Linguistic and Cultural Conceptualisations of Time in Huni Kui, Awety´, and Kamaiurá Communities in Brazil* shows this for some Amazonian tribes, and Benjamin Lee Whorf in "The Relation of Habitual Thought and Behavior to Language" makes this point comparing how Hopi and Western European speakers talk.